A THEORY OF THE PERMANENT MAGNETIC FIELDS OF THE SUN AND EARTH*

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ABSTRACT

The motion of ions, executing short free paths under the influence of thermal agitation in an inhomogeneous magnetic field, in crossed magnetic and gravitational fields or in crossed magnetic and electric fields are shown to produce drift currents. The ion drifts are found to be opposite in direction to the drifts produced in the analogous cases of long free path. Under the condition of radial symmetry and a closed circuit the magnetic gradient gives rise to circular currents which flow in such a direction that magnetic regeneration takes place. Regeneration is limited by demagnetizing currents arising from the thermal motions of the ions interacting with the resultant magnetic field and an internal electric or gravitational field. The magnetic moments of the sun and earth are calculated from data which are approximately known and the correct magnitudes obtained. The permanent fields arise from the thermal energy of the body and would be maintained if the bodies ceased their rotation. The asymmetry of the earth's magnetic field indicates that the hemisphere embraced by the Pacific Ocean is at a higher mean internal temperature than the rest of the earth.

ONE of the unsolved problems of cosmical physics has been the problem relating to the origin and maintenance of the permanent magnetic fields of the sun and the earth. Many theories[1,2] of the field have been proposed, most of which depend in some manner on the rotation of the body. It is generally accepted that those theories which have been definitely formulated lead to values for the magnetic moments which are far too small to account satisfactorily for the observed facts. It is the purpose of the present paper to outline briefly certain electromagnetic effects which are of such a magnitude as to be able to account for the observed magnetic moments of the earth and sun. The present theory attributes the magnetic field to electrical currents set up in the high temperature regions of the sun and earth as a result of the thermal motion of the ions interacting with a self produced magnetic field and an inhomogeneity of the magnetic field *or* an electric field produced by any one of several phenomena. Analysis of the motions of ions and electrons spiralling about an inhomogeneous impressed magnetic field shows that two important effects are produced,[3,4,5,6] (1), a diamagnetic

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[1] Dictionary of Physics, Glazebrook, Vol. **II**.
[2] Handbuch der Physik, Vol. **15**.
[3] Schroedinger, Wien, Ber. **66**, 1305 (1912) also Bulletin 16 of Nat'l. Res. Council.
[4] R. Gunn, Phys. Rev. **32**, 133 (1928)
[5] R. Gunn, Phys. Rev. **33**, 614 (1929).
[6] R. Gunn, Phys. Rev. **33**, 832 (1929).

effect dependent on the magnitude of field; (2), a drift velocity dependent on the gradient of the field which gives rise to a steady current under suitable conditions. Expressions have been derived[6] for the current density resulting from the drift velocity imposed on the ions due to thermal agitation and an inhomogeneous magnetic field. The resulting currents that flow in a highly ionized region when the free paths are long have been shown to account for the magnetic fields of sun-spots[7] and to account for a portion of the general solar magnetic field.[6] The extension of the calculations to the case where the free paths of the ions are short leads immediately to a consideration of the currents and resulting magnetic fields which are produced in the interior of the sun and earth.

In the earlier work an expression was given, without formal proof, for the drift current due to an impressed magnetic gradient when the ion free paths were short. It now seems of sufficient interest to carry through the

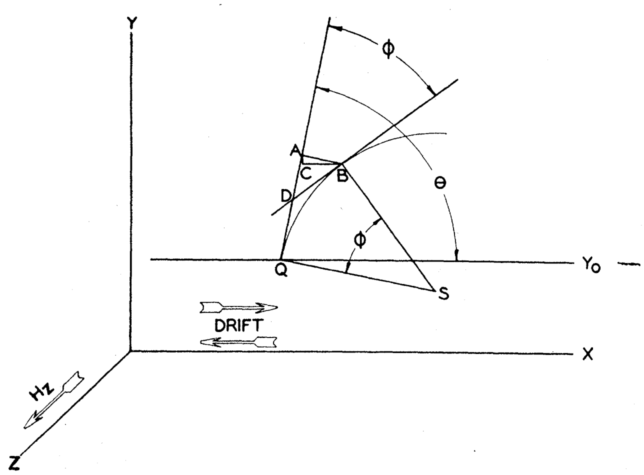

Fig. 1.

calculation somewhat more carefully and take into account the effect of small electric or gravitational fields which may be present. Consider an ionized region having a magnetic field impressed in the positive Z direction and having a uniform magnetic gradient dH_z/dy, a uniform electric gradient E_y, or a uniform gravitational field g_y. An ion after collision at Q (Fig. 1), begins a free path at random, making initially an angle θ with the x axis. In the presence of the magnetic field the ion traces out a portion of a circle in the xy plane and terminates its free path at a point B. The radius of the circle R traced by the ion is given by

$$R = mV/He \tag{1}$$

where m is the mass of the ion, V its velocity, e its charge and H the impressed magnetic field. All quantities are expressed in c.g.s. electromagnetic units.

