path. The rotor was turning at a rate of 6500 r.p.m. and the green path extended from one fixed electrode to another (one-half a revolution), representing a time interval of about 0.0045 sec. Over this distance the intensity dropped by about one-half (a visual estimate) of its value at the starting point of the path.

By cleaning (scraping) the ends of the electrodes the luminescence described above disappears but returns after the discharge passes for a few minutes and finally builds up to its former intensity. At the same time the ends of the electrodes show a marked degree of corrosion. By substituting iron for the aluminum of the fixed electrodes the phenomenon disappears and does not reappear until the aluminum is replaced. Increasing the capacity across the gap increases the intensity of the green radiation. The above treatment of the electrodes seems to indicate that aluminum oxide is the emitter of this radiation. The intensity of radiation given off by the ends of the rotating electrodes is so low as to make it difficult to photograph its spectrum with high dispersion; although it can be observed with a direct vision spectroscope which shows it to extend from around 4800Å to 5300Å. Examination of the spark indicates that a green "flame" "follows" the rotating electrodes out of the white spark for a distance of about one-quarter of an inch. This "flame" is much more intense than the radiation emitted from the arc described by the rotating electrodes but in the direct vision spectroscope it appears identical in structure (a series of unresolved bands in the blue-green and green). This "flame" has been photographed with a grating spectrograph of about 4Å per millimeter dispersion. From the position of the heads of the bands and the structure of two of them they have been identified as the bands of aluminum oxide which were described by Pomeroy.[1] An effort is being made to photograph the spectral structure of this radiation at a point along the arc 90° from the spark.

The observations recorded above indicate that during the discharge aluminum oxide is formed and that this molecule is raised to some particular energy state in which it can remain, without radiating the excess energy, for at least four thousandths of a second. Work now in progress will determine the maximum life of this condition and probably the life of the aluminum oxide molecule.

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June 7, 1929.

[1] W. C. Pomeroy, Phys. Rev. **29**, 59 (1927).

Electron Scattering in Hydrogen

The experiments on the angular scattering of electrons described in an article in the PHYSICAL REVIEW (Vol. 33, page 559, 1929) have been extended both in angular range and accuracy. The scattering gas most extensively used in this recent work has been hydrogen.

The apparatus, though new, was essentially the same as that described in the article referred to. The changes consisted in refinements, such as narrower slits, and more accurate methods of measuring the angles and potentials. The hydrogen was admitted to the scattering chamber from the center of a Wood's tube. The distance from the Wood's tube to the region where the scattering occurred was made as small as possible. The flow of hydrogen was as rapid as the conditions allowed in order to favor the presence of a large percentage of atomic hydrogen in the scattering chamber. The actual amount of atomic hydrogen present in this region when the discharge was in operation could not be determined accurately, but from the work of Wood, Bonhoeffer, Kaplan, and others probably in excess of 75 percent of the hydrogen was in the atomic condition.

A further check on the presence of atomic hydrogen in the scattering chamber was afforded by the scattering curves themselves. Several differences were apparent between the curves obtained when the discharge was on and when it was not. The magnitude of the peak due to elastic impacts dropped slightly in the presence of the discharge. The peak representing those electrons which had suffered an inelastic collision with a loss of approximately 12.5 volts shifted over till the peak occurred between 10.5 and 11 volts. This has been taken to indicate the presence of those lower energy losses due to collisions with atomic hydrogen. In addition, another peak, representing an energy loss of from 8 to 9

volts dropped very greatly in magnitude though it never completely disappeared. This critical potential has been previously observed by Jones and Whiddington (Phil. Mag. **6,** 889, 1928). The conditions under which they observed it were quite different from those existing in the present apparatus but the behavior of the peak was quite similar. They attributed it to an energy loss characteristic of the hydrogen molecule and this hypothesis seems to gain additional support from the present work. Unfortunately the most recent theoretical work on the hydrogen molecule contains no explanation for an energy loss of this value. However, it is not contrary to any experimental work as hydrogen has not previously been investigated under the conditions necessary to bring the effect into evidence.

Accurate data has been obtained for the angular distribution of scattered electrons with from one hundred to two hundred volts energy in both atomic and molecular hydrogen. The experimental points obtained for elastically scattered electrons in atomic hydrogen lie within the limits of experimental error on the theoretical curve predicted by the new quantum mechanics. This is of a different order of magnitude from the scattering which would be expected classically. The scattering in molecular hydrogen is somewhat greater than in atomic hydrogen. This is in general what would be expected from theoretical considerations. No accurate comparison with the theory is possible, however, for molecular hydrogen, as the scattering to be expected has not been calculated for this case.

The inelastic scattering in hydrogen has also yielded some very interesting results which will be discussed more fully in a forthcoming article. But one point should be mentioned here in as much as one statement made in the PHYSICAL REVIEW article previously referred to has been found to be in error. The decrease in the number of electrons scattered inelastically as one proceeds from small to large angles is more rapid than the similar decrease in the number of elastically scattered electrons. The reason for this is not clear, but the same thing has been observed by Dymond (Proc. Roy. Soc. **122,** 571, 1929) for electrons scattered in helium.

These results will be reported in full in an article to appear shortly in the PHYSICAL REVIEW.

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June 15, 1929.

The Uncertainty Principle

The uncertainty principle is one of the most characteristic and important consequences of the new quantum mechanics. This principle, as formulated by Heisenberg for two conjugate quantum-mechanical variables, states that the accuracy with which two such variables can be measured simultaneously is subject to the restriction that the product of the uncertainties in the two measurements is at least of order h (Planck's constant). Condon* has remarked that an uncertainty relation of this type can not hold in the general case where the two variables under consideration are not conjugate, and has stressed the desirability of obtaining a general formulation of the principle. It is the purpose of the present letter to give such a general formulation, and to apply it in particular to the case of angular momentum.

We define the "mean value" A_0 of an (Hermitean) operator A in a system whose state is described by the (normal) function ψ as

$$A_0=\int\bar{\psi}A\psi d\tau$$

where the integral is extended over the entire coordinate space. The Hermitean character of A (i.e.

$$\int\bar{\phi}A\psi d\tau=\int\bar{\psi}A\phi d\tau$$

for arbitrary ϕ, ψ) insures the reality of A_0. The "uncertainty" ΔA in the value of A is then defined, in accordance with statistical usage, as the root mean square of the deviation of A from this mean, i.e.

$$(\Delta A)^2=\int\bar{\psi}(A-A_0)^2\psi d\tau.$$

The uncertainty principle for two such variables A, B, whose commutator $AB-BA=hC/2\pi i$, is expressed by

$$\Delta A\cdot\Delta B\geqq h|C_0|/4\pi$$

i.e. the product of the uncertainties in A, B is not less than half the absolute value of the mean of their commutator.

* E. U. Condon "Remarks on Uncertainty Principles" Science LXIX, p. 573 (May 31, 1929), and in conversations with the writer on this topic.