THE SATELLITES OF THE X-RAY LINES $L\alpha$, $L\beta_1$ and $L\beta_2$

BY F. K. RICHTMYER* AND R. D. RICHTMYER
DEPARTMENT OF PHYSICS, CORNELL UNIVERSITY

(Received July 10, 1929)

ABSTRACT

The satellite structure accompanying the x-ray diagram lines $L\alpha$, $L\beta_1$ and $L\beta_2$ has been investigated for the elements Rb(37) to Sn(50). This structure is more complex than has heretofore been assumed. Not only is the number of satellites greater than previously reported, but a continuous spectrum seems to extend for some distance toward shorter wave-lengths beyond the satellites. Tables are given showing wave-lengths, values of ν/R and of $\Delta\nu/R$ ($\Delta\nu$ = difference in frequency between satellite and parent line) for five (seven for some elements) satellites of $L\alpha$; four of $L\beta_1$; and five of $L\beta_2$. The empirical relation previously reported for the $K\alpha$ satellites is found to hold for these L satellites: namely, the square root of the difference in frequency between a satellite and its parent line is a linear function of atomic number —similar to Moseley's law for diagram lines. The difficulties of obtaining values of the wave-lengths of satellites as accurate as are the measured wave-lengths of the corresponding diagram lines, are discussed. It is shown that some of these difficulties may be eliminated by obtaining a densitometer curve for the energy distribution through the parent line at a voltage just below the excitation voltage of the satellites. From measurements on a plate taken at high dispersion it appears that satellites may be resolved into fine-structure components.

X-RAY satellites are those comparatively faint lines in x-ray spectra, which do not fit the usual energy-level diagram, for which reason these lines are frequently called "non-diagram lines." They were first discovered by Siegbahn and Stenstrom[1] in their studies of the wave-lengths of the K-series lines of the elements Na (11) to Zn (30). On the short wave-length side of the $K\alpha$ doublet of Zn (30) a faint line was observed, which became more prominent for elements of lower atomic number, split up into a doublet at Ca (20) and was followed as far as Na (11).

Although the literature on x-ray satellites is not extensive, investigations since the above observation of Siegbahn and Larsen have shown that many x-ray lines have such faint companions, usually on the high-frequency side. Thus, satellites are known to accompany $K\alpha$, $K\beta$, $L\alpha$, $L\beta_1$, $L\beta_2$, $L\gamma_1$, $L\gamma_{2,3}$ and $M\alpha$. However, there seems to have been no systematic search to prove that satellites of other lines do *not* exist; there is some evidence to indicate that there are more satellites accompanying the above-mentioned lines than have been heretofore reported; little is known regarding the excitation potentials of satellites; the range of atomic numbers of the elements emitting a given satellite has not been absolutely fixed; and the current theories of

* The senior author wishes to express his thanks to the Heckscher Research Council of Cornell University for a grant in aid of this research.

[1] Siegbahn and Stenstrom, Phys. Zeits. **17**, 48 and 318 (1916).

the origin of satellites agree only partially with experimental facts. There is need for more complete data on this subject. The present paper[2] is a preliminary report, largely empirical, of measurements of wave-lengths of the satellites of certain of the L lines: $L\alpha$, $L\beta_1$ and $L\beta_2$.

The spectrum plates were made with a Siegbahn vacuum spectrometer. A calcite crystal was used throughout, except for wave-lengths longer than about 5.8A, for which a quartz crystal was used. The voltage applied to the x-ray tube varied somewhat from element to element, but was in the neighborhood of 13 to 15 kv, unidirectional and with a ripple of several percent. Exposures necessary to bring out the satellite structure varied from thirty minutes to three hours, depending on the nature of the cathode material, the lines under investigation, and the permissible tube current.

The wave-lengths of the satellites of a given diagram (or "parent") line were determined by comparison with the known wave-length of the parent line (Siegbahn's values). This measurement presents some difficulties, if precise values of the wave-lengths of the satellites are desired. Either of two prodecures may be followed:

(1) The distance between parent line and satellite on the plate may be determined directly by a micrometer microscope. This method is open to two sources of error. First, the exposures necessary to bring out the satellite structure are so long that the parent line is much over exposed. Not only is it difficult to determine its exact center, but, the parent line thus being very broad, the satellites lie "in its shadow." Because of the rapidly varying density of the photographic plate in this neighborhood, the eye is subject to the well-known errors of psychological contrast in estimating the position of maximum density corresponding to the satellite line on which to set the cross hair of the microscope. Second, even though this position of maximum density on the plate be correctly estimated, such position is *not* the position of maximum *intensity* of the satellite line, since, as before mentioned, the satellite is superimposed on the rapidly varying density at the side of the parent line.

