To be absolutely sure that these bands might not perhaps be due to some other molecule, e.g. $(NO)_2$, the absorption tube was replaced by a tube 1.2 cm in length and filled up to 75 cm pressure with NO. The light passing through this tube has to traverse about the same number of molecules of NO as in the case of the 92 cm tube with 1 cm pressure. If $(NO)_2$ molecules were present, their concentration should increase with the square of the pressure and the path being 75 times shorter, one would expect the effect due to these molecules to become 75 times as intense. The absorption of the isotope heads found is quite similar to that obtained with the 92 cm tube, except that they are a little more diffuse, as one might expect. The measured shift agrees accurately with the above results.

The effect on the (0, 0) band is very difficult to observe owing to its smallness. The wave-lengths of the displaced heads found are 2269.605($N^{15}O^{16}$) and 2269.747($N^{14}O^{18}$) which correspond within experimental error with the calculated shifts. The displaced head due to the $N^{14}O^{17}$ molecule could not be observed.

At this stage it can only be said that $N^{15}O^{16}$ is about as abundant as $N^{14}O^{18}$. The work is being carried on and it is hoped to find the abundance of N^{15} isotope more accurately. At higher pressures faint absorption lines appear beyond the isotope heads whose positions seem to agree with those calculated for $N^{16}O^{16}$, but this must be considered doubtful until further work has been done. This point will also be subject for further research.

I wish to thank Professor R. S. Mulliken who suggested the search for isotopes in these NO absorption bands and at the same time wish to thank him and Dr. A. Christy for their valuable advice.

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The Paschen-Back Effect of Hyperfine Structure

The study of the Paschen-Back effect in hyperfine structure is of particular interest as it is the only possiblity to verify the complete theory of the gradual change of the Zeeman effect from weak to strong fields,as there are no suitable ordinary multiplets available for this purpose. The theory of the Zeeman effect for any field strength has been studied by Heisenberg and Jordan [Zeits. f. Physik. **37,** 263 (1926)] and by Darwin [Proc. Roy. Soc. **A115,** 1(1927)] for the case of ordinary multiplets. There may be shown to be a very close analogy between ordinary multiplets and the hyperfine structure separations, the former being due to the interaction between the resultant orbital moment and the resultant spin moment and the latter to the interaction between the total extranuclear moment andthe nuclear moment. The roles of s the resultant spin moment, l resultant orbital moment, and j total moment in the ordinary multiplet structure are taken by i nuclear moment, j total extranuclear moment, and f resultant moment in the hyperfine structure.[1] This correlation together with the fact that the addition to the energy in a very strong magnetic field, due to the nuclear spin (E_{ij}) and to the presence of a magnetic field (E_m) is given by

$$E_m+E_{ij}=(m_jg+m_ig_i)\omega H+am_jm_i \qquad (1)$$

allows the immediate adaption of the results of Darwin to the hyperfine structure problem. One arrives at a system of equations which are given by the key equation[2]

$$\begin{aligned}&-X_{m_j-1,m_i+1}\frac{a}{2}(j-m_j+1)(i+m_i+1)\\&+X_{m_j,m_i}(E-am_jm_i-gm_j\omega H)\\&-X_{m_j+1,m_i-1}\frac{a}{2}(j+m_j+1)(i-m_i+1)=0\end{aligned} \qquad (2)$$

where a is the interval unit of hyperfine structure; g is the Landé factor for the extranuclear electrons; ω is the frequency of the Larmor precession $e/4\pi mc^2$ in cm^{-1}; H is the magnetic field strength in gauss; E is the energy displacement in wave-number units from the hypothetical level which would exist if there were no nuclear spin and no applied magnetic field; the X's are the coefficients in the expansion of the wave function

[1] Completely analogous to the case of multiplets one can obtain selection rules, intensity relations, interval rule, and expressions for the magnitude of the hyperfine structure for different levels.

[2] The interaction energy between the nuclear moment and the external field is omitted because it is much too small to be detected.

and must be determined and used for the calculations of intensities. The roots of the determinant of this system of equations give the values of E for any given $m=m_j+m_i$.

