IONS AND ELECTRICAL CURRENTS IN THE UPPER ATMOSPHERE OF THE EARTH*

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Abstract

It is assumed that the ionization in the upper atmosphere is caused by the ultra-violet light of the sun and that the ion and electron densities at noon at the equator are in keeping with the facts of wireless waves. From the laws of recombination of the ions and of diffusion and drift of the ions in the earth's magnetic, gravitational and electric fields the distribution of the ions over the earth is worked out. The distribution is found to agree with wireless data over the earth, and with Gunn's diamagnetic theory of the solar diurnal variation of the earth's magnetism, (Phys. Rev. **32**, 133 (1928). The gravitational drift currents are found to flow mainly along the parallels of latitude in the following way: (1) a current sheet in the daylight hemisphere flowing eastward in the levels above 150 km which at the sunrise and sunset longitudes divides into two sheets; (2) one of these flows westward on the day side of the earth underneath (1) in the levels below 150 km, and (3) the other sheet continues eastward around on the night side of the earth. The current is mainly (4/5) between the 40'th parallels of latitude north and south, and falls to lower values at the higher latitudes. The total currents in the three sheets are about 1.16×10^7, 8.7×10^6 and 2.9×10^6 amperes, respectively. The east and west daytime current sheets subtract from each other leaving in effect an eastward current of about 2.9×10^6 amperes flowing around the earth all the time. This causes a magnetic field agreeing in magnitude and type with that obtained by Bauer in his 1922 analysis of the magnetic field of the earth of external origin, (Terr. Mag. **28,** 1 (1923)). The current sheets are not of the type required by Chapman's drift current theory of the diurnal magnetic variation, (Proc. Roy. Soc. **A122,** 369 (1929)). As a result of the drift currents, the sunset longitude of the earth is at a potential around 2000 volts above that of the sunrise longitude. This electric field combined with the earth's magnetic field causes the ions and electrons on the night side of the earth to drift upward with velocities between 100 and 200 cm $\sec^{-1}$. The ions and electrons move into regions of lower pressure and therefore do not recombine as fast as they otherwise would. This removes a difficulty from an earlier calculation which yielded a slightly too great rate of disappearance of the free charges at night. The upward drift of the ionization causes a rise of the Kennelly-Heaviside layer which is, partially at least, compensated for by the fall due to the cooling and contraction of the atmosphere at night, and is complicated by the diffusion of the ions. It is difficult to say how much of the nighttime rise of the layer observed in experiments with wireless rays may be a genuine rise and how much may be an apparent rise due to delayed group velocities, or to other causes.

THE development during the past five years in the physical theory of the upper atmosphere of the earth guided by the facts of wireless wave propagation phenomena has yielded information about the electrons in the upper atmosphere. The conclusion was reached[1] that the ionization was

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[1] Hulbert, Phys. Rev. **31,** 1018 (1928).

caused by the ultra-violet light of the sun and that the electron density increased with the height above the earth to a maximum value of about 3×10^5 electrons cm^{-3} at about 190 km for the sun directly overhead, the sun being quiet and at an epoch midway between the periods of minimum and maximum solar activity. The wireless waves indicated that ionization of some sort existed below 190 km down to about 100 km and put an upper limit upon the amount of ionization in this region, but did not decide whether the ionization was predominately electronic or ionic. Exact theoretical calculations of the ionization below 190 km were not possible because many facts about the photo-ionization of the atmospheric gases were, and are still, unknown.