(2) The first of these two sources of error can be eliminated by making a densitometer record through the parent line and its satellite structure, and from this record the correct positions of maximum density determined. Such a record is shown by the full curve A of Fig. 1, which is a tracing of the original densitometer record for $L\alpha$ of Ag (47). (The spectrum plate was exposed thirty minutes at 12.0 kv and 16 m. a.). The four satellites, $L\alpha'$ to $L\alpha^{iv}$ inclusive, are clearly observable at the left of $L\alpha_1$ and the distance between the maximum of $L\alpha_1$ and that of each of the satellites may be readily determined, although with no very high precision. However, such determination is open to the second of the two sources of error mentioned above. Since curve A is a composite of parent line plus satellites, the *correct* positions of maximum *intensity* of the several satellites can be obtained *only* by subtracting from the ordinates of curve A the ordinates of a densito-

[2] A part of this work was done at Uppsala, Sweden, in the laboratory of Professor Siegbahn, to whom the authors wish to express their best thanks for the many courtesies extended.

meter curve of the parent line $L\alpha$ without its satellites. This latter curve cannot, of course, be obtained directly. A procedure frequently followed is to make an estimate of its shape, and therefrom to determine the desired wave-lengths of the satellites. This procedure at best is open to obvious objections.

An alternative, though far from satisfactory, procedure is the following: Existing data seem to show that the excitation potentials of satellites are some twenty to thirty percent higher than those of the corresponding parent lines. Thus the excitation potential of the $L\alpha$ lines of Ag (47) is 3.34 kv. The satellites of this line should not be produced at 3.8 kv. Accordingly, a plate was made of $L\alpha$ of Ag (47) at 3.8 kv and a densitometer record of this plate is shown as curve B of Fig. 1. No satellite structure is observable, although it is to be noted that the curve is unsymmetrical, there being a "foot" on the high frequency side of the line where the satellites appear at higher voltages. This curve enables one to make a somewhat better *guess* as to the correct shape of the curve for $L\alpha$ alone if it could be obtained at 12.0 kv. Such a curve is shown as the dotted curve C of Fig. 1. Finally one obtains curve D by subtracting the ordinates of curve C from those of A. Curve D gives the satellite structure accompanying the diagram line $L\alpha$.

It is observed that this satellite structure seems to consist of two parts: (1) a kind of continuous spectrum c_1 c_2 c_3 upon which are superposed[3] the satellites α' to α^v. Even curve D would have to be analyzed into its component parts in order to give unambiguously the position of the respective maxima. The procedure here outlined seems to be necessary if one wishes to obtain values of the wave-lengths of satellite lines comparable in precision with corresponding values for the parent lines.

However, the number and approximate location of satellites is sufficient for many purposes. Accordingly, the data herein recorded were obtained from the spectrum plates by direct measurement with a micrometer microscope.[4] The order of magnitude of the differences between such direct measurements with the micrometer microscope and those obtained from a densitometer record such as curve A Fig. 1, are indicated in Table I, the second

TABLE I. *Comparison of micrometer microscope with densitometer method of measuring position of satellites with reference to the parent line.*

Satellite	Distance from $L\alpha_1$		
	By micrometer microscope	By densitometer record	Difference, percent
$L\alpha_1'$	2.01 mm	2.01 mm	0.
$L\alpha_1''$	2.77	2.64	4.8
$L\alpha_1'''$	3.34	3.16	5.5
$L\alpha_1^{iv}$	4.64	4.59	1.1

[3] The curves in Fig. 1 should be regarded as diagrammatic, rather than as representing accurately to scale the relative intensities of the component spectral parts.

[4] A very low power microscope—magnification not more than 1.5× or 2.0×—is necessary to render the fainter satellites visible.

column of which gives (in mm) the distances between $L\alpha$ and its first four satellites (for Zr (40)). The third column gives similar data determined from a densitometer record. From the last column it is seen that these differences amount to a few percent. They vary from satellite to satellite, and with the density of the spectrum plate. A careful study of this point would yield valuable information.

