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THE PIERCE ACOUSTIC INTERFEROMETER AS AN INSTRUMENT FOR THE DETERMINATION OF VELOCITY AND ABSORPTION

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Abstract

Sputtered quartz crystals of known natural frequency were used as sources of high frequency sound in a Pierce acoustic interferometer. The interfering sound waves react on the crystal driver in such a way as to change periodically the plate current, I_p, when the mirror is displaced a half wave-length or more. The galvanometer-deflection, mirror-displacement curves show that multiple reflections take place at the movable mirror and at the crystal surface. The general agreement of these curves and those which represent the resultant pressure amplitude in the gas at the crystal surface as ordinates and the mirror displacement as abscissa shows that the changes in galvanometer deflection are a direct function of the changes in pressure amplitude. Check runs made with a torsion vane detector indicate that this function expresses a proportionality between the deflection change and the square of the change in the pressure amplitude, provided the latter change remains small.

Observed facts indicating a variation of wave velocity with intensity are: *1st*, the asymmetry of the deflection peaks; *2nd*, the change in the type of this asymmetry with increasing mirror displacement; *3rd*, non uniform spacing of deflection maxima; *4th*, sudden changes in the logarithmic decrement of the I_p-maxima. The precision in the value of the wave velocity is diminished considerably by this velocity variation.

Measurements are made on air and CO_2 at frequencies ranging from $3(10)^5$ to $14(10)^5$ cycles per sec. The apparent velocity obtained from the average spacing of I_p-maxima is slightly higher than the commonly accepted value for audible frequencies.

The absorption by air and by CO_2 increases with frequency through this range, CO_2 being nearly opaque at $14(10)^5$ vps. Lebedew's constant, A, has a value of 0.00037 for air at 20°C. The observed value obtained with the interferometer at $12(10)^5$ vps. was 0.00039 which agrees well with a torsion vane determination. At the lower frequencies the observed value is higher, partly because of the multiple reflections. The torsion vane arrangement used with CO_2 and with the highest frequency in air was not suitable for the lower frequencies in air. The observed value of A for CO_2 was 0.0073 at $12(10)^5$ and 0.012 at $3(10)^5$ vps. A careful torsion vane determination at $6.555(10)^5$ gave 0.0096. Abello's data at $6.12(10)^5$ extrapolated to 100 percent CO_2 give 0.0063. The humidity has a marked effect on the absorption in CO_2.

I. Introduction

The absorption of audible sound in a gas medium is very small. However it has long been known that the absorption increases with increasing frequency of the sound. In 1911 Lebedew[1] published a paper in which the absorption of high frequency sound in air was treated from a theoretical point of view. This article and one by Neklapajeff[2] on an experimental de-

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[1] Peter Lebedew, Ann. d. Physik **35**, 71 (1911).

[2] Neklapajeff, Ann. d. Physik **35**, 175 (1911).

termination of the absorption helped suggest that more accurate data on absorption might throw some light on molecular structure. In an unpublished investigation by G. E. Raburn it was discovered that CO_2 absorbs sound of frequency 100,000 to 300,000 per sec. to such an extent that it is almost opaque to it. This excessive absorption has since been verified by the author by two independent methods. G. W. Pierce[3] and T. P. Abello[4] refer to it also.

It is a familiar fact that the *velocity* of sound varies with the intensity. Such a change in velocity was observed and recorded by Regnault in connection with his experiments in 1863 on the velocity of sound in the newly laid Paris water pipes. Regnault states that the velocity decreases as the intensity diminishes but that a limiting velocity of 330.6 m per sec. for feeble sounds is soon reached. A. L. Foley[5] observed a similar decrease in velocity in 1920. H. O. Taylor[6] records a diminished wave-length in a highly absorbing tube. Hitchcock[7] reported that the velocity of the waves from a vibrating crystal is a function of the energy which the crystal radiates and he obtained a value in air nearly twice the velocity of ordinary sound waves. No other record of a variation in the velocity with intensity for a continuous low or high frequency sound has been found by the author. Both Regnault's and Foley's observations were on sound *pulses.*

Briefly stated, the problem is (a) that of testing the possibilities of the Pierce interferometer as an instrument for the determination of the absorption of high frequency sound in air and in CO_2 and (b) investigating the relations which the observations have to the velocity of sound at the frequencies used.

II. History of the Work on Absorption

Although considerable theoretical work had been done on the absorption of sound previous to 1911, that date may be considered as the beginning of the experimental work on the problem. In a preliminary report in 1924 on the author's[8] work, reference was made to Lebedew's theoretical value of A, the so-called absorption constant, which is defined by the equation

$$I = I_0 e^{-(A/\lambda^2)x} \tag{1}$$

where I is the sound intensity at the position x, measured from the source, I_0 is the intensity at the source and λ is the wave-length. The equation is written for plane waves. The theoretical value of A, given by

$$A = \frac{4\pi^2}{a\rho}\left[\frac{4}{3}\mu' + \left(\frac{C_p}{C_v} - 1\right)\frac{K}{C_p}\right] \tag{2}$$

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[3] G. W. Pierce, Proc. Amer. Acad. Arts and Sciences **60,** 271 (1925).

[4] T. P. Abello, Proc. Nat. Acad. Sci. **13,** 699 (1927); also Phys. Rev. **31,** 1083 (1928).

[5] A. L. Foley, Phys. Rev. **16,** 449 (1920).

[6] H. O. Taylor, Phys. Rev., Vol. 2, p. 270 (1913).

[7] R. C. Hitchcock, Proc. Inst. R.E. **15,** 907 (1927).

[8] D. L. Rich, University of Michigan and W. H. Pielemeier, Pennsylvania State College, "Absorption of High Frequency Sound," Phys. Rev. **25,** 117 (1925).

is approximately 0.00037 for air at ordinary room temperature. In this equation, a is the velocity of sound, ρ, is the density of the medium, $\mu' = \mu(1+0.146)$, where μ is the coefficient viscosity of the medium, C_p and C_v are the two specific heats and K is the coefficient of thermal conductivity. The value of A should increase about 1 percent for each 5°C rise in temperature.

From Equation (1)

$$A = \left(\log \frac{I_1}{I_2}\right) \lambda^2 \Big/ (x_2 - x_1). \tag{3}$$

Neklapajeff,[2] under Lebedew's direction, attempted to check the value of A, experimentally. The mean of all his values was approximately twice the theoretical (0.00073 as against 0.00037). Considering, however, the crude apparatus used, the work was remarkably good.

Work on the problem was first taken up at the University of Michigan Physical Laboratory in 1917 by D. L. Rich and G. E. Raburn. Echelette gratings were used. The excessive absorption of CO_2 to which reference has already been made was observed with this form of apparatus. Pressure vanes were used as detectors and the source was an electric spark which was placed at the principal focus of a movable mirror. The source spark was in series with a rotary gap. The set differed from that of Neklapajeff chiefly in the type of grating and the electrical circuit containing the source spark gap. When the author began work on the problem the difficulty was that of finding a suitable source. Several types of electric spark, a Poulson arc, a direct current with superimposed a.c. of proper frequency, a Hewlett tone generator, and a glow microphone were tried and discarded. The glow microphone which was kindly loaned by the Westinghouse Electric and Manufacturing Company, had some excellent features, such as maximum frequency possibilities and convenience of adjustment in the collimating system but the intensity was too small and unsteady for absorption measurements.

