THE RANGES OF IONIZING ELECTRONS IN HELIUM

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Abstract

The paper shows that a fourth power law for the absorption of ionizing electrons in helium represents experimental results only for initial energies greater than 250 electron volts. For initial energies less than 200 electron volts, the formula

$$x=(a/c^2)\left[cE-f+f\log\,(cE-f)\right]$$

is deduced, where x is the range of an electron of initial energy E electron volts, and the remaining quantities are constants. The formula is in agreement with the available experimental evidence, and appears to be applicable, with suitable changes in the constants, to other gases.

INVESTIGATIONS of the behavior of ionizing electrons in passing through gases are concerned with three interdependent quantities; the initial energy of the electrons, the distance which they travel and the number of ions they produce. Most workers[1] have endeavoured to find relationships between the energy of an electron and the average number of ions produced by it per unit distance travelled; a few,[2] in more recent years, have found the total number of ions in the whole distance travelled as a function of the initial energy of the electron, and have made some direct measurements of the ranges of the particles. It is necessary to distinguish between what, in this paper, is called the range of a particle and its track-length. The former is taken to mean the straight line distance from its origin to the point at which it ceases to ionize; the latter denotes the distance travelled by the electron measured along the actual path, as can be done for beta particles in a cloud expansion photograph.

To represent the absorption of cathode rays by matter, J. J. Thomson formulated the fourth power law, which has been verified experimentally by Whiddington,[3] C. T. R. Wilson,[4] Nuttall and Williams[5] and others. This law is concerned with the track-length, not with the range of the particles. Electrons with small initial energy, below about 2500 electron volts, cannot be studied adequately by cloud expansion methods, for they produce only

[1] For example: W. Kossel, Ann. d. Physik **37,** 393 (1912); S. Bloch, Ann. d. Physik **38,** 559 (1912); F. Mayer, Ann. d. Physik **45,** 1 (1914); Hughes and Klein, Phys. Rev. **23, 450** (1924); K. T. Compton and Van Voorhis, Phys. Rev. **26,** 436 (1925).

[2] Johnson, Phys. Rev. **10,** 609 (1917); Anslow, Phys. Rev. **25,** 484 (1925); Lehmann and Osgood, Proc. Roy. Soc. **A115,** 609 (1927); Lehmann, Proc. Roy. Soc. **A115,** 624 (1927).

[3] Whiddington, Proc. Roy. Soc. **A86,** 360 (1912).

[4] C. T. R. Wilson, Proc. Roy. Soc. **A104,** 1 (1923).

[5] Nuttall and Williams, Phil. Mag. **2,** 1109 (1926).

the "sphere" tracks described by C. T. R. Wilson. Electrical methods, with gas at low pressures must therefore be used for their investigation. It is the purpose of this paper to deduce from the experimental evidence now available, relationships between the initial energies and the ranges in helium of ionizing electrons which cannot be studied by cloud expansion photographs.

Purely electrical experiments devised to measure the number of ions produced per unit distance in a gas by moving electrons determine the ionization along the range (as defined above), not along the track. The faster the electron moves, however, the more nearly the ionization per unit distance along the range approaches the same quantity per unit distance along the track, since fast electrons suffer comparatively infrequent abrupt deviations from their original direction. The ionization per unit distance along the range is, of course, greater than the ionization per unit distance along the track.

Since in a stream of moving electrons no individual particle pursues exactly the same track as its neighbour, the quantities with which we are concerned are the average values for a large number. It is possible to produce a beam of electrons whose initial energies differ by only a few percent.[6] With these restrictions, let E be the initial energy of a moving electron, measured in electron volts, n the total number of ions which it makes before ceasing to ionize, and x its range. Then dn/dx represents the ionization per unit distance along the range. Experiments by Lehmann[2] have shown that for values of E between 190 and 1060 volts there exists a linear relation between E and n; that is

$$n = aE + b \tag{1}$$

where a and b are constants. There is good reason for believing that this equation is true also for all high-speed beta particles. The fourth power law, which may be expressed as

$$\frac{1}{4}\,\frac{m^2v^4}{e^2} = E^2 = \frac{NR}{K} \qquad \text{(Nuttall and Williams)}$$