2. The facts of terrestrial magnetism and their physical interpretation offered further clues to the ionization below 190 km. Gunn[2] remarked that long free path ions in a magnetic field were diamagnetic and worked out his diamagnetic theory of the quiet day solar diurnal variations in the earth's magnetic field. He was able to account for the prominent features of the solar diurnal variation on the assumption that the diamagnetism of the atmosphere due to the ions was a maximum directly underneath the sun and fell off roughly with the cosine of the angular distance from high noon, becoming at night about 1/10 or less of its noon maximum value. At the noon maximum the theory required in a 1 cm^2 column of the atmosphere 5×10^{16} ions at a temperature of 300°K, or, since the diamagnetism of an ion depends on its kinetic energy and hence on the temperature, 3×10^{16} ions at 500°K, or 1.5×10^{16} at 1000°K. Below 150 km the free paths are so short that the ions do not contribute to the diamagnetism. Therefore the ions were taken to be mainly in the levels from 150 to 180 km, although the exact height to which they extended was not very important as long as it was not too great, below 200 km, say. Maris[3] had previously shown that the noonday temperatures in the atmosphere above 100 km were probably 300°K or greater and Maris and Hulburt[4] that the temperatures might be as much as 1000°K. Maris' tables[3] of the molecular densities in the upper atmosphere indicated that the 150 km levels of the atmosphere, because of the expansion of the lower levels of the atmosphere due to the heat of the sun, moved upward about 20 km in the morning and descended in the afternoon. Gunn assumed that because of the lack of thermal equilibrium in the atmosphere (reference 3, page 243) the maximum height was attained about 2 P.M., local time, and thereby secured agreement with the observed occurrence at about 11 A.M., local time, of the maximum of the horizontal field variation in temperate latitudes.

3. Maris and Hulburt,[4] and simultaneously and independently Chapman,[5] recognized that an ion of mass m and charge e, c.g.s.e.m.u., in a gravita-

[2] Gunn, Phys. Rev. **32,** 133 (1928); Terr. Mag. and Atmos Elect. **34,** 17 (1929).
[3] Maris, Terr. Mag. and Atmos. Elect. **33,** 233 (1928).
[4] Maris and Hulburt, Phys. Rev. **33,** 412 (1929).
[5] Chapman, Proc. Roy. Soc. **A122,** 369 (1929).

tional field g and magnetic field H, drifts perpendicular to these two vectors with a velocity w given approximately by

$$w = mg \sin \phi / He, \tag{1}$$

the positive and negative ions going in opposite directions; ϕ is the angle between H and g. For a uniformly magnetized sphere, and the earth approximates this quite closely, H at latitude θ is given by

$$H = H_0(1 + 3 \sin^2 \theta)^{1/2}, \tag{2}$$

where $\cot \theta = 2 \tan \phi$, and H_0 is 0.3227 gauss the value of H at the equator at sea-level. (2) is taken to give H on the 100 to 200 km levels of the atmosphere. With these expressions (1) becomes

$$w = mgA/H_0 e, \tag{3}$$

where

$$A = (\cos \theta)/(1 + 3 \sin^2 \theta). \tag{4}$$

4. In the case of the earth the positive ions drift eastward and the negative ions westward; this constitutes an eastward electric current in the upper atmosphere. The drift velocity given by (3) occurs only when the free path of the ion is sufficiently long, i.e. above 150 km at summer noon. Maris and Hulburt[4] used the gravitational drift current to explain certain features of a magnetic storm, Chapman[5] suggested that the current might explain the quiet day magnetic variation. The type of atmospheric current necessary to explain the diurnal variation had already been worked out by Bartels.[6] His diagram is shown in Fig. 1, in which the abscissas are the longitudes in hours and the ordinates the latitudes; the numbers along the lines of current flow are in 10^3 amperes. Thus the figure indicates in the northern hemisphere in the day a current sheet of 62,000 amperes wheeling counter-clockwise around a center at about 40° latitude and 11 A.M., and at night a current sheet of 32,000 amperes wheeling in the opposite direction. The maximum current density occurs at 11 A.M. at the equator and has the value 3×10^{-5} c.g.s.e.m.u., excess of the day over the night value, referring to a section of the atmosphere 1 cm wide and of a thickness made up of the total number of long free path ions in a vertical 1 cm^2 column of the atmophere. Let us call this number $2D$, so that D is the total number of ion pairs in the 1 cm^2 column; if the positive and negative ions each carry the elementary charge e and are present in equal numbers, D represents these numbers. To be specific we shall assume throughout that the ions are singly charged positive and negative nitrogen atomic ions at a temperature of 500°K at equatorial noon; other possibilities will be mentioned later. At the equator $A = 1$, $m = 2.3 \times 10^{-23}$ grams, and from (3) $w = 4.6$ cm sec^{-1}. The drift current i is $2\,Dew$, and upon equating this to 3×10^{-5} we get $D = 2 \times 10^{14}$ nitrogen atomic ion pairs, or 4×10^{14} ions. This is 1/75 of the 3×10^{16} ions required

