THE CATHODE DROP IN AN ELECTRIC ARC

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Abstract

By making the assumption that the total cathode drop occurs in a distance less than one mean free path from the cathode, Poisson's equation can be solved. If the experimental values of 4000 amperes per square centimeter, and 10 volts are used for the current density at the cathode, and the cathode drop in a mercury arc, values for the electric field existing at the surface of the cathode can be determined for varying percentages of the current carried by positive ions. If 5% of the current at the cathode is carried by positive ions, the field existing at the surface of the cathode exceeds 5×10^5 volts per cm. This is probably sufficient to furnish the necessary electron current by "field" currents produced by this high field. The whole cathode drop occurs within a distance of approximately 2×10^{-5} centimeters, so that the original assumption is justified.

An arc is differentiated from a glow discharge primarily by the current density at the cathode and the magnitude of the cathode drop. In a glow discharge the current density at the cathode is normally a small fraction of an ampere per square centimeter, and the cathode drop is usually greater than 100 volts. In an arc, on the other hand, the current density at that portion of the cathode which is carrying current, is of the order of hundreds or thousands of amperes per square centimeter and the cathode drop is of the order of 10 volts. From these facts it can readily be concluded that in the case of a glow discharge most of the current at the cathode is carried by positive ions and only a small percentage by free electrons leaving the cathode. On the other hand in the arc the current is carried primarily by free electrons leaving the cathode and a smaller amount by the positive ions striking the cathode.

In the case of an arc, in which the cathode is not a heated filament, the free electrons have generally been thought of as thermions emitted by the cathode "hot spot."[1] Recently, however, Langmuir[2] has suggested that the positive space charge, causing the cathode drop, may exert a strong enough electric field at the surface of the cathode to cause a large "field" current from the cathode, even though the latter may be too cool to emit thermions. Compton[3] has developed this idea further and from energy considerations believes that in the mercury arc the electrons emanating from the cathode are due primarily to a high electric field and not to thermal emission. The purpose of the present analyses is to show that such a theory is consistent with such data as are available.

[1] Compton, Phys. Rev. **32**, 492 (1928).
[2] Langmuir, G. E. Rev. **26**, 735 (1923); Science **58**, 290 (1923).
[3] K. T. Compton, Jour. A. I. E. E. **46**, 1192 (1927).

The electric field existing at the surface of the cathode, for any given current density of electrons and positive ions, and for a given cathode drop can be determined by using Poisson's equation. If distance and potential difference are measured from the boundary of the cathode drop farthest from the cathode, then Poisson's equation gives

$$\frac{d^2V}{dx^2}=-4\pi\rho=-4\pi\left(\frac{j}{v}-\frac{i}{u}\right) \tag{1}$$

where j and i are the current density carried by the positive ions and electrons respectively, and v and u are the velocities of the positive ions and electrons. We will assume now that the whole cathode drop occurs within one mean free path. This assumption will be found to be consistent with the results obtained from the solution of this equation. Then both j and i are constant and independent of x. Moreover the velocities v, and u, are given within the cathode drop by the following equations:

$$\tfrac{1}{2}Mv^2=Ve \qquad \tfrac{1}{2}mu^2=(V_c-V)e \tag{2), (3}$$

where M is the mass of the positive ion, m, the mass of the electron and V_c is the potential of the cathode. Substituting in Eq. (1) the values of v, and u from equations (2) and (3) we obtain

$$\frac{d^2V}{dx^2}=-4\pi\left[j\left(\frac{M}{2Ve}\right)^{1/2}-i\left(\frac{m}{2(V_c-V)e}\right)^{1/2}\right] \tag{4}$$

This can readily be integrated once and gives

$$E^2=\left(\frac{dV}{dx}\right)^2=16\pi\left\{j\left(\frac{MV}{2e}\right)^{1/2}+i\left(\frac{m(V_c-V)}{2e}\right)^{1/2}-i\left(\frac{mV_c}{2e}\right)^{1/2}\right\} \tag{5}$$

when the boundary condition, that $\partial V/\partial x=E=0$ when $x=0$, is assumed. This expression does not integrate into a simple function and it is simpler and better to proceed with a graphical integration. Equation (5) can be rewritten as

$$E^2=7.57\times10^5\{j(1845WV)^{1/2}-i[(V_c)^{1/2}-(V_c-V)^{1/2}]\} \tag{6}$$

where E is measured in volts per cm, j and i in amperes per cm^2, V and V_c in volts, and W is the ordinary atomic weight of the positive ion.

The value of the electric field at the cathode is given by

$$E_c^2=7.57\times10^5(V_c)^{1/2}\{j(1845W)^{1/2}-i\}. \tag{7}$$

This equation will serve to determine the field existing at the cathode provided the values of the cathode drop (V_c), the density of the positive ion current reaching the cathode (j) and the density of the electron current from the cathode (i), are known.

For the case of the mercury arc no reliable data are available to determine the values of j and i, accurately. The cathode drop V_c, lies close to 10 volts.[4]

[4] Killian, Phys. Rev. **31**, 1122 (1928).