where N is the atomic number of the absorbing gas, K is a constant, R the track length of the electron, v its velocity and e and m its charge and mass, has been shown above not to be strictly applicable unless the track length can be measured, yet it should be at least an approximation to the truth if we write

$$E^2 = kx. \tag{2}$$

Here x is the range of the electron and k another constant. Eqs. (1) and (2) can be combined, for

$$dn/dx = a(dE/dx) \tag{1a}$$

and

$$2E(dE/dx) = k. \qquad \text{That is}$$

$$dn/dx = (ka)/2E. \tag{3}$$

[6] Lehmann and Osgood, Proc. Camb. Phil. Soc. **22**, 731 (1925), and Ref. 2.

If dn/dx be plotted as a function of $(1/E)$, the resulting graph should be a straight line, passing through the origin. This linear relation was shown to be true for high-speed cathode rays in air many years ago.[7]

Bearing this in mind, it seemed worth while to plot values of dn/dx as a function of $1/E$ for slower moving electrons. The most reliable experimental values of dn/dx are those of K. T. Compton and Van Voorhis[1] as corrected by their critique[8] of June, 1926. They expressed their results as a graph in which the ionization per unit distance along the range was shown as a function of E, over the range 25–300 volts. In Fig. 1, which deals with helium at 1 mm pressure, values of dn/dx are plotted as ordinates and those of $1/E$ as abscissas; Fig. 2 shows, on a different scale, part of the same curve

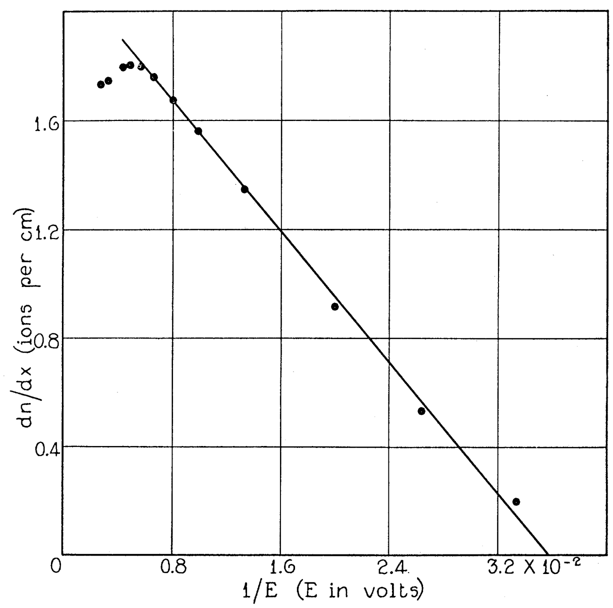

Fig. 1. Showing the relation between the ionization produced by an electron per unit distance along its range (dn/dx) and the reciprocal of its initial energy ($1/E$) measured in electron volts; based on the experimental results of K. T. Compton and Van Voorhis.

amplified by a few points from the work of Kossel, Glasson and Durack on high-speed electrons. These were obtained experimentally for air, but have been reduced to a helium standard by assuming from the mass absorption law that this gas is seven times as transparent as air. Fig. 2 also shows, as a full line, a plot of Eq. (3) using the constant for the fourth power law as given by Nuttall and Williams. The following interesting features are apparent: (*a*) For initial energies greater than 250 electron volts, the ranges of electrons are approximately representable by a fourth power law (dotted line, Fig. 2), the constant, as is to be expected, differing from that given by

[7] See S. Bloch, Ref. 2.

[8] K. T. Compton and Van Voorhis, Phys. Rev. **27**, 724 (1926)

Nuttall and Williams; (*b*) for energies less than 200 electron volts, a fourth power law is quite inadequate to represent the experimental results; (*c*) from 30 to 200 electron volts, the relation between dn/dx and $1/E$ is linear (Fig.1); (*d*) the transition between the two types of absorption is sharp, covering approximately the range 200–250 electron volts.