[7] R. Gunn, Astrophys. J. May (1929).

The departure δ of the ion path from its normal straight line under no magnetic field is given by (AB) or $\delta \doteq \lambda/2 \sin\phi$; where λ is the length of the free path (QB) and ϕ is the angle swept off by the ion between collisions or $\phi = \lambda/R$ so that $\delta = \lambda/2 \sin \lambda/R$. The component δ_x of the departure δ in the x direction represented in the figure by $\overline{BC}$, is the departure effective in producing drift currents and

$$\delta_x = \lambda/2 \sin \lambda/R \sin\theta. \tag{2}$$

Since the magnetic field is non-uniform and an electric or gravitational field is present the instantaneous value of R is a continuous function of y. The path of an ion is approximately a true circle whose radius is determined by the average velocity and the average value of the magnetic and electric field impressed on the ion as it executes a free path. It will be convenient to discuss for the present the velocity of the ion and its impressed magnetic field rather than the electric and magnetic fields. Moreover, we shall assume (correctly) that the change in scalar velocity of an ion between any two collisions is small compared to the thermal velocity of agitation. Let the mean thermal velocity of the ions and their impressed magnetic field at $y = y_0$ be V_0 and H_0 respectively, then the mean value of $1/R$ for a given ion describing a free path λ which originated on y_0 is

$$\frac{1}{R} = \frac{e}{m}\left[\frac{H_0}{V_0} + \frac{\lambda}{2}\sin\theta \frac{d(H_z/V)}{dy}\right] \tag{3}$$

so that the departure in the x direction is

$$\delta_x = \frac{\lambda}{2}\sin\theta \sin\left[\frac{\lambda e}{m}\left(\frac{H_0}{V_0} + \frac{\lambda}{2}\sin\theta\frac{d(H_z/V)}{dy}\right)\right]. \tag{4}$$

In the present discussion we shall confine our attention to the case of short free paths so that $\lambda << R$. If we expand (4) and neglect product terms containing the sines of small angles and set the cosines of the same small angles equal to unity, then (4) becomes

$$\delta_x = \frac{\lambda}{2}\sin\theta \sin\left[\frac{\lambda^2 e}{2m}\sin\theta\frac{d(H_z/V)}{dy}\right]. \tag{5}$$

Expanding again and neglecting second order terms

$$\delta_x = \frac{\lambda^3 e}{4m}\sin^2\theta\frac{d(H_z/V)}{dy} \tag{6}$$

In order to obtain V_x the mean value of the departure per unit time for a typical ion with random distribution in θ we have only to multiply (6) by the number of collisions per second and average the values with respect to θ; thus (6) becomes

$$V_x = \frac{\lambda^2 eV}{8m}\frac{d(H_z/V)}{dy} = \frac{\lambda^2 e}{8m}\left[\frac{dH_z}{dy} - \frac{H_z}{V}\frac{dV}{dy}\right] \tag{7}$$

and the current density $i_x = Ne\ V_x$ is given by

$$i_x = \frac{Ne^2\lambda^2}{8m}\left[\frac{dH_z}{dy} - \frac{H_z}{V}\frac{dV}{dy}\right] \qquad (8)$$

where dH_z/dy is the impressed magnetic gradient, N is the number of ions partaking of the thermal motion, e their charge, λ their mean free path and V the scalar velocity of the ion in the xy plane. The second term in equation (8) involving the velocity V is restricted to a *continuous* velocity change; that is, the velocity must change *during* the free path since otherwise the ion drift vanishes.

Inside the sun or earth apparently the only factors which could give rise to a continuous change of velocity along a radial axis are electric and magnetic fields and gravity. In these regions the electric and gravitational forces are small and the thermal velocities are large compared to any change of velocity produced between collisions by the impressed electric or gravitational fields. Remembering that the mobile ion is the electron, Eq. (8) may be transformed to

$$i_x = \frac{Ne^2\lambda^2}{8m}\left[\frac{dH_z}{dy} - \frac{H_z m g_y}{kT} - \frac{eH_z E_y}{kT}\right] \qquad (9)$$

where E_y is an impressed electric field, k the Boltzmann constant, T the absolute temperature and g_y by gravitational acceleration. It will be noted that the sign of each term in this equation is opposite to that resulting when the ion paths are long or when it is tacitly assumed that, on the average, an ion executing a short free path starts from rest. The bearing of the present derivation on the anomalous Hall effect, and on drift currents arising from an inhomogeneous magnetic field will be discussed in a future paper. Application of these effects to an explanation of the sun's and earth's magnetic field shows that the first term contributes in such a manner as to magnetize the bodies, the second term tends to demagnetize the bodies and the last term may magnetize or demagnetize according to the direction of the impressed electric field. It will be convenient to treat each of these effects separately and estimate their relative importance in the production of the observed magnetic fields.