The current density at the cathode $(i+j)$ is given as 4000 amperes per square centimeter.[5] Using these values we can calculate E_c for varying ratios of j to i. This has been done and the results plotted in Fig. 1. This curve shows that for values of $j=0.05i$ the field at the cathode exceeds 5×10^5 volts per centimeter and for $j=0.30i$ this field exceeds 1.3×10^6 volts per centimeter. There are no data available for the electric field necessary to produce an electron current of 4000 amperes per square centimeter from a mercury surface. In this laboratory it has been found that the field at which current begins to appear from a tungsten wire in a vacuum is variable, being very much lower for the case of a surface contaminated by impurities than for a clean surface that has been "conditioned." Moreover the current increases extremely rapidly as the field is increased beyond that necessary to produce the first perceptible current. Since mercury has a lower work function than tungsten, it is probable that an electron current could be produced by a lower field in the case of mercury than in the case of tungsten. Since the surface of

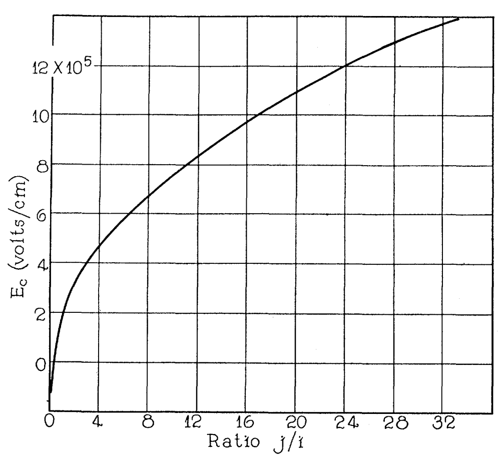

FIG. 1

the mercury is covered with impurities it is reasonable to suppose that a field of the order of 5×10^5 volts per centimeter is sufficient to produce an electron current of 4000 amperes per centimeter square. It is probable that the ratio of j to i does not stay constant at the cathode but may vary, increasing as the surface of the cathode becomes "conditioned," and thus producing a larger field. The fact that the cathode spot moves so rapidly about the surface of the mercury may be explained by the fact that the surface acting as cathode is quickly conditioned by the discharge and that the spot moves to a neighboring position where the mercury surface is contaminated by impurities, and where electrons can be extracted more readily by the electric field. Not only does the cathode spot move rapidly over the surface of the mercury but the mercury surface is depressed. Very probably the electrons come from the irregular edge of this crater which then is immediately depressed, due to the vapor pressure of the mercury which is heated by the bombardment of positive ions at the cathode spot. This would

[5] Güntherschulze, Zeits. f. Physik **11,** 74 (1922).

also account for the rapid movement of the cathode spot, since the electrons always come from the edge of the crater, which is then depressed by the increased vapor pressure at that point.

Equation (5) has been solved graphically on the assumption that $j=0.05i$. This graphical solution gives both the potential V and the electric field E as a function of the distance from the cathode. In Fig. 2 both the electric field and the potential, measured now from the surface of the cathode have been plotted against the distance from the surface of the cathode. It is seen that practically the whole cathode drop occurs within less than 3×10^{-5} centimeters from the surface of the cathode. As this distance is less than the mean free path of a mercury molecule at the temperature generally existing in the mercury arc, the original assumption is justified.

If it is assumed that a field as great as 1.3×10^{6} volts per centimeter is necessary to produce an electron "field" current of 4000 amperes per square centimeter, then $j=0.30i$ and the cathode drop will occur within a distance of 10^{-5} centimeters from the surface of the cathode.

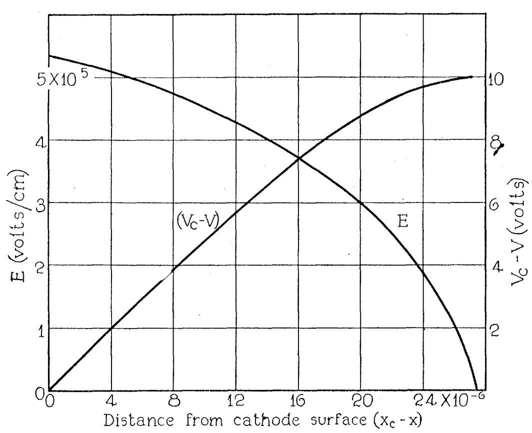

FIG. 2

It should be noted that the assumption has been made that the density of electricity ρ is continuous. This is not true when we consider distances as small as 10^{-5} centimeters. Poisson's equation however may be used if ρ is considered as a time average of the density of electricity. Actually the field existing at the surface of the cathode will be greater than that calculated due to the fact that ρ is not continuous.

Probably in most electric arcs, even in air, electrons are pulled from the surface of the cathode by the high electric field existing there. In certain cases, as for instance in the carbon or tungsten arc, the temperature of the cathode spot is high enough so that probably thermions contributed also to the "field" currents produced by the high electric field existing at the cathode.

In conclusion it may be stated that if five percent of the current reaching the cathode of a mercury arc, is carried by positive ions, there must exist at the surface of the cathode an electric field of approximately 5×10^{5} volts per centimeter. It is believed that this field is adequate to produce the electron current necessary for the maintenance of the arc.