[6] Bartels, "Handbuch Exp. Physik," Vol. 25 (1928).

by the diamagnetic theory. If the temperature is 1000°K the ratio becomes 1/38, if the ions are molecular nitrogen at 300°K the ratio is 1/250. Therefore Chapman concluded that the gravitational drift action of the ions was much more effective than their diamagnetism in causing the earth's magnetic field variation.

5. The calculation, however, is not adequate to support such a conclusion; one can not say what the value of a current may be until the entire circuit is known, and the foregoing calculation considers only the current producing part of the circuit and neglects the return path. With equal propriety we may argue as follows: diamagnetism owes its effect to 2/3 of

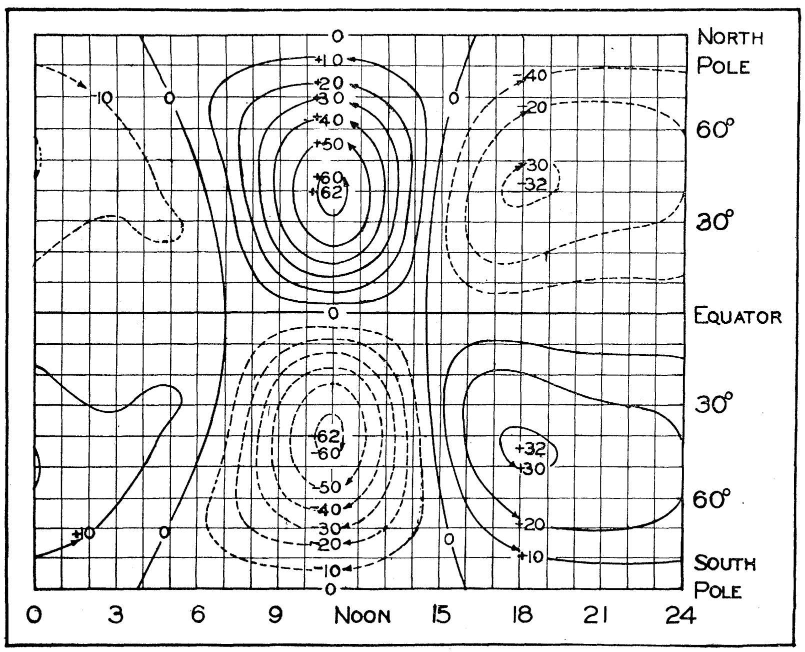

Fig. 1. The theoretical current system, due to Bartels,[6] in the upper atmosphere necessary to produce the solar diurnal variation in the earth's magnetic field. The chart is taken from Chapman's paper.[5]

the temperature energy of the ion, or kT, where k is 1.372×10^{-16} erg deg^{-1} and T is the temperature Kelvin. The factor 2/3 enters because it is only the components of the random temperature velocities in planes normal to H which contribute to the diamagnetism. With $T=500°$, kT is 7×10^{-14} erg. The drift current derives its energy from mgh, the energy of the ion falling through a height h. With $h=30$ km mgh is 6×10^{-14}, and we might conclude that diamagnetism is equal to the drift current action. The point to be made is that neither of the arguments is sufficient. A solution of the complete problem is required before a decision is possible. This is attempted in the following pages. An ion distribution over the earth due to the ultra-violet light of the sun is worked out. This distribution is found to be in agreement with wireless observations and with the diamagnetic theory. It possesses drift currents. The drift currents were not similar to those of Fig. 1 but were such as to explain a certain part of the permanent magnetic field of the earth.