Considering first the electric currents arising from direct gravitational action we find that these currents flow in such a direction that they demagnetize the earth. A calculation of the value of the second term of Eq. (9) in the brackets using $H_z = 0.5$ gauss, $m = 10^{-27}$ grams, $g = 10^3$ cm/sec^2, $k = 1.37 \times 10^{-16}$ and $T = 10^4$ degrees, yields a value of 2×10^{-11} gauss/cm which is small compared to the other quantities inside the bracket. We may then conclude that the direct gravitational action on the ion is unimportant and in the following we shall confine our attention to the two more important effects which are due (1) to a magnetic gradient and (2) an electric field.

All the phenomena which have been mentioned depend on the presence of a magnetic field and we must account in some manner for an initial field which was in the correct direction relative to the direction of rotation of the sun and earth to regenerate and produce the observed fields. Several possibilities present themselves, but perhaps the most reasonable arises from the considerations of Pannekoek,[8] who has considered the gravitational separation of the positive ions and electrons in the sun and calculated the mean volume charge density. A similar calculation shows that the earth, at the time of its formation, possessed a positive charge of 10^{-5} coulombs with an appropriate electron atmosphere which rendered the whole electrically neutral. This separation of charge combined with the rotation of the earth provided a very small magnetic field which was in the correct direction to grow and form our present field.

We shall consider the currents produced inside the sun or earth as a result of thermal motion of the ions and their interaction with an impressed non-uniform magnetic field. Let us suppose for simplicity that when the earth was whirled off the sun by tidal or other forces that its rotation and static charge produced a non-uniform, but radially symmetrical magnetic field that was directed from north to south and parallel to the axis of rotation at all points inside the earth, the field being greater at the center than at the surface. By choosing the direction of the magnetic flux as the positive z axis, applying the first term of Eq. (9) and remembering that everything is symmetrical about the axis of rotation, it is found that a circular current is produced which *increases* the total magnetic flux enclosed by the current. This increase in magnetic flux increases the gradient which in turn increases the current. Thus such a radially symmetrical system is regenerative and can build up large magnetic fields from insignificant beginnings. In order to examine under what condition this type of regenerative action can give rise to steady currents, consider a line integral around a rectangular circuit which lies in a plane perpendicular to the current. Neglecting the contribution of the ends of the rectangle to the integral, we have approximately

$$i_x = \frac{1}{4\pi}\left(\frac{dH_z}{dy}\right)_1 \tag{10}$$

where $(dH_z/dy)_1$ is the magnetic gradient produced by the current i_x as a result of the impressed gradient. If we neglect temporarily the effects of electric and gravitational fields a comparison of Eqs. (10) and (9) shows that to a rough approximation

$$1/4\pi = Ne^2\lambda^2/8m \tag{11}$$

This relation serves to specify the magnitudes of the quantities necessary to produce a current under the assumed conditions. By assuming that the charged particle effective in producing the drift current is the electron, and assuming the number of electrons partaking of thermal agitation at high temperature is equal to the number of free electrons in the region or 10^{23} ions/cm^3 then the substitution of $m = 10^{-27}$ gms, $e = 1.59 \times 10^{-20}$ e.m.u. in

[8] Pannekoek, Bull. Astr. Inst. Netherlands, No. 19 (1922).

Eq. (11) shows that λ the free path of the electron must approximate 3×10^{-6} cm. The requirements are less stringent if the variable permeability of the medium is taken into account, but this factor will redistribute the resulting magnetic field in an unknown manner and will not be considered. Calculations made by Eddington[9] indicate that in the center of Capella the electrons of most atoms are stripped off and mean free path of the ion or electron increases to well over 10^{-2} cm. We cannot estimate exactly at the present time how far the stripping of the atoms progresses, but there is considerable evidence supporting the view that the free paths of the ions within the sun are much longer than 10^{-6} cm, and that the electron density is greater than 10^{23}. Moreover, it is known that the internal temperature of the earth is 10,000° or more, so the assumption of a free path length of 10^{-6} cm in that region also seems warranted.