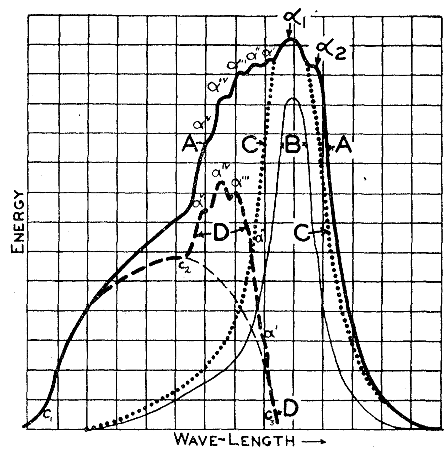

Fig. 1. A: Ag $L\alpha$ with satellites, at 12,000 volts; B: Ag $L\alpha$ at 3,800 volts; C: Estimated shape of Ag $L\alpha$ without satellites at 12,000 volts; D: Estimated structure of Ag $L\alpha$, at 12,000 volts.

RESULTS

Satellites of $L\alpha$. In Siegbahn's tables of x-ray spectral lines[5] only one satellite of $L\alpha$ is listed. This line is called by Siegbahn $L\alpha^3$, and is recorded over the range of elements, As (33) to In (49). However, Siegbahn and Larson[6] later found five satellites of $L\alpha$ of Mo (42), with a possible sixth somewhat doubtful. Druyvesteyn[7] reports five satellites over the range of elements Nb (41) to Ag (47). The present authors have found five satellites for each of the elements from Rb (37) to In (49). On the plates for some of the elements two others, and in the case of one element (Mo (42)) three, are clearly ob-

TABLE II. *Wave-lengths of the satellites of $L\alpha$, in X-units.*

	Rb 37	Sr 38	Zr 40	Nb 41	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49
$L\alpha_1$	7302.7	6847.8	6055.9	5711.3	5394.3	4835.7	4587.8	4356.5	4145.6	3947.8	3763.7
α'	7280.8	6827.3	6038.5	5695.4	5380.0	4823.2	4576.2	4345.3	4134.9	3938.0	3754.5
α''	7274.1	6821.4	6032.5	5689.6	5374.0	4818.1	4571.3	4341.2	4130.6	3934.0	3750.9
α'''	7263.0	6814.3	6026.5	5684.6	5368.9	4812.9	4566.5	4335.9	4125.6	3929.0	3746.2
α^{iv}	7752.0	6800.7	6015.9	5672.1	5359.8	4804.5	4559.3	4329.1	4119.3	3922.2	3740.0
α^{v}	7241.4	6793.1	6008.4	5667.0	5351.4	4797.2	4551.9	4322.2	4112.5	3915.8	3733.2
α^{vi}				5661.0	5343.6	4788.7		4314.2			3730.1
α^{vii}					5336.6						
α^{viii}					5324.0						

[5] The Spectroscopy of X-Rays, p. 118.

[6] Siegbahn and Larson, Arkiv för Matematik, Astronomi och Fysik **18,** 1 (1924).

[7] Druyvesteyn, Het Röntgenspectrum van de tweede Soort. Dissertation, Groningen, 1928.

served. It seems probable that carefully chosen exposure-times would show at least seven satellites for all these elements.

Tables II, III and IV give, respectively, the wave-lengths, values of ν/R and of $\Delta\nu/R$ for these satellites. (ν = frequency; R = Rydberg's constant; $\Delta\nu$ = difference in frequence between satellite and parent line). Only those lines are recorded for which satisfactory micrometer-microscope settings could be made.

TABLE III. *Values of ν/R for the satellites of $L\alpha$.*

	Rb 37	Sr 38	Zr 40	Nb 41	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49
$L\alpha_1$	124.79	133.07	150.48	159.56	168.93	188.45	198.63	209.08	219.81	230.83	242.12
α'	125.17	133.47	150.91	160.01	169.39	188.94	199.14	209.62	220.38	231.41	242.72
α''	125.28	133.59	151.06	160.17	169.57	189.14	199.35	209.82	220.61	231.64	242.95
α'''	125.47	133.72	151.21	160.31	169.73	189.35	199.56	210.09	220.89	231.93	243.25
α^{iv}	125.66	133.99	151.48	160.66	170.02	189.67	199.88	210.41	221.22	232.33	243.66
α^{v}	125.84	134.14	151.66	160.80	170.29	189.98	200.20	210.75	221.59	232.70	244.00
α^{vi}				160.97	170.53	190.29		211.14			244.30
α^{vii}					170.75						
α^{viii}					171.15						

A word about terminology: Following Druyvesteyn and others, there seems to be a tendency to indicate satellites by primes (′), double primes (″), etc. The present authors have adopted this system whenever possible. (Certain exceptions will be noted below). Accordingly the satellites of $L\alpha$ are called $L\alpha'$, $L\alpha''$, $L\alpha'''$, $L\alpha^{iv}$ . . . . This terminology can be at best only tentative as is indicated by the data in Table V below.