6. Maris' tables[3] of the pressures in the upper atmosphere are used, omitting hydrogen. The summer day molecular densities are conveniently expressed by the relation

$$n = n_0 \epsilon^{-pz}, \tag{5}$$

where n is the number cm^{-3} of molecules of all kinds at a height z cms above the surface of the earth. $p = 0.872 \times 10^{-6}$ and for $z = 150$ km $n = 1.86 \times 10^{12}$. With these values (5) is approximately valid from 100 to 250 km, for other heights Maris' tables should be used directly. The tables are based on an upper atmospheric temperature of 300°K and in using them in calculations for 500° we are not quite consistent. We should recalculate all of the tables for the higher temperature, this would put the molecular density at about 165 km which is at 150 km on the 300° basis. The difference is not very important here, especially as the height z enters into the calculations mainly as a parameter, the molecular density being the fundamental quantity. (There is another inconsistency. I think that the arguments for noon temperatures greater than 500°, as much as 1000°, are too strong to be put aside, and yet I use temperatures of 500° and below. The same procedure occurred in the earlier paper[1] in which a temperature of 219° was used at times in spite of the fact that the 300° temperature seemed a better value. The reason was that 300° seemed then as strange and speculative as 1000° does now.)

7. The diamagnetism and the drift of the ions decrease with the free path γ according to the same ratio

$$\gamma^2/(\gamma^2 + r^2), \tag{6}$$

where r is the radius of gyration of the ion in the magnetic field. r is given by

$$r = mu/He, \tag{7}$$

where u is the velocity of thermal agitation of the ion. From (2), (5), (6), and (7) it is seen that the diamagnetism and the drift are complete at 180 km and are reduced to 60% and 4% at 170 and 150 km, respectively, at noon at the equator. For simplicity we shall assume that the diamagnetism and the drift are complete above a critical level z_c and are absent below z_c; this amounts to assuming that the change from the long to the short free path region occurs abruptly. The critical level z_c is taken to be 150 km at the equator, it varies with the latitude. The assumption is made that the temperature T of the high atmosphere is 500°K at noon at the equator and that T along the noon meridian varies with cosine of the latitude θ until at the poles T is 220°K. It is assumed throughout that the geographic and magnetic poles of the earth coincide. Thus at any latitude $T = 220 + 280$

cos θ. The values of T at each latitude are given in column 2, Table I. From the values of H and T at each latitude r was determined from (7) and z_c was calculated by means of (6). The values of z_c are given in column 3, Table I.

TABLE I.

1	2	3	4	5	6	7	8	9	10
θ	T	z_c	α	$z-z_1$	σ	D	i	v	E
0°	500°K	150.0 km	0.69×10^{-25}	0.0 km	13.3×10^{-6}	1.50×10^{16}	21.9×10^{-4}	360	116
10	494	149.8	0.71	0.2	12.6	1.53	20.3	308	110
20	483	148.6	0.74	1.0	7.72	1.54	15.6	245	114
30	464	146.8	0.83	2.0	4.58	1.48	10.6	216	140
40	435	145.2	0.98	3.0	2.02	1.36	6.7	207	195
50	399	143.5	1.21	5.4	0.86	1.21	4.1	210	290
60	360	142.0	1.66	8.0	0.33	1.02	2.3	231	480
70	315	140.5	2.19	12.6	0.07	0.71	1.0	405	1380
80	268	139.0	3.32	20.5	0.00	0.32	0.2	710	5200
90	220	—	5.43	—	—	—	—	—	—