The best source used previous to the present one was a piezo-electric tone generator constructed of two 6×6 inch aluminum plates which were driven by small squares of quartz cemented between the aluminum plates with molten shellac. Langevin[9] and others used similar oscillators under water for submarine detection, etc., during the recent war. The results obtained with this source were presented in a report[8] at the November meeting of the American Physical Society. This source was very limited as to the frequencies which could be obtained with it. Moreover it behaved as if certain portions of the oscillator had a slightly different resonance frequency than others. This might have been expected because each quartz square would differ

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[9] The experiments along this line made by the Columbia University Group (Professors M. I. Pupin, A. P. Wills and J. H. Morecroft) in 1918, have never been published. The experiments of Langevin and Chilowsky are given in *Nature* May 9, 1925, pp. 698–90, in an article on "Echo Sounding." A more complete description of Langevin's apparatus is given in "Ultra Sonic Waves for Echo Sounding," Hydrographic Review, (Monaco) II, No. I, 1924, p. 57. A brief account of such apparatus is given by Crandall, p. 142 (See reference 11).

slightly in natural frequency from all the rest. Those portions of the entire oscillator for which the natural frequency was below that of the driver would be exactly out of phase with the parts for which the natural frequency was above that of the driver. This would result in unequal distribution of sound energy in the beam.

The determinations of A with this source lay between Lebedew's theoretical value and Neklapajeff's experimental value.

III. Experimental Arrangement

The apparatus used in obtaining the data presented in this paper is a modification of that used by Pierce.[3] It consists of a piezo-electric oscillator connected into a thermionic vacuum tube circuit. The oscillator is a quartz plate sputtered on both faces with platinum. The orientation of the axes in natural quartz crystal and the method of sectioning for an oscillator is shown by Pierce[3] and also by Cady.[10]

Most of the crystals were cut by A. Espositor, 64 W. 48th Street, New York. One was loaned by the Alfred Loomis Laboratory. They were sputtered by Dr. Shohat, Department of Physics, University of Michigan. In operation one of the prepared crystals is placed on a horizontal brass base so that it radiates a parallel beam of sound vertically upward. The brass base is attached to the lower end of the ways of a micrometer screw. It is fitted with leveling screws. A reflecting mirror is operated by the screw. The mirror can be displaced upward or downward in the direction of the sound beam. It is a piece of plate glass a trifle larger than the largest crystal used. It is fitted with three leveling screws. It is always oriented with its face parallel to that of the crystal. Its position can be read on the screw scales. The whole assembly, which Crandall[11] calls an acoustic interferometer, is mounted against the back wall of a cast iron box fitted with leveling screws and two glass windows.

The electric circuit for producing vibrations. This circuit is approximately the same as that used by Pierce.[3] It is one for which the frequency of oscillation is determined by the crystal and not by the electrical constants of the circuit. It consists of a 135-volt B-Battery (see Fig. 2) in series with a load coil of large inductance, a UX-201-A thermionic tube, a lamp and scale type galvanometer with adjustable shunts, and a group consisting of a telephone and a small fixed condenser in parallel. In most cases the crystal is connected between the plate and the grid of the tube. Usually no grid leak was used. Connected to the galvanometer and its shunts is a bias battery and a high variable resistance. This makes it possible to reduce the current through the galvanometer to approximately zero. Then the sensitivity of the galvanometer is increased to the desired value by selecting the proper shunt. By merely operating the rotating shunt switch the sensitivity can be changed by factors of ten from approximately 10^{-7} to 10^{-4} amperes per mm deflection.

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[10] G. W. Cady, Proc. Inst. Radio Eng. **10,** 83 (1922).

[11] Irving B. Crandall, Theory of Vibrating Systems and Sound, p. 244.

The load coil is mounted so that it can be coupled with a wave meter. The frequency of each crystal was also accurately determined by Dr. N. H. Williams.

A final check was made on some of the absorption measurements with a very sensitive torsion vane, which was substituted for the mirror, no other change in the apparatus being involved.

Procedure. The position, x_0, of the mirror is read when in contact with a crystal. Then the mirror is raised and the contact wire is placed on the sputtered surface of the crystal. With most of the crystals I_p falls from 5 or 6 ma. to 1 or 2 ma. when the oscillations begin. The bias circuit is then closed and the galvanometer deflection is brought to nearly zero. Then the sensitivity of the galvanometer is increased and the two currents are again adjusted for nearly zero deflection. This is repeated until the desired sensitivity is obtained. Now the mirror is displaced slowly and the deflection changes periodically. (See Figs. 2–5). At a sharp maximum the system is quite sensitive to about 1/5 of a least division on the screw head (1/200 mm). The temperature is recorded every 5 or 10 minutes. Usually the wave meter is loosely coupled to the load coil and the reading is taken so that the frequency can be computed. This reading is indispensible for those crystals which have two neighboring natural frequencies. With one of the crystals a tuned circuit had to be used in order to suppress the undesired frequency. If the galvanometer current is steady the peak and trough deflections are plotted against the number of half wave-lengths between the crystal and the mirror. A smooth curve is drawn through the peak values and another through the

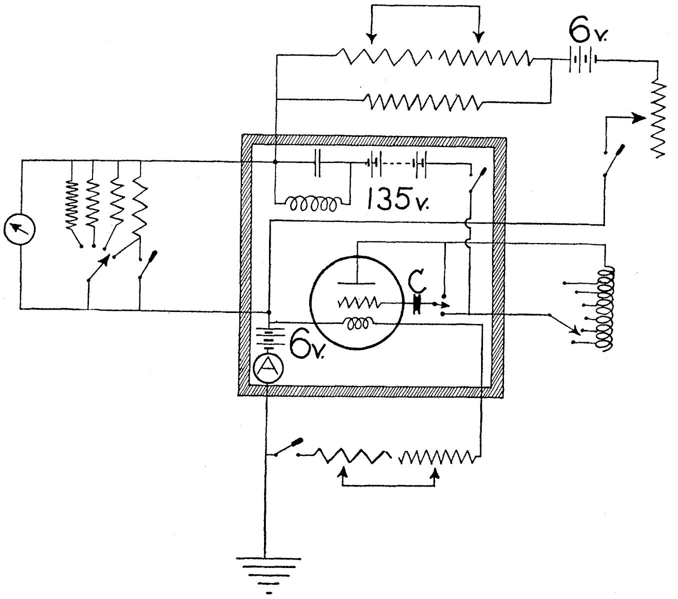

Fig. 1. Circuit diagram.

trough values. A third curve is drawn of which the ordinates are the logarithms of the vertical separation of the first two curves. This separation, α, is called the I_p amplitude. If the set is not running steadily enough for this method conditions are sometimes good enough to obtain reliable I_p amplitudes in spite of an occasional shift. In such a case the I_p amplitudes are plotted as ordinates directly and the logarithms of these ordinates are plotted for a second curve. In either case the change in logarithm per half cm of mirror displacement is computed from the curve. Assuming a single round trip is made by the sound waves from crystal to mirror and back again this would be equivalent to the decrease in logarithm per centimeter of sound path, which would be equal to the factor A/λ^2 in the exponent of e in Eq. (1), provided the decrease in α is proportional to the absorbed sound. This method of calculating the value of A is similar to that used by Neklapajeff[2] (see Eq. 3). The value of A, found in this manner, is much larger for small mirror displacements or for the lower frequencies than the value of A found by Neklapajeff.[2] The reason for this will be discussed in Part V. However, the higher frequency crystals give approximately the theoretical value (0.00037) for all except the smaller mirror displacements.