TABLE IV. *($\Delta\nu/R$) for the satellites of $L\alpha$. $\Delta\nu/R=(\nu/R)_s-(\nu/R)_{L\alpha_1}$*

	Rb 37	Sr 38	Zr 40	Nb 41	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49
$L\alpha'$	0.375	0.400	0.432	0.447	0.464	0.490	0.507	0.542	0.573	0.577	0.598
α''	0.490	0.517	0.583	0.609	0.640	0.691	0.721	0.745	0.805	0.810	0.831
α'''	0.680	0.654	0.733	0.749	0.803	0.895	0.932	1.010	1.076	1.104	1.134
α^{iv}	0.868	0.920	0.996	1.100	1.090	1.220	1.248	1.334	1.415	1.500	1.540
α^{v}	1.048	1.066	1.182	1.243	1.355	1.508	1.572	1.668	1.780	1.875	1.975
α^{vi}				1.412	1.600	1.84		2.06			
α^{vii}					1.82						
α^{viii}					2.22						

$L\alpha'$ is faint. $L\alpha''$ and $L\alpha'''$ are quite strong and have the appearance of a doublet. $L\alpha^{iv}$ is about as strong as $L\alpha'$. Beginning with $L\alpha^{v}$ the lines become increasingly fainter.

The satellites of In (49) are not well separated and at Sn (50) the satellites seem to be replaced by a band showing little or no structure. Whether higher resolving power and properly chosen exposures will show that this band can be resolved for elements above In (49) remains to be seen.

That greater detail of structure may be revealed by using higher resolving power is clearly indicated by a plate of $L\alpha$ of Ag (47) taken with a quartz crystal in second order, for which the dispersion, at $\lambda = 4.15$A, is nearly four and one half times as great as for a calcite crystal in first order. On this plate the more intense satellites, $L\alpha''$ and $L\alpha'''$ *are each clearly split up into two components*. $L\alpha'$ seems to appear complex, with possibly three components. And several faint lines appear between $L\alpha'$ and $L\alpha_1$. One of these, designated in Table V as $L\alpha_1$(a) is not separated from $L\alpha_1$ but appears as a "shoulder" on the latter. Table V shows the wave-lengths of these fine structure components as observed with the quartz crystal.

TABLE V. *Fine structure of the satellites of $L\alpha$ of Ag (47), as observed with a quartz crystal in second order. (Wave-lengths, in X-units).*

Line	Quartz crystal Second order	Calcite crystal First order*
$L\alpha_1$	(4145.64)	(4145.64)
$L\alpha_1$ (a)	4142.25	
$L\alpha_1$ (b₁)	4140.03	
$L\alpha_1$ (b₂)	4139.29	
$L\alpha_1$ (b₃)	4138.56	
$L\alpha_1$ (c) (?)	4136.58	
$L\alpha'$ (a)	4135.41	
$L\alpha'$ (b)	4134.63	4134.9
$L\alpha'$ (c)	4133.90	
$L\alpha''$ (a)	4131.64	4130.6
$L\alpha''$ (b)	4130.11	
$L\alpha'''$ (a)	4125.84	4125.6
$L\alpha'''$ (b)	4124.63	
$L\alpha^{iv}$	4119.21	4119.3

* From Table II, for comparison.

The difference in frequency between a satellite and its parent line increases in a regular way with atomic number, as is shown in Fig. 2 (a) in which $\Delta\nu/R$ for the several satellites of $L\alpha$ are plotted as ordinates against atomic number as abscissa. The resulting straight lines justify the conclusion[8] that for the satellites of $L\alpha$, the *square root of the frequency difference between a given satellite and the parent line is a linear function af atomic number.*

Satellites of $L\beta_1$. Two satellites of $L\beta_1$ have been previously reported. They were called $L\beta_1$ and $L\beta_{13}$ by Coster,[9] who measured them for the elements Rb (37) to Sb (51). Thoreaus[10] reports two satellites for the elements Cu (29) to Br (35) inclusive, which he calls $L\beta'$ and $L\beta''$ and which are identical with Coster's $L\beta_1'$ and $L\beta_{13}$. Druyvesteyn[7] calls these lines $L\beta_1'$ and $L\beta_1''$ respectively. On the plates of $L\beta_1$ taken by the present authors for the elements Nb (41) to Sn (50) these two lines were clearly visible, and