There is one point on which the correlation is not exact; for the multiplets, l has the selection rule $l\rightarrow l\pm 1$ while corresponding to it in the hyperfine structure j has the selection rule $j\rightarrow j\pm 1, j$. The intensity relations derived by Darwin do not contain the transition $l\rightarrow l$ corresponding to $j\rightarrow j$ but the relations may be found simply by analogy. After having obtained the different values of E, the X's may be calculated from the Eqs. (2) with the aid of certain normalizing relations which are of the form

$$N_m^{if}=\sum_m (X_{mjmi}^{if})^2(i+m_i)!(i-m_i)!(j+m_j)!(j-m_j)!=1$$

For a transition $j\rightarrow j'$, the intensities are for the perpendicular components $m\rightarrow m-1$

$$I=\left[\sum_m X_{mjmi}^{if}X_{mj-1,mi}^{i'f'}(i+m_i)!(i-m_i)!(j+m_j)!(j-m_j)!\right]^2\div N_m^{if}N_{m-1}^{i'f}$$

for the other perpendicular components $m\rightarrow m+1$

$$I=\left[\sum_m X_{mjmi}^{if}X_{mj+1,mi}^{i'f'}(i+m_i)!(i-m_i)!(j+m_j)!(j-m_j)!\right]^2\div N_m^{if}N_{m+1}^{i'f'}$$

for the parallel components $m\rightarrow m$

$$I=4\left[\sum_m X_{mjmi}^{if}X_{mjmi}^{i'f'}(i+m_i)!(i-m_i)!(j+m_j)!(j-m_j)!\right]^2\div N_m^{if}N_m^{i'f'}$$

In the case $j\rightarrow j$, the factor 4 must be omitted. In these relations $\sum_m$ means the sum over all values of m_j and m_i such that $m=m_j+m_i$ is constant.

For very weak fields the procedure is not so complicated. One can easily derive a g formula for each of the hyperfine structure levels,

$$g(f)=g(j)\frac{j}{f}\cos(jf)=g(j)\frac{f(f+1)+j(j+1)-i(i+1)}{2f(f+1)}$$

The intensities will be given by the well-known ordinary intensity rules for the Zeeman effect, applied to each transition between the hyperfine levels of the initial and final state.

If the field is not very weak, the Zeeman effect will show distortions and at the same time lines representing transitions which would have been forbidden according to the selection rule for f, occur. This is analogous to the observations of Paschen and Back [Physica **1,** 261 (1921)] in zinc for ordinary multiplets.

For very strong fields for which the Zeeman effect is larger than the hyperfine structure the treatment is also not so complicated and it has been given in detail for the case of bismuth. [Back and Goudsmit, Zeits. f. Physik. **47,** 174, (1928)]. In such a strong field those transitions will be found for which m_i does not change and for which m_j changes by ± 1 or 0. If however, the field is not yet "very strong," slight asymmetries in the Paschen-Back effect will appear and at the same time "forbidden" transitions will occur for which m_i changes but the total projection $m=m_j+m_i$ changes by ± 1 or 0. A detailed calculation can be given only with the help of the complete set of relations (1) and (2). The occurrence of such forbidden components has been observed in the case of bismuth.

The thallium line λ3776 in a field of 43,350 gauss is an example of a "not yet complete" Paschen-Back effect. The following table gives the calculated positions and ntensities of the Zeeman components.

Parallel		Perpendicular	
Pos.	Int.	Pos.	Int.
-2.74cm^{-1}	0.4	-3.00cm^{-1}	10.
−1.43	10.	−2.52	9.4
−1.20	9.8	−1.65	0.02
+1.28	10.	−0.99	0.6
+1.36	9.8	+1.09	0.02
+2.89	0.4	+1.55	0.6
		+2.45	10.
		+3.08	9.4

We are very much indebted to Professor E. Back and Dr. J. Wulff for providing us with the results of their excellent work on this line. The calculated values of the positions and intensities are in complete agreement with the observed values, the two weakest of the "forbidden" components not being observed due to their small intensity and two of the strong parallel components not being resolved due to their proximity.

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