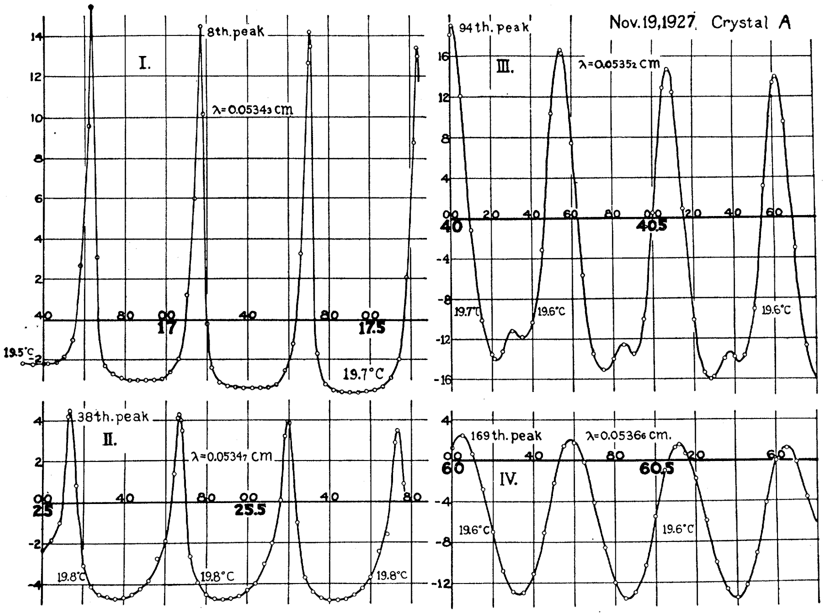

Fig. 2. Wave-form curves for crystal A in air. Galvanometer deflections in cm are plotted as a function of mirror position in mm and 200ths mm.

Both with air and with CO_2 some runs were made with the gas having been dried either by P_2O_5 or by passing it through H_2SO_4. The cast iron box which had a coat of shellac on the inside was washed with commercial CO_2 for about 10 or 15 minutes before the stop-cocks were closed. Then the set was allowed to run for some time before readings were taken so that it might become steady. A few runs were made with air which had the CO_2 removed from it. The wave-form runs were obtained in the same general manner as the absorption runs except that the deflection was read for many points in a single cycle, ($\lambda/2$), and only a few wave-lengths were covered in this way.

IV. Data and Curves

The data herein presented consist chiefly of two general types. One type, which will be referred to as *stationary wave-form data* or merely as *wave-form data*, gives the galvanometer deflection for each setting of the reflecting mirror, the distance between settings being small fractions of a half wave-length of the reflected sound. The chief value of this type is that it throws light on the kind of vibrations which the crystal is executing and the manner in which the components of the stationary waves produce their resultant at the face of the crystal.

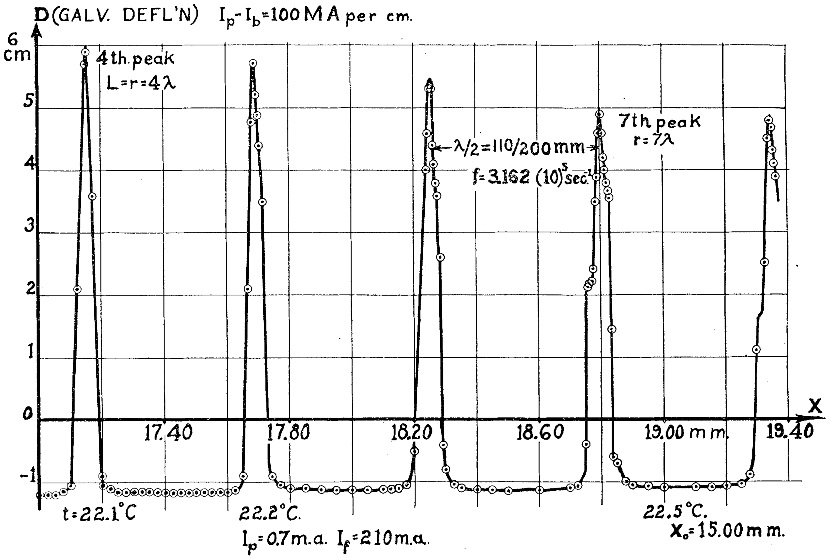

Fig. 3. Wave-form curve for crystal *D* in air.

The other type of data which will be termed *velocity and absorption data*, or sometimes merely absorption data, gives merely the maximum and minimum galvanometer deflections and the corresponding mirror positions. This type is used in calculating the approximate velocity and absorption of the sound in the space between the crystal and the mirror. With each crystal the position of the mirror was observed when in contact with the crystal survace. Henceforth, x_0 will be used to designate this position. Great care was exercised in orienting the two so they were normal to the *path* of the mirror (or screw axis) and also parallel to each other. With every crystal it was found that a maximum galvanometer deflection fell at x_0 if the curve was extrapolated back to this point.

The torsion vane data are presented in the form of a curve. (See Fig. 9).

With the frequencies used and with the short sound paths for which observations were taken, diffraction effects[12] were negligible for one return trip from source to mirror and back. (The length of this return trip will be designated by L.) For the shorter paths the same is true for *several* return trips. In other words, the wave fronts for these cases remain confined approximately to the geometrical beam. It is evident that stationary sound waves will exist between the source and the mirror for certain mirror positions one half wave-length apart. For these positions the intervening gas acts as a resonator. Since the mirror positions corresponding to the observed galvanometer deflection *maxima* and the mirror positions for which one would expect stationary waves are identical it is also evident that a deflection maximum occurs when a minimum displacement amplitude (a node in the stationary wave pattern) falls at the crystal surface, that is, when the gas column resonates. A maximum *pressure* amplitude corresponds to a displacement node in a stationary longitudinal wave system. Then the d.c. component of the plate current, I_p, is a maximum when a maximum pressure amplitude exists at the crystal surface. This and other considerations brought out in Part V imply that the changes in I_p are a simple direct function of the changes