TABLE VI. *Wave-lengths of the satellites of $L\beta_1$, in X-units.*

	Nb 41	Mo 42	Ru 44	Rh45	Pd 46	Ag 47	Cd 48	In 49	Sn 50
$L\beta_1$	5479.6	5165.8	4611.0	4364.0	4137.3	3926.6	3730.1	3547.8	3377.9
β_1'	5465.7	5153.5	4600.4	4353.0	4126.8	3915.0	3720.4	3539.5	3369.7
β_1''	5451.5	5139.9	4588.5	4343.1	4116.9	3907.1	3712.5	3531.6	3361.5
β_1'''		5132.5	4581.6	4335.5	4109.5	3899.6	3703.9	3523.5	3352.6
β_1^{iv}		5125.2	4572.2	4327.4	4103.0	3893.5	3696.4		

[8] F. K. Richtmyer, Phil. Mag. **6,** 64 (1928).
[9] Coster, Phil. Mag. **43,** 1088 and 1105 (1922).
[10] Thoreaus, Phil. Mag. **2,** 107 (1926).

in addition there were two others at still shorter wave-lengths, which we may designate as $L\beta_1'''$ and $L\beta_1^{iv}$. The wave-lengths, values of ν/R and of $\Delta\nu/R$ for these four lines are shown in Tables VI, VII and VIII. The plot

TABLE VII. *ν/R for the satellites of $L\beta_1$.*

	Nb 41	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50
$L\beta_1$	166.30	176.40	197.63	208.82	220.26	232.07	244.30	256.85	269.77
β_1'	166.72	176.82	198.08	209.35	220.82	232.76	244.94	257.45	270.43
β_1''	167.15	177.28	198.59	209.82	221.35	233.23	245.45	258.02	271.09
β_1'''		177.53	198.89	210.18	221.74	233.67	246.02	258.62	271.59
β_1^{iv}		177.78	199.29	210.57	222.09	234.03	246.51		

TABLE VIII. *$\Delta\nu/R$ for the satellites of $L\beta_1$.*

	Nb 41	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50
$L\beta_1'$	0.421	0.422	0.455	0.526	0.560	0.688	0.636	0.603	0.655
β_1''	0.853	0.883	0.964	0.999	1.086	1.156	1.152	1.174	1.315
β_1'''		1.130	1.261	1.363	1.481	1.597	1.715	1.765	1.822
β_1^{iv}		1.386	1.663	1.752	1.825	1.959	2.209		

of $\Delta\nu/R$ as a function of atomic number is shown in Fig. 2 (b). These satellites are relatively weaker, compared to $L\beta_1$, than are the satellites of $L\alpha$ and are hence more difficult to measure. This may account for the greater scattering of the points in Fig. 2 (b) than in Fig. 2 (a).

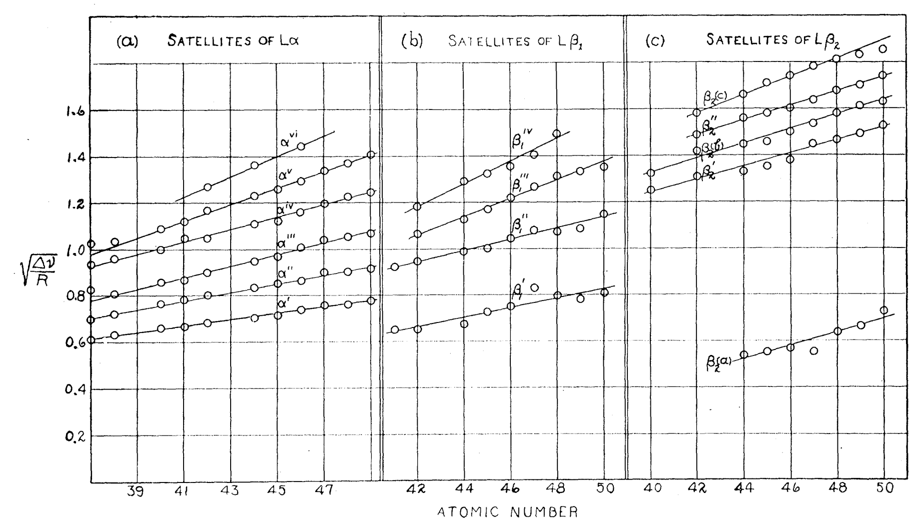

Fig. 2. Linear relation between $(\Delta\nu/R)^{1/2}$ and atomic number.