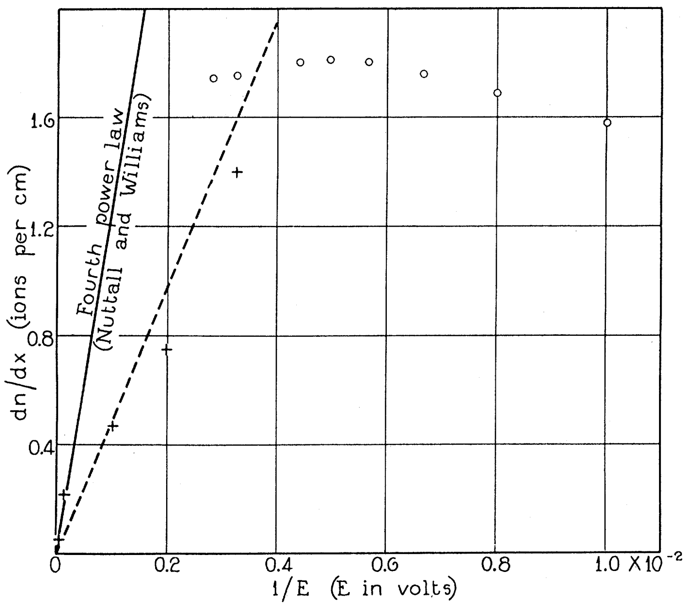

Fig 2. Showing part of the curve of Fig. 1 amplified by results from the work of Kossel, Glasson and Durack.

The observation (*c*) can be used to find a formula for the ranges of electrons between the limits specified, provided Eq. (1) is also true between these limits. This was experimentally proved by Johnson.[2] The equation to the straight line in Fig. 1 may be written

$$dn/dx = c - f/E,$$

where c and f are constants. Combining this with Eq. (1a),

$$dx = [(aE)/(cE - f)]dE,$$

which integrates to

$$x + h = (a/c^2)[cE - f + f \log (cE - f)], \tag{4}$$

where h is a constant of integration. This is small, and may for the present be neglected, since the straight line in Fig. 1 cuts the horizontal axis very near the point corresponding to the ionization potential of helium. The values of the constants are $a = 0.0328$ per volt, $c = 2.17$ per cm, $f = 61.5$ volts per cm. The value of a has been taken by extrapolating a curve given by Lehmann, which was obtained under more reliable conditions than Johnson's. Putting $h=0$, the Eq. (4) is represented graphically in Fig. 3.

Since the formula (4) is based directly on experimental evidence, there is no reason to doubt its validity. It would be interesting, however, to check it by comparison with actual measurements of the ranges of electrons, as this would test the consistency of the experimental observations obtained by different methods. The available data are scanty. The value of x for

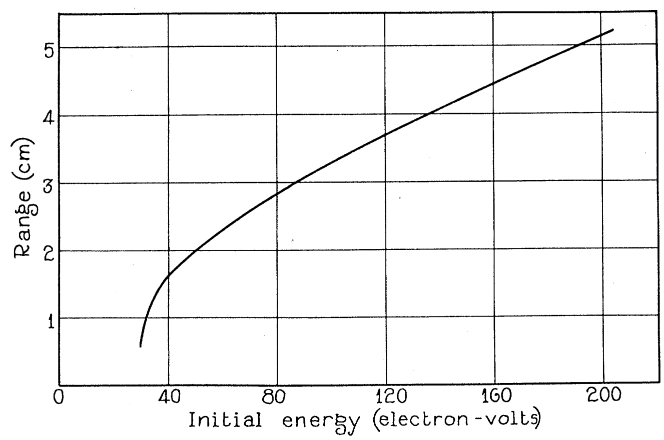

Fig. 3. Graph of Eq. (4) showing the ranges of electrons in helium at 1 mm pressure, as a function of their initial energy.

$E=190$ electron volts agrees well with the result obtained by Lehmann.[9] The curves given by Johnson are not definite enough to provide a comparison.

K. T. Compton and Van Voorhis have given results for many gases. The general trend of their curves is the same for all, including helium, and hence the conclusions arrived at in this paper are applicable, with suitable changes in the constants, to other gases.

Experiments are in progress in this laboratory to verify Eq. (1) for slow moving ionizing electrons, and to find their ranges by direct measurement.

[9] Lehmann, Ref. 2, p. 625, Fig. 1.