ERRORS IN THE USE OF GRATINGS WITH X-RAYS DUE TO THE DIVERGENCE OF THE RADIATION

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Abstract

The errors in the measurement of x-ray wave-lengths by means of gratings that are to be expected on account of the divergence of the incident beam are considered in more detail than has been done before. In particular, the displacement of the diffracted line, the displacement of the zero order, or reflected, line, the displacement of the center of the incident beam relative to the center of the grating, and the effect of the height of the slits are considered and the expressions for the errors are obtained. These are applied to one of Bearden's spectrograms, and the magnitude of the errors calculated. In this particular case, and apparently in general, the errors are less than other experimental errors.

THE use of plane gratings makes possible the direct determination of x-ray wave-lengths with a high degree of accuracy. Unfortunately, unlike the case in optics, non-parallel radiation must be used with the gratings. It is possible that this condition may introduce errors into the measurements if the ordinary formula for parallel radiation is used in the calculations. One source of error is, as Porter[1] points out, the displacement of the spectrum lines. Bearden,[2] in his measurements of the K lines of copper, finds a discrepancy between his results and Siegbahn's that is greater than the probable error. For the K_β line he finds a value of 1.3926 ± 0.0002A as compared with 1.3893. Prins[3] thinks that part of the difference may be due to the use of a divergent beam of x-rays. Therefore, it is desirable that a detailed study of the errors that may arise from this cause be made. Two sources of error are to be expected. One is the divergence of radiation in what is assumed to be the plane of incidence, i.e., the plane through the source that is normal to the elements of the grating. The other is the divergence in the plane through the source and the central element of the grating—a plane which is normal to the other one considered.

In connection with the first group of errors, Porter shows that the ordinary grating formula must be replaced by

$$n\lambda = r(\cos\theta_0 - \cos\theta_0')\left(1+\frac{3X^2}{20l_0{}^2}\right)$$

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[1] A. W. Porter, Phil. Mag. **5**, 1067 (1928).

[2] J. A. Bearden, Proc. Nat. Acad. Sci. **15**, 528 (1929). Similar results have been found by Bäcklin, but the author has not seen a detailed account of his work.

[3] J. A. Prins, Nature **124**, 370 (1929).

where r is the grating space, θ_0 the angle of incidence, θ_0' that of diffraction, X the length of the grating, and l_0 the distance from the source to the grating or from the grating to the point of maximum intensity. This formula is a special case, applicable only when the two lengths mentioned are equal. The general case can be obtained readily, however, by the same methods and with the same assumptions and is

$$n\lambda = r(\cos\theta_0 - \cos\theta_0')\left(1 + \frac{3X^2}{20(\theta_0'^2 - \theta_0^2)}\left[\frac{\theta_0'^2}{l_2^2} - \frac{\theta_0^2}{l_1^2}\right]\right) \tag{1}$$

where l_1 is the distance from the source to the center of the grating and l_2 that from the grating to the point of maximum intensity.[4] When $l_2 \neq l_1$, the correction term may be positive or negative, and differs for each line and each order in the spectrum.

The derivation of this formula assumes a point source, which can only be the radiating atom in the target of the x-ray tube. If a slit is considered as a source, the effects due to all parts must be integrated, as in the case of parallel radiation, but the problem is more complex. The errors due to the divergence of the x-rays are so small that there is no reason for developing a new formula.

Eq. (1) shows that the reflected beam, or the line of zero order, will also be displaced if $l_2 \neq l_1$. When the left side is zero, $\cos\theta_0$ cannot equal $\cos\theta_0'$ for then the second factor becomes infinite. Therefore this second term vanishes, or

$$1 + \frac{3X^2}{20(\theta_0'^2 - \theta_0^2)}\left(\frac{\theta_0'^2}{l_2^2} - \frac{\theta_0^2}{l_1^2}\right) = 0$$

This reduces to

$$\theta_0'^2 = \theta_0^2\left[1 + \frac{3X^2}{20}\left(\frac{1}{l_1^2} - \frac{1}{l_2^2}\right)\right]. \tag{2}$$

When l_1 and l_2 are greatly different, the displacement of the reflected beam may become important. The displacement may affect the value of λ in two ways. The position of the reflected line enters directly into all the angle mea-

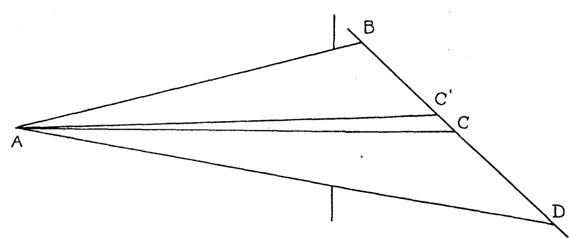

Fig. 1.

surements. It may also enter into the length l_2 if this is determined from two exposures at different distances fom the grating. In this case the error in the

[4] This expression is general, but negative values of n must be used for the negative orders.

plate nearer the grating may be considerable, but from the nature of the dependence of l_2 upon the measurement of this plate, it is readily seen that a large error in the angle will make only a small error in l_2.