$L\beta_1''$ is the strongest of this group. Next in order of intensity come $L\beta_1'$ and $L\beta_1'''$. $L\beta_1^{iv}$ is very weak and is difficult to measure, although its existence seems to be definitely established.

A comparison of Figs. 2 (a) and 2 (b) shows that, although the two satellite structures are quite dissimilar in general appearance, yet the frequency differences between satellites and the respective parent lines are all included within the same range of values of $\Delta\nu/R$.

Satellites of $L\beta_2$. The satellite structure accompanying $L\beta_2$ is rather remarkable. Two prominent satellites have long been known, namely: $L\beta_2'$ and $L\beta_2''$ (Siegbahn $L\beta_{11}$ and $L\beta_{12}$). These lines are nearly equal in intensity, are sharper than other L-satellites and are separated farther from the parent line. Our plates show, in addition to these, three other lines. Close to $L\beta_2$ is a very faint, yet fairly well defined line which we have called $L\beta_2$ (a). Between $L\beta_2'$ and $L\beta_2''$ appears a broad, diffuse line. This is much weaker than either $L\beta_2'$ or $L\beta_2''$ and extends nearly from one to the other. We have designated this $L\beta_2$ (b). Beyond $L\beta_2''$ is still another very faint line, $L\beta_2$(c) in our notation. The wave-lengths, values of ν/R and of $\Delta\nu/R$ for these five satellites are given in Tables IX, X and XI. Fig. 2 (c) shows the plot of $\Delta\nu/R$ against atomic number.

TABLE IX. *Wave-lengths of the satellites of $L\beta_2$ in X-units.*

	Zr 40	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50
$L\beta_2$	5573.4	4909.2	4361.9	4122.1	3900.7	3693.8	3506.4	3331.2	3167.9
$L\beta_2$(a)			4355.9	4116.5	3895.3	3689.3	3501.0	3325.9	3162.1
$L\beta_2'$	5520.2	4864.9	4325.0	4088.0	3869.0	3663.7	3477.3	3304.1	3142.3
$L\beta_2$(b)	5514.2	4856.1	4318.1	4082.4	3863.2	3658.7	3472.6	3299.7	3138.8
$L\beta_2''$	5504.8	4850.8	4311.4	4075.6	3857.8	3653.9	3468.2	3296.0	3134.4
$L\beta_2$(c)		4843.0	4304.3	4067.9	3850.2	3646.5	3462.2	3290.5	3130.1

TABLE X. *Values of ν/R for the satellites of $L\beta_2$.*

	Zr 40	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50
$L\beta_2$	163.50	185.62	208.92	221.07	233.62	246.70	259.89	273.56	287.66
$L\beta_2$(a)			209.21	221.37	233.94	247.01	260.29	274.00	288.19
$L\beta_2'$	165.06	187.33	210.69	222.90	235.54	248.71	262.04	275.79	289.98
$L\beta_2$(b)	165.24	187.62	211.02	223.20	235.87	249.05	262.39	276.15	290.31
$L\beta_2''$	165.63	187.83	211.34	223.56	236.19	249.37	262.72	276.45	290.70
$L\beta_2$(c)		188.13	211.68	223.98	236.65	249.86	263.16	276.90	291.09

TABLE XI. *$\Delta\nu/R$ for the satellites of $L\beta_2$.*

	Zr 40	Mo 42	Ru 44	Rh 45	Pd 46	Ag 47	Cd 48	In 49	Sn 50
$L\beta_2$(a)			0.289	0.302	0.325	0.307	0.402	0.437	0.530
$L\beta_2'$	1.562	1.714	1.768	1.830	1.900	2.009	2.154	2.230	2.321
$L\beta_2$(*b*)	1.739	2.00	2.10	2.13	2.25	2.35	2.50	2.59	2.65
$L\beta_2''$	2.13	2.21	2.42	2.49	2.571	2.668	2.829	2.891	3.044
$L\beta_2$(c)		2.51	2.76	2.91	3.03	3.16	3.27	3.34	3.43

It is not the purpose of the present paper to discuss the bearing of these data on theories concerning the origin of satellite lines.[11] Indeed it seems probable that these theories cannot be advanced much farther until much more information is available. Particularly important are such questions as the excitation potentials of satellites, their relative intensities, and the fine structure which may be revealed by using the highest attainable resolution. Investigations along these lines are in progress in this laboratory.

[11] For a general discussion of this question see F. K. Richtmyer, Franklin Institute, Journal, (paper in press).