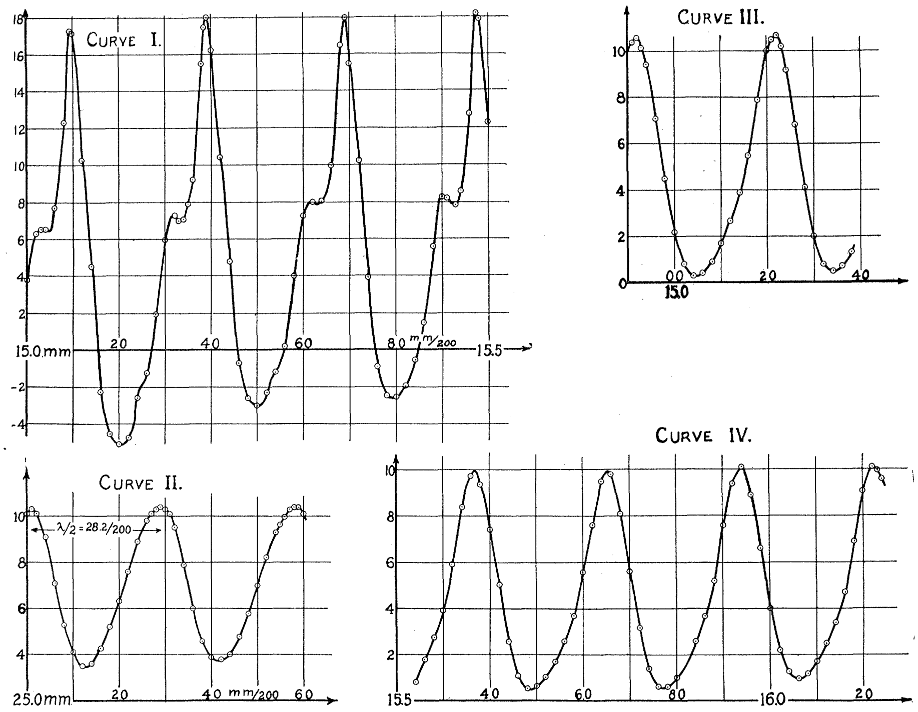

Fig. 4. Wave-form curve for crystal E in air.

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[12] See p. 138 of Reference 11.

in pressure amplitude in the resultant wave at the crystal surface as the mirror takes different positions.

Observations show that the wave-meter current fluctuates also when the mirror is moved. The maxima correspond to the minima of I_p.

Table I summarizes the results from the given data and also from data not included in this article in any other form.

In Figures 2–5 galvanometer deflections are shown as functions of mirror position for the conditions indicated in the figure captions. The galvanometer deflections in Figs. 2–8 give the excess of the d.c. component of the plate current, I_p, over the bias current, I_b, namely $(I_p - I_b)$.

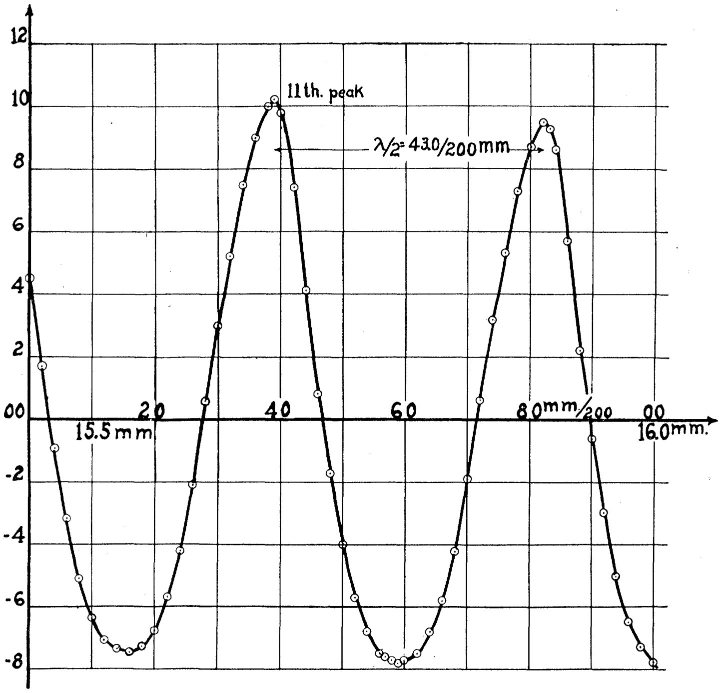

Fig. 5. Wave-form curve for crystal A in CO_2.

For Fig. 2 the crystal frequency, f, is 6.555 $(10)^5$ cycles per sec, $x_0 =$ 14.95 mm, $I_p = 2.2$ ma., $I_f = 220$ ma., $E_p = 135$V. The ordinates are in cm; the unit for the heavy numbers along the x-axis is the mm; the light numbers (screw head readings) give the number of 200th mm. $(I_p - I_b)/D = 10$ micro-amperes per cm (μA/cm) for Curves III and IV. There is a close similarity between Figs. 2–5 and Fig. 10. However the effect of the decreasing velocity is noticeable in the former. In Curve I the rise to a maximum is more gentle than the drop to a minimum. This asymmetry is much greater in the peaks of Curve II. Along the left slopes of the 2nd and 4th peaks of this curve there appears to be a slight tendency toward a hump. On account of the diminished velocity this hump lags more and more behind until it

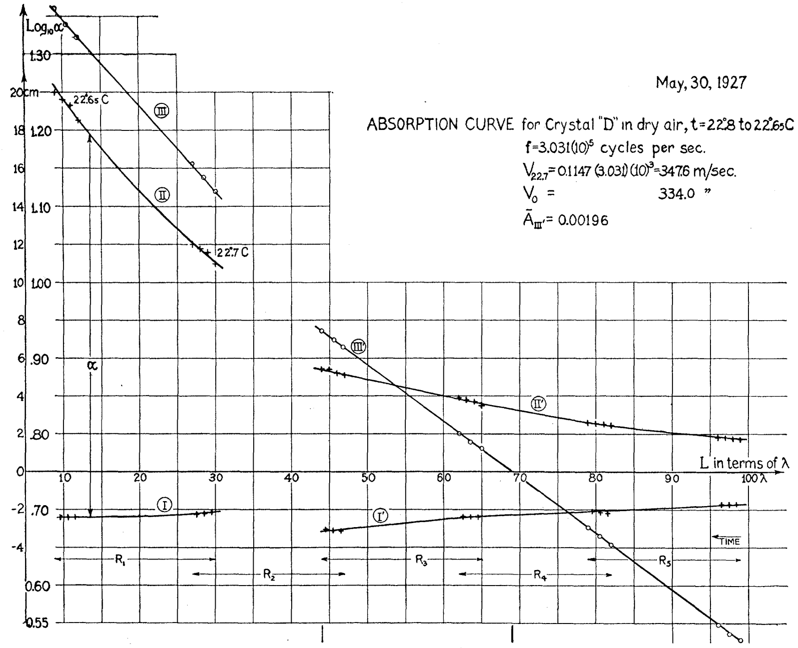

Fig. 6. Absorption curve for crystal *D* in air.

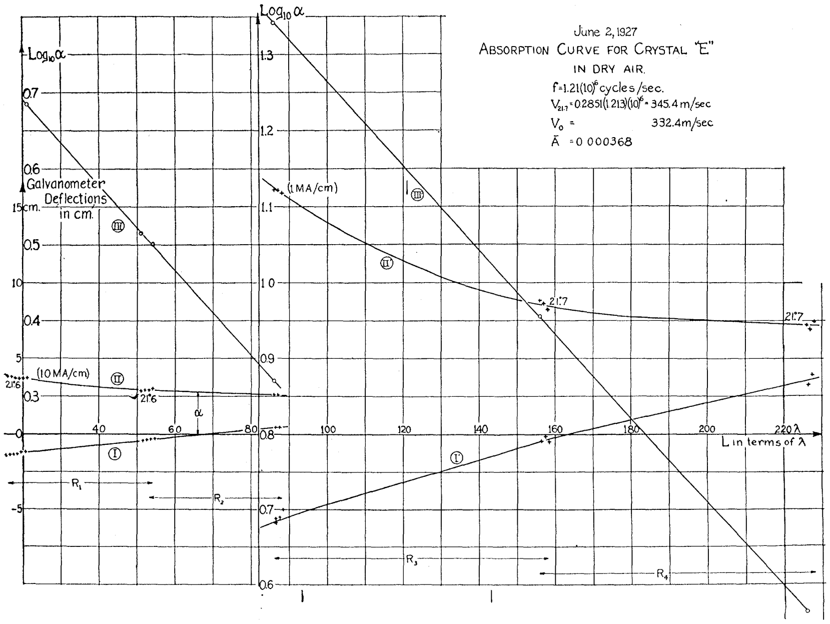

Fig. 7. Absorption curve for crystal *E* in air.

falls near the trough as shown in Curve III. In Curve IV this hump has apparently fallen behind by nearly a cycle. The data for these curves were taken as rapidly as possible and without any interruption of the oscillations. The average plate current slowly diminished during the run.