It seems probable that electric currents rising from the inhomogeneity of the self produced magnetic field are the currents important in the production of the magnetic fields associated with the sun and earth. However, if a theory of the growth of the magnetic field similar to the foregoing is to be accepted, the resulting currents in the steady state must be great enough to account for the observed magnetic moment. A calculation of the magnetic moment is readily made if the effective magnetic gradient and free path length for the ions are known at all points within the earth or sun. Actually such information is lacking and on the earth we must content ourselves by assuming values for the free paths consistent with Eq. (11) and assuming that the average gradient effective in producing currents is nearly the same or slightly larger than that observed at the surface of the earth; namely, 1.6×10^{-9} gauss/cm. The cross sectional area of the solar and terrestrial current sheet is roughly $\pi R^2/2$ where R is the radius of the earth or sun and the mean area of the current circuit is $\pi R^2/4$ so that the total magnetic moment M is given roughly by

$$M=\frac{\pi^2 R^4}{8}\cdot i_\theta \tag{12}$$

where i_θ is the mean westward current density. By aid of Eq. (10) this becomes

$$M=\frac{\pi^2 N e^2 \lambda^2 R^4}{64m}\,\frac{dH}{dr}\,. \tag{13}$$

If we assume that the carrier effective in producing the current is the electron and take $N=10^{23}$, $e=1.57\times 10^{-20}$ e.m.u., $\lambda=3\times 10^{-6}$ cm, $R=6.4\times 10^8$ cm, $m=10^{-27}$ gms and $dB/dr=1.6\times 10^{-9}$, we find that the magnetic moment of the earth is 12×10^{24}. This value is taken to be in agreement with the observed values of 8.24×10^{24}, since as we shall show later, demagnetizing currents may exist which are of the same order of magnitude as the magnetizing currents. It seems probable that the average values taken for dH/dr and R

[9] Eddington, The Internal Constitution of the Stars, p. 222 (1926).

as well as the free path λ will require revision. The regions where the currents become large cannot as yet be specified exactly but it seems reasonable to expect moderate currents to appear when the temperature of the ionized region approaches a few thousand degrees. Geophysicists believe that inside the earth a temperature of 2,500° is attained at 300 km below the surface and we are therefore probably not far in error when we assume that currents circulate through the total volume of the earth.

A calculation of the magnetic moment of the sun cannot be made directly since no data are available regarding the mean gradient. A lower limit to the magnetic moment may be set, however, since the simplest consideration indicates that in the high temperature regions of the sun, the free path and mean gradient will be larger than on the earth. If we assume that the gradients and free paths are the same, then the solar magnetic moment will be larger than that of the earth by the fourth power of the ratio of the diameters. On this basis the minimum possible value of the solar magnetic moment is 5.0×10^{33}. Direct spectroscopic evidence indicates that the moment is roughly 8.6×10^{33}, while the following extrapolation of diamagnetic data into regions well below the photosphere indicates that the solar magnetic moment is several hundred times the smallest allowable value given above. It has been shown[5] that at a certain critical ion density, an ion gas is no longer strongly diamagnetic and as we penetrate deeper and deeper into the sun's atmosphere we find a critical pressure which is such that the outward radial magnetic gradient drops to moderate and normal values. It has also been shown[5] that the outward radial magnetic gradient of the sun could be explained if we assumed that the effective permeability of the diamagnetic medium was nearly zero. This assumption which was shown to agree moderately well with observational data provided a further relation between the magnetic field and the ion density. Moreover, the discussion indicated that the limitation of the solar magnetic field was to be attributed to the high temperature and ionization of the solar atmosphere and not to a special peculiarity of the manner by which the field was produced. We would then expect the solar and terrestrial magnetic fields to arise from the same causes as indicated by the present work. The critical ion density which we have discussed is given by

$$N_{critical}=\frac{4eB}{\pi\sigma^2(2mkT)^{1/2}} \tag{14}$$

where e is the ion charge in e.m.u., B the magnetic field strength, σ the kinetic theory diameter of the ions, m the mass of an electron, k the Boltzmann constant and T the absolute temperature. When the free paths are long, the ion density N is related to the field by

$$N\doteq B^2/4\pi kT. \tag{15}$$

If we restrict ourselves to the critical value of B we may equate (4) and (5) which yields

$$B_{critical}=\frac{16e}{\sigma^2}\left(\frac{kT}{2m}\right)^{1/2} \tag{16}$$

taking $e=1.57\times10^{-20}$; $k=1.37\times10^{-16}$; $T=6{,}000°$; $\sigma=2\times10^{-8}$ cm and $m=10^{-27}$ gm B critical approximates 12,000 gauss. This value of the field is found some distance below the photosphere where the ion pressure is nearly one half an atmosphere. If we calculate the magnetic moment of the sun using the above figures for the equatorial field the magnetic moment turns out to be 4×10^{36} a value nearly 800 times the smallest allowable value which indicates reasonably enough, that the mean magnetic gradient and ion free paths are much larger in the sun than in the earth.