The angles used in the previous formulae are measured to the center of the grating. In measuring the photographic plates, the center of the direct beam is used. In cases where slits prevent the beam from covering the grating, the center of the beam and the center of the grating do not coincide. The angle between the two, CAC' in Fig. 1, is readily determined. If $BCA=\theta$, $BC'A=\theta'$, and $BDA=\theta''$

$$BC'=l_1\alpha/\theta' \text{ where } \alpha \text{ is half the slit width}/l_1$$

$$BD=2l_1\alpha/\theta''=2l_1\alpha\left(\frac{1}{\theta'}+\frac{\alpha}{\theta'^2}\right), \quad \text{since } \theta'=\alpha+\theta''$$

$$BC=BD/2=l_1\alpha\left(\frac{1}{\theta'}+\frac{\alpha}{\theta'^2}\right)$$

$$C'C=BC-BC'=l_1\alpha^2/\theta'^2$$

$$CAC'=CC'\cdot\theta/l_1=\alpha^2\theta/\theta'^2\sim\alpha^2/\theta=s^2/4\theta l_1^2 \tag{3}$$

where s is the slit-width.

The fact that the source is not a single point, but is extended, will make the diffracted lines asymmetrical because the value of θ_0 will be different for each point in the source. The effect is measured by $d\theta_0'/d\theta_0$ and can be determined accurately enough from the ordinary grating formula, $n\lambda=r(\cos\theta_0-\cos\theta_0')$.

Thus

$$\frac{d\theta_0'}{d\theta_0}=\frac{\sin\theta_0}{\sin\theta_0'}=\frac{\theta_0}{\theta_0'}\,.$$

For the small values of $d\theta_0$ that are used in practice, the ratio θ_0/θ_0' is practically a constant, and hence $d\theta_0'/d\theta_0$ also, and the diffracted beam is symmetrical within the limits of observation.

The second type of divergence, that in a plane through the source and the central element of the grating, involves three-dimensional geometry, and hence the work will only be indicated here. If radiation from a point strikes the grating at a height h above the plane through the point normal to the grating elements, the angle of incidence is $(\theta_0^2+h^2/l^2)^{1/2}$. In the plane of incidence of this beam the grating space is $r/\cos\beta$ where $\beta=h/l_1$. On the photographic plate the angle of diffraction is not measured, but its projection on a plane perpendicular to the lines of the grating. The net result is to produce an error in the wave-length given by

$$n(\lambda'-\lambda)=\frac{r}{2}\left(\frac{\theta_0^2h^2}{2l_1^2}-\frac{\theta_0'^2\,h'^2}{2l_2^2}\right) \tag{4}$$

where $h+h'$ is the height of the photographic trace above the source, and h' is l_2h/l_1.

The order of these various errors can be determined for Bearden's experiment.[2] In the case of the upper left hand spectrogram in his figure, the spacing of the lines fits approximately the values, $\theta_0=0.00247$, $\theta_0'=0.00459$ for the first order, and $\theta_0'=0.00988$ for the sixth order. These values seem high, but they will be used. "l_1" is about 60 cm and l_2 about 215. Then X must be 4 mm. The errors to be expected from Eq. (1) for these figures are -2.1×10^{-6} for the first order and $+1.1\times10^{-7}$ for the sixth, or -0.0002 % and $+0.00001$ % respectively in the wave-length. The greatest uncertainty in these figures lies in the value of X. If this is assumed to be 1 cm instead of 0.4 cm, the errors are increased to -0.0013 % and and $+.00007$ %. The error to be expected in θ_2 in Eq. (2) is $3.2\times10^{-6}\times\theta$, or 0.0003 % of θ. If X were 1 cm, the error would be 0.0020 %. This latter corresponds to a displacement on the photographic film of 0.0001 mm. The values of CAC' in Eq. (3) is 2.81×10^{-8}. This represents 0.0006 mm on the photographic film. In order to determine the error due to the divergence in a vertical plane, it will be assumed that the spectrum lines are 1 cm high. The maximum error in λ (for the K_α line) is then -9.66×10^{-14} for the first order and -1.0×10^{-13} for the sixth, or errors in λ of 0.0006 % and 0.0007 % respectively. The actual errors are less because any point on a spectrum line receives radiation from a large part of the source. It is evident that in this particular experiment all the errors considered above are less than other experimental errors and are negligible in their effect upon the accuracy of the experiment. It would seem, although it cannot be dogmatically stated to be always true, that in most cases the errors due to assuming the formula for parallel radiation are less than the experimental errors of measurement.