Fig. 3 resembles curve I of Fig. 10 even more than Fig. 2 does. This is due to the smaller absorption in connection with the longer waves emitted by crystal *D*.

For Fig. 4, $f = 1.125(10)^6$; for Curves I and II $x_0 = 11.85$ mm and $t = 21.7$°C. Approximately the same values of x_0 and t hold for Curves III and IV. In each $(I_p - I_b)/D = 1\mu A/\text{cm}$. The ordinates of Curves III and IV vary approximately as the pressure amplitude varies when merely *one* reflection at the crystal takes place. With more intense oscillations, as for Curves I and II, the mirror must be farther removed to obtain such results. See Part V for a discussion of the secondary peaks such as those in Curve I.

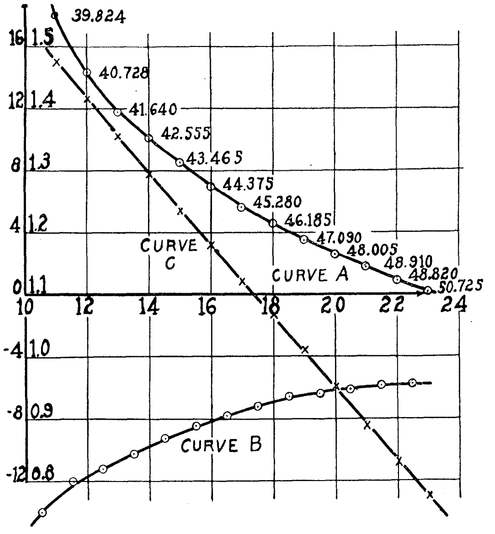

Fig. 8. Absorption curve for crystal *D* in CO_2 (Mirror positions are recorded along curve *A*).

For Fig. 5, $x_0 = 13.34$ mm, $t = 21.1$°C, $I_p - I_b/D = 1\mu A/\text{cm}$. This approximates a simple harmonic curve although the mirror is less than 2.5 mm from the crystal. CO_2 absorbs the sound at $6(10)^5$ more than air does at $12(10)^5$ cps. This curve indicates that the higher order reflections are nearly negligible.

Peak and trough values of *D*, the galvanometer deflection, are plotted as functions of *L* in Figs. 6–8, where *L* is the path-length for the sound waves which have made one return trip, i.e. $L = 2(x - x_0)$. In these figures *L* is given in terms of λ. The curve for $\log \alpha$ is included also. The latter is of constant slope for the entire range in Figs. 7 and 8. This indicates that the sound intensity is very much reduced in the first return trip.

In Fig. 6 the troughs are on Curves I and I′, the peaks are on II and II′ and log α points are on III and III′ where α is the vertical separation of I and II or of I′ and II′. $(I_p - I_b)/D = 10\mu A/\text{cm}$. The break between the two sets of curves is probably due to the smaller wave velocity for the higher order return trips. (See Part V) The mirror was moved toward the crystal for these data. A check run was made before the set was stopped. In this the mirror was displaced in the opposite direction. The break came at the same place in both runs. There was also a change in the *spacing* of the maxima near $L = 30\lambda$ for both runs. Since this change is small it was determined by taking averages. The ranges R_1, R_2, etc., in the figure each furnish four spaces for which the end points are determined independently of those for the other three spaces in the same range. Each of the four spaces has the same number of half wave-lengths (17 for some ranges and 18 for the others). Since the mirror position is recorded for each peak the average wave-length, $\bar{\lambda}$, can be computed for each range. For R_1 $\bar{\lambda} = 0.1142$ cm; for each of the other ranges $\bar{\lambda} = 0.1147$ cm. The only definite change is that of approximately 1/2 percent from R_1 to R_2. Curve III represents a change of 0.408 in $\log_{10}\alpha$ and a change of 6.309 cm in L whence c, the change per cm is 0.0647. This gives 0.00196 as the value of A, where $A = \lambda^2 C \log_e 10$. This is approximtely 5 times the theoretical value. Multiple reflections, even at $L = 100\lambda$ probably cause part of the large logarithmic decrement.

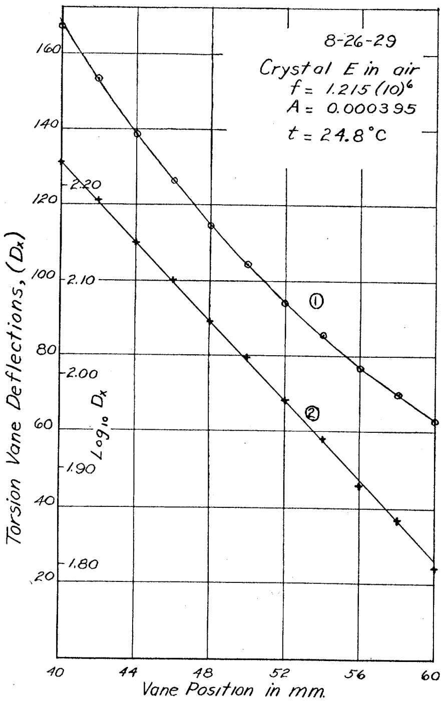

Fig. 9. Absorption curve for crystal E in air by the radiometer method. (D_x in mm) Curve 1 is a D_x curve; curve 2 is a log D_x curve.

In Fig. 7 the break between Curves I, II, III and I′, II′, III′ is caused by a tenfold increase in galvanometer sensitivity. There is no change in the slope of the log α curves as there was in Fig. 6. Moreover the value of A from Fig. 7 agrees well with the theoretical value. (0.000368 as against 0.00037) The absence of any break in the slope is ascribed to the fact that the intensity is small and no great range of wave velocity exists. The enormous absorption makes the effect of the higher order reflections small even though there may be some phase lag. The value of $\bar{\lambda}$ for each range is as follows: For R_1, 0.02843 cm; for R_2, 0.02840 cm; for R_3, 0.02848 cm; for R_4, 0.02851 cm.

For Fig. 8 the value of A is 0.0118. This is about 42 times the value obtained from Lebedew's equation. A check run was made with a radiometer and the value of A for 93 percent CO_2, not dried, was 0.0161, therefore multiple reflections do not seem to be responsible for the excess over the theoretical value. It was found that the presence of water vapor increases the absorption, however. The wave velocity falls approximately on the extrapolation of Pierce's curve for velocity of sound in CO_2.