In the foregoing theory no mechanism has been provided to limit the magnetic fields or gradients and the field would be expected to build up to much larger values by regenerative action than are actually observed. In a search for a means to control the final magnetic field we are led to a consideration of effects which could demagnetize the body. We have seen in the development of Eq. (9) that currents can be produced by an electric field of the proper sign which are opposite in direction to the currents produced by magnetic gradients. Several effects within the sun and earth could give rise to radial electric fields which upon interacting with an impressed magnetic field and thermal agitation will produce drift currents. Among those that seem important we may mention temperature gradients, difference in ion concentration due to temperature ionization or non-uniform distribution of radioactive material and gravitational separation of charge such as considered by Pannekoek.[8]

The electric field arising from a temperature gradient assuming that the mobile ion is the electron, is given on classical theory by

$$E_y=\frac{k}{2e}\,\frac{dT}{dy} \tag{17}$$

where dT/dy is the temperature gradient, k the Boltzmann constant, e the ion charge and E_y the resulting electric field. Similarly the electric field arising from a difference in ion concentration C is given by

$$E_y=\frac{kT}{eC}\,\frac{dC}{dy} \tag{18}$$

and the electric field arising from the gravitational separation of charge[8] is

$$E_y=(\mu/e)g_y \tag{19}$$

where μ is the average molecular mass and g_y the gravitational acceleration.

Leaving out of consideration ion concentrations due to radioactive material, which may be important, it is possible to calculate the electric fields due to the above effects. Using such data as are available it turns out that the electric field due to the gravitational separation of the ions is the largest and this will be considered in the present discussion for the purpose of illustration. The electric field due to the gravitational separation of charge is non-uniform, drops to zero at the center of the body, and is always directed radially outward. The mean atomic mass of the particles inside the earth may be taken to be 1.0×10^{-23} grams as compared to 3.7×10^{-24} grams in the sun and the mean gravitational acceleration as 5×10^2 cm/sec^2. Substitution of these quantities in Eq. (19) gives E_y as 0.3 e.m.u/cm which on substitution in the last term of Eq. (9) gives a value for $eH_z E_y/kT$ of 1.2×10^{-9} if we take $H_z=0.5$ gauss and $T=10{,}000°$. Comparison of this value with the assumed value for the surface magnetic gradient of 1.6×10^{-9} gauss/cm shows that the effects due to the gradients or magnetizing action are larger, but almost identical in magnitude with the demagnetizing action arising from the electric field. We are thus led to conclude that the two effects play a part in the production and maintainance of a steady field and that the absolute value of the resulting magnetic moment will depend on the size and temperature of the body and on the internal distribution of the gravitational, magnetic and electric fields. With an exact knowledge of the internal distribution of the magnetic field of the earth it would be possible to calculate the consequences of the present theory and in this manner study the requirements relating to the internal distribution of the electric field. A complete theory, moreover, must take account of a variable free path which when applied to the sun or earth would tend to reduce the demagnetizing effects and increase the relative magnetization. It seems quite possible that a portion of the magnetic flux directed from north to south near the axis, returns somewhere inside the earth. Such a distribution or a distribution taking into account diamagnetic effects would give rise to a system of currents which would be enormously more complicated than the systems we have just considered.

The eccentricity of the magnetic fields of the earth and sun relative to their rotational axes indicates that the values of the free paths and temperatures are not symmetrical about these axes and the magnetic axis would therefore be expected to be displaced toward the hotter regions as viewed from the axis of rotation. The present theory indicates that at the time of formation of the earth the area now embraced by the Pacific Ocean was much hotter than the other half of the earth and that at great depths this asymmetry still persists today. These considerations are in keeping with the view that the planets were formed as a result of tremendous tides on the sun since such a mechanism readily explains why one side of the planet would be much hotter than the other and would account for the reflected shift in the magnetic poles of the sun which are known to be slightly displaced from the geographic poles.

The theory which has been developed provides a regenerative magnetizing mechanism which is capable of producing internal circulating currents in the sun and earth of a magnitude and distribution suitable for the production of the observed fields. The theory further provides a demagnetizing mechanism which is capable of controlling the regenerative phenomenon and limits the resultant magnetic field to finite values which are dependent on the physical constants of the hot body. It is not to be supposed that the theory is complete and will not need modification for the complete solution to the problem will undoubtedly be very complicated.