8. Denote the long and short free path regions of the atmosphere by D and S, respectively. Let y be the density of ion pairs at a height z and D be the total number of ion pairs in a 1 cm² vertical column of the D region. Then

$$D=\int_{z_c}^{\infty} y dz. \tag{8}$$

It is assumed that in each cm³ of the atmosphere there are approximately equal numbers of positive and negative ions. Therefore D and y are also the numbers of positive or negative ions. The general procedure is as follows: at one point on the earth, namely, the point directly underneath the sun D is assumed to be 1.5×10^{16}, as required by the diamagnetic theory of the diurnal magnetic variation, and the y,z curve is assumed to be in accord with the facts of wireless wave propagation. On the assumption that the ions are caused by the ultra-violet light of the sun, the y,z curves were then calculated for all latitudes and longitudes of the earth.

9. We shall need certain formulas which are derived in reference 1. At a height z the number of ions N which diffuse cm^{-2} sec^{-1} vertically under gravity is $N=0.41\gamma u\ (py+dy/dz)$. Putting $\gamma=1/2^{1/2}\pi n\sigma^2$, where the molecular diameter σ is taken to be 3×10^{-8} cm, and $u=3\times10^5$ cm sec^{-1} approximately, the expression becomes

$$N=10^{19}(py+dy/dz)/n. \tag{9}$$

For equilibrium conditions in the atmosphere in each cm^{-3} the rate of production q of the ions is equal to the rate of loss, and the relation between y and z is therefore

$$q=\alpha ny^2+10^{19}(d^2y/dz^2+2pdy/dz+p^2y)/n, \tag{10}$$

[7] Thomson, Phil. Mag. **47,** 337 (1924); see also the discussion on page 1023, reference 1.

where $\alpha n y^2$ is the rate of loss due to recombination and the last term in (10) is the rate of loss due to gravity diffusion. We assume Sir J. J. Thomson's formula[7] for the recombination coefficient α. At low pressures α varies approximately with the $-5/2$ power of the temperature; the values of α for nitrogen atomic ions calculated from Thomson's formula are given in column 4, Table I. The formula was derived on the theory that for a positive and negative ion to combine a third body, as a neutral molecule, must be present to carry away the energy of recombination. Thus the theory considers in essence only the spatial position of the ions. It neglects at least two factors which may be of importance, namely, that recombination is probably a function of the energy states of the ions as well as of their positions in space, and that recombination may occur by a direct two body collision

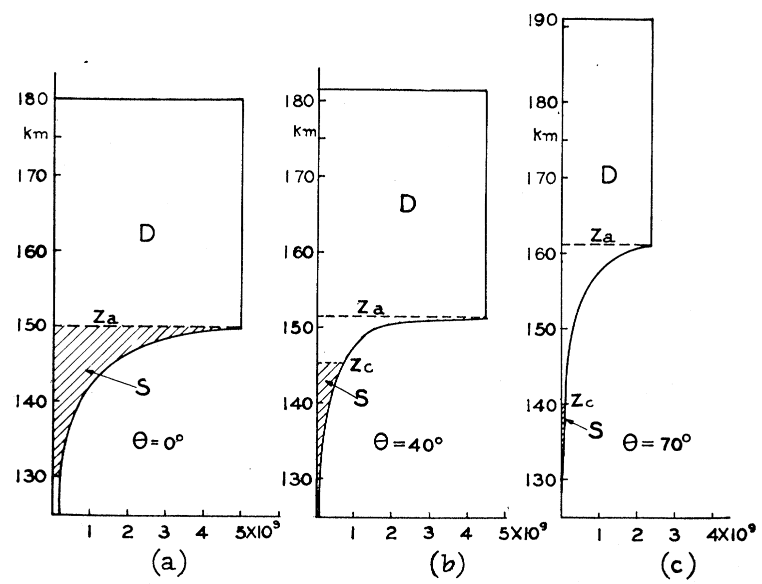

Fig. 2. Theoretical ion density curves in the upper atmosphere for latitudes 0°, 40° and 70° at noon equinox.

between the ions with a transfer of the energy into excitation or the emission of