TABLE I. *Average values of V_0 and A.*

Crystal	Frequency	Gas	Apparent V_0	A
F	$1.408(10)^6$	air	333.3	0.00044
F	$1.408(10)^6$	CO_2		0.0014
E	$1.215(10)^6$	air	333.3	0.00039
E	$1.215(10)^6$	CO_2	265.5	0.0073
G	$1.167(10)^6$	air	332.8	0.00043
H	$1.158(10)^6$	air	334.5	0.00039
A	$6.555(10)^5$	air	333.8	0.00050
A	$6.555(10)^5$	CO_2	269.9	0.0058
C	$3.893(10)^5$	air	333.7	0.00053
C	$3.893(10)^5$	CO_2	270.	0.0064
D	$3.031(10)^5$	air	333.7	0.0014
		CO_2	265.	0.012

Table I gives the average value of V_0 and of A for each of the cases listed. Probably all these values of V_0, the velocity at 0°C, are slightly too high on account of the slightly large spacing of the I_p maxima. The velocity is a function of intensity, therefore the effect is small for CO_2. However there was probably a trace of air present with the CO_2. This would tend to make V_0 too large and A too small. The presence of water vapor with the CO_2 increases the absorption. Further data are being taken in this connection. Water vapor with *air* has little effect on the value of A. Multiple reflections, however, cause an apparently large absorption in air. With the higher frequencies this effect is slight as is indicated by the close check with the radiometer. A large number of runs were made with each crystal; for example, the tabulation for crystal E in air represents 14 independent runs in which the maximum variation from the mean V_0 was 0.3 m/sec and from the mean A it was 0.00002. The author's reliance in the values of A for air is greatest for the following crystals; E, F, G, H. The straight log α curves for CO_2 by the mirror and the radiometer methods indicate that A is exact for the particular amount of water vapor present but the uncertainty in this factor makes the CO_2 values less reliable.

V. Discussion of Results

In Part IV some evidence was given that the periodic changes in the plate current, I_p, are a simple and direct function of the changes in pressure amplitude in the gas at the surface of the crystal as the mirror is moved. This pressure amplitude and I_p reach their maximum values for the *same* position of the mirror. They are not only both periodic in space or displacement but both have the same space period, $\lambda/2$, and they are in phase. Let us now consider more in detail how the pressure amplitude should vary at the surface of the crystal as the mirror is moved. This type of variation should not be confused with the point-to-point variation along the resonating gas column between the crystal and the fixed mirror. From graphical analysis it was found that in the case of a system of undamped waves, produced by a plane source and a single reflection at a moving reflector which remains parallel to the source, the pressure amplitude at the source, resulting from the two oppositely travelling waves, goes through the same changes as those represented by the point-to-point variation along this wave system as it would exist if the reflector were to remain stationary at a resonance position. It is as if an imaginary plane, (parallel to the wave fronts) were to move through the stationary wave pattern. The same applies to displacement and velocity amplitudes. For damped waves none of these statements would hold. H. O. Taylor[6] has made excellent observations on the point-to-point variation of amplitude in *audible* stationary sound waves. The results to be expected from the theoretical considerations can then be compared with the observed details of the I_p-variation.

Let us investigate, from theoretical considerations, what the pressure amplitude at the crystal should be for several representative mirror positions. Let us assume that the emitted waves represent a *simple harmonic variation in pressure* (that p_0, the excess pressure due to the emitted wave at the crystal is a simple harmonic function of the time). For the sake of simplicity let us also assume, at first, that V, the wave velocity, is constant (independent of the sound intensity) and that the reflection is perfect at the mirror and also at the crystal. Let us first consider a *resonance* position of the mirror. For this position when an equilibrium condition has been reached the resultant pressure at the crystal at the instant when a condensation is being emitted will be the sum of a large number of condensations or pressure maxima, each of which is the result of reflection. (See Fig. 11) These components of the resultant pressure form a sequence of terms of decreasing magnitude and their sum is a series which would be infinite except for diffraction effects. Let p_R be the *resultant* instantaneous excess above normal pressure at the crystal. Let p_n be the instantaneous excess above normal pressure at the crystal, due to the nth reflection. Let P_R and P_n be the corresponding maxima or condensations, which would also equal the pressure amplitude.[13] Then $p_n = P_n \cos(\omega t - \epsilon)$ where $(\omega t - \epsilon)$ is the phase angle.

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[13] K. S. Johnson and others use p to represent the r.m.s. value of the pressure variation. See K. S. Johnson, Transmission Circuits for Telephone Communication.

At a resonance position all the condensations, P_n, return to and are being reflected by the source at the instant it is emitting a condensation. For this instant

$$p_R = P_R = P_0 + P_1 + P_2 + \cdots + P_n = \sum_0^n P_n \qquad (4)$$

For *any* position of the mirror Eq. (4) is of the form

$$y = \sum_{i=1}^{i=n} a_i \cos\left(\frac{2\pi}{T}t + \epsilon_i\right) = a \cos\left(\frac{2\pi}{T}t + \epsilon\right). \qquad (5)$$

If $y = p_R$; $a_i = P_0 c^{nL} = p_n$, $a = P_R$, $\omega = 2\pi f$, $t =$ time elapsed since a condensation in the n^{th} reflected wave was in phase with an emitted condensation at the crystal, $\epsilon_i = -\omega nL/V$, $P_0 =$ the constant amplitude in the emitted wave; $c =$ a constant, less than one, expressing the decrement; $V =$ assumed constant wave velocity, and $L =$ length of one return trip of the waves, then Eq. (5) is adapted to the quantities under discussion and takes the form

$$p_R = \sum_0^n P_0 c^{nL} \cos \omega(t - nL/V) = P_R \cos(\omega t + \epsilon) \qquad (6)$$

where

$$P_R{}^2 = \left[\sum_0^n P_0\, c^{nL} \cos(-\omega nL/V)\right]^2 + \left[\sum_0^n P_0\, c^{nL} \sin(-\omega nL/V)\right]^2. \qquad (7)$$

Since it is pressure *amplitudes* which were to be investigated Eq. (7) offers the solution. As equation (6) stands it involves four variables, i.e. $p_R = \Phi(n, Lt)$. By fixing L this may be represented graphically in three dimensions. (See Fig. 11) For $L = r$ a reasonance length, Eq. (7) becomes

$$P_R = \sum_0^n P_0\, c^{nL} = P_0/(1 - c^{nL}) = P_0\,(1 - c)^{-nL}$$

In order to plot a $P_R{}^2 - L$ curve or a $P_R{}^2$-mirror position curve it will be necessary to evaluate $P_R{}^2$ in terms of $P_0{}^2$ at a number of points between two successive r-values. The values of $P_R{}^2$ and P_R for two representative values of c and several values of r are given below.

1st. $c = 0.9$; $r = 1$cm. (This value of c is approximately correct for crystal "A" provided Lebedew's theoretical value of the absorption constant holds for this frequency.)

For $L = r$	$P_R{}^2 = 100.\ P_0{}^2$;	$P_R = 10.0\ P_0$
For $L = r + \lambda/8$	$P_R{}^2 = 1.86\ P_0{}^2$;	$P_R = 1.36\ P_0$
For $L = r + \lambda/4$	$P_R{}^2 = 0.552\ P_0{}^2$;	$P_R = 0.743\ P_0$
For $L = r + \lambda/2$	$P_R{}^2 = 0.277\ P_0{}^2$;	$P_R = 0.526\ P_0$
For $L = r + 3\lambda/4$	$P_R{}^2 = 0.552\ P_0{}^2$;	$P_R = 0.743\ P_0$
For $L = r + 7\lambda/8$	$P_R{}^2 = 1.86 P_0{}^2$;	$P_R = 1.36\ P_0$
For $L = r + \lambda$	$P_R{}^2 = 100.\ P_0{}^2$;	$P_R = 10.0 P_0$

In all except the first of these values of $P_R{}^2$ and P_R the slight increase in L, the exponent of c, is neglected. The only set of values appreciably affected thereby is the last set (for $L = r + \lambda$). The value of λ for crystal

"A" in air is very nearly 1/2 mm consequently the exponent of c should be increased by 5 percent for $L=r+\lambda$. With this correction $P_R^2=91.3P_0^2$ and $P_R=9.56P_0$.

It is thus seen that, except for the damping of P_R, due to the extra distance, λ, the values of P_R are periodic in space. The values of the period is λ in terms of the length of the first return trip. In terms of mirror displacement it is $\lambda/2$, which agrees with the period of I_p. Moreover it was found by extrapolating the very short distance back to the crystal position that, in every case, an I_p- maximum should fall at this point. The same applies to the P_R- maxima, as will be seen by setting L in equation (7) equal to zero. The variations in P_R and I_p are therefore in phase.

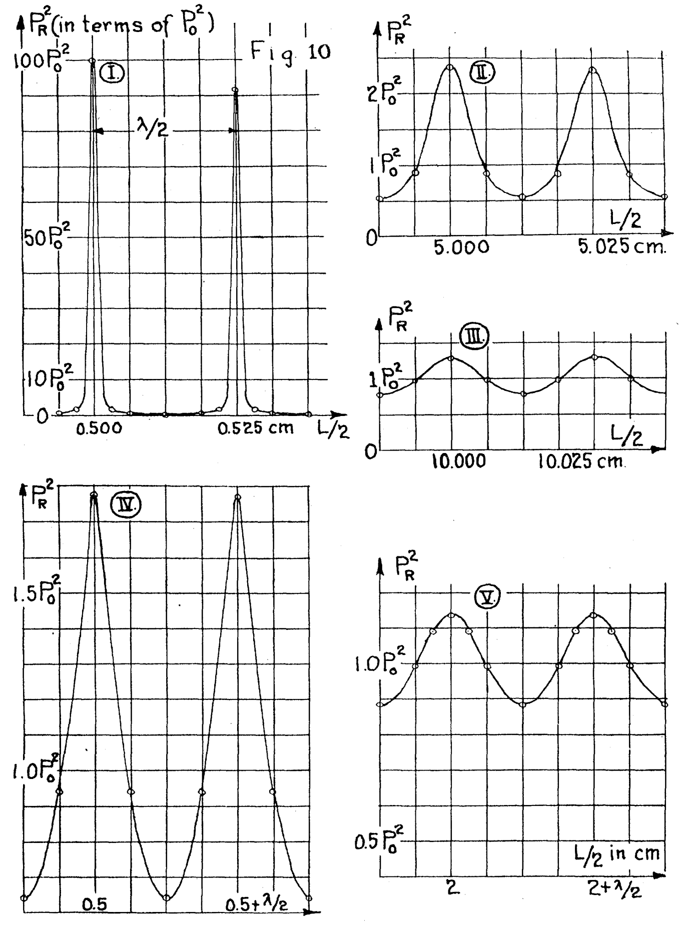

Fig. 10. Calculated pressure amplitude curves. Compare with Fig. 3–5.

The above values of P_R^2 are plotted as a function of $L/2$ in Fig. 10, Curve I. For $c=0.9$ and $r=10$ cm the corresponding values of P_R^2 are plotted to a finer scale in Curve II. Curve III is for $c=0.9$ and $r=20$ cm. These curves resemble the observed galvanometer-deflection, mirror-position curves for Crystal "*A*". (See Fig. 2). The difference between Curve I and the corresponding observed curve (Curve I) is probably largely due to the variable velocity of the sound waves.

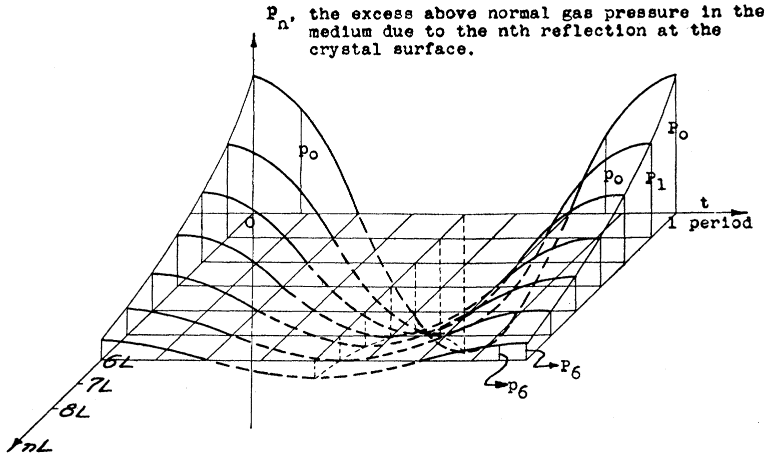

Fig. 11. Pressure time curves for successive reflections at the crystal.

If c, the fraction of P_0 which remains after 1 cm of path has been traversed, is chosen as 0.5, curves IV and V, Fig. 10, show the variation in $P_R{}^2$ near $L=1$ cm and $L=4$ cm. Compare these with the wave form curves Fig. 4.

Fig. 12 shows the positions of the maxima when projected on a plane perpendicular to the pressure axis. (See Fig. 11). The wave velocity is assumed to fall off with intensity approximately as shown by Foley's[5] curves. Such a velocity distribution places the maxima along a curved line instead of a straight one. Thus *no* position of the mirror exists for which all the maxima reach the mirror simultaneously. Consequently, the peaks corresponding to those in Fig. 10 will be somewhat lower and the entire

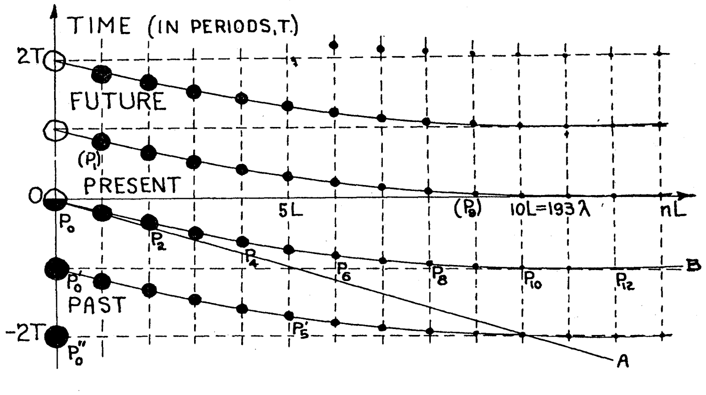

Fig. 12. Time and path length diagram for wave velocity decreasing with intensity.

curve will be modified. Moreover the extent of this modification will depend partly on L. As L increases slightly the curved line joining the maxima must rotate counter clockwise about point, P_0, through a corresponding angle. For some definite angle and time the sum of the ordinates along a line parallel to the nL axis will reach a maximum which is less than the amplitude would be if the velocity had remained constant. The higher order reflections shown farther along the curved line are brought into phase with P_0 at a slightly smaller mirror displacement hence the peaks should have a more gentle slope on the right than on the left side. For certain cases even a minor peak may be developed. Fig. 12 shows such a case. In this figure 100 percent reflection is assumed. When a higher order reflection is retarded a half period the reflection is probably far from perfect but as the retardation increases reflection improves again. Thus the curved line may have nearly straight segments and the minor peaks may be magnified. However p_R will still be a simple harmonic function of the time if each of the components, p_n remains simple harmonic.

Fig. 12 shows the location of some of the pressure maxima at the crystal, in a *time* and *path length* diagram. The size of the dots is intended to represent the magnitude of the pressure maxima. P_1, P_2, P_3 etc. have already been reflected. They were emitted as P_0 's 20, 40, 60 etc. periods ago, respectively, and they would lie along the straight line P_0A if the wave velocity, V, were constant. This line would rotate counterclockwise about P_0 as shown by curved arrow if L increases. When $L=20\lambda$ it would coincide with the nL axis. The passage of *time* may be represented by moving the boundary line between past and future in the direction of the time axis (toward the future). Since $L=19.2\lambda$, P_1 was *reflected* approximately 1/5 period ago. However, P_3 was not reflected 3 times this long ago. On account of the decreasing velocity it has lagged behind so that it has been reflected merely 1/2 period ago.

The minor peak caused by (P_9) and following P's would fall approximately 1/5 space to the left of a major peak if it were represented in Fig. 10. If L were doubled this lag would become about 1/3 space. Thus the system of minor peaks moves back through the system of major peaks as L increases. This process probably helps cause the abrupt changes in the slope of the log α curves.

Possibility of other causes for such unexpected results as those shown in the wave form curves (Fig. 2–5) were investigated but no other adequate cause was found. A lowering of the crystal's frequency of oscillation due to the reaction of the resonating gas column on the crystal, does not explain the observed facts. To test whether the type of crystal mounting was responsible metal blocks of different thickness were successively placed under the same crystal. Several crystals were tested this way. It was found that the peculiarities of the wave-forms still persisted. Harmonics were also suspected. A sensitive wave meter was used to search for them but none were discovered. Finally the calibration of the screw was checked with a set of precision guages but it was found that the irregularities in the screw were much less than those in the I_p spacing. Thus far the evidence points to a variable sound velocity as a cause for the observed peculiarities.

It is assumed that the logarithmic decrement of α may be used in computing A, in case the reflections of higher order than the first may be neglected This implies that α is proportional to that $(\Delta P_R)^2$ which corresponds to a mirror displacement of $\lambda/4$ from a resonance position. Such an assumption requires an experimental justification, hence a torsion vane determination of of A was made. Crystal E was used first because most confidence was placed in its value of A as determined with the interferometer. The data for one of the runs are given in the form of a curve (Fig. 9).

With the lower frequencies in air the deflections fall off so little with increasing mirror displacement that the results are not very decisive. With different percentages of CO_2 the logarithmic decrement was increased and it was found to be very uniform in any one run. It was found, however, that the humidity modifies the absorption enormously at these frequencies. Further measurements on this effect are being made.

Another crystal (Crystal F) was procured and tested. Its frequency is 1.408 $(10)^6$cps. The vane response was large enough to determine the value of A for air to two significant figures. (See Table I). The interferometer response was too small to obtain the same precision ($A = 0.0004$). With a mixture of 56 percent CO_2 in air, partly dried, the vane method gave $A = 0.0014$.

All the measurements on CO_2 with the interferometer and also with the pressure vane gave points falling very close to a straight log α curve but the slope of this curve was found to depend very much on the moisture content.

The wave velocity is not strictly constant as was assumed for Fig. 10. This modifies the wave form (as outlined above) and the log α curve, instead of approaching a constant slope *gradually*, may do so through comparatively straight sections, the last of which gives the correct value of A. Not much loss of energy takes place for those reflections in which the wave trains leave the crystal in phase with the emitted wave train. The higher order reflections involve considerable phase lag and loss of energy. As the phase lag approaches a complete cycle the reflection takes place with less loss of energy again. The energy losses also involve a further reduction in wave velocity. The effect of phase relationship on the reflecting power may cause the curved line in Fig. 12 to have more or less abrupt changes in direction. It may be seen from the figure that the rotation of such a broken line about the point, P_0, would result in a different wave form and a different value of P_R^2. A more or less abrupt change in the apparent value of A with increasing value of L may be expected when a minor peak is receding through a trough of the main wave-form. (See Curve III, Fig. 2).

The several runs made with the torsion vane detector of the progressive waves emitted by crystal "E" resulted in values of A that agreed well with each other and with the values found by the interferometer method. This agreement is additional evidence that the interferometer method is satisfactory for absorption measurements under certain circumstances. Moreover the latter method has the advantages of furnishing wave-length data also. Herzfeld and Rice[14] point out the need of such a method.

CONCLUSIONS

1. The galvanometer-deflection, mirror-displacement curve obtained with an acoustic interferometer is an index of the variation of the resultant pressure amplitude, P_R, with increasing mirror displacement, $(x-x_0)$.

2. For air as the transmitting medium the logarithmic decrement of the I_p-amplitude is not suitable for the determination of A, the absorption constant, at small values of L unless the frequency is of the order of 10^6 cycles per sec. or more. For a frequency of $3(10)^5$, L must be more than 10 cm.

3. The spacing of the I_p maxima indicates that the velocity of high frequency sound varies with intensity. The precision which G. W. Pierce[3] claims for the velocity has no significance for waves which suffer a decrease in velocity as their intensity diminishes. The best that can be done is to speak of the average velocity over a given path-length. Since Pierce[3] used the edges of the crystals as sound radiating surfaces in many of his tests he was probably dealing with fewer multiple reflections; nevertheless it is predicted that he would find a slight difference in the average spacing at large and at small values of L. All of the author's determinations for the velocity at 0°C are slightly greater than those found by Pierce. This is probably due to greater intensity of the emitted waves and to more reflections. Other things being equal, the spacing increases with intensity.

4. With CO_2 the reaction at the crystal is due almost entirely to the first reflection, hence the logarithmic decrement of the I_p-amplitude is much more suitable for the determination of A than is the case with air. The variation of velocity with frequency checks approximately with that found by Pierce.[3] Herzfeld and Rice[14] state the need of dispersion and absorption measurements on the same sample of gas. For cases of great absorption this method satisfies such a need.

The sonic interferometer as a research tool in acoustics is rapidly gaining in importance. This paper is being published at this time because it has a direct bearing on the work being done. There are many additional problems which suggest themselves as possibilities, problems to which the sonic interferometer appears well adapted.

Additional experimental work will probably increase the precision of the calculated quantities, V_0 and A, not because the author's direct measurements themselves lack precision but rather because these measurements are connected with the calculated quantities in such a complicated manner. Equally precise data taken under a great range of conditions should follow.

The writer wishes to express his appreciation to all who have aided him in his work; especially to Professor H. M. Randall and to Professor D. L. Rich, at whose suggestion the work was undertaken and partly carried out at the University of Michigan; also to Professor W. R. Ham, of Pennsylvania State College, where these investigations were completed. The work was directed by Professor Rich.

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[14] K. F. Herzfeld and F. O. Rice, Phys. Rev. **31**, 691 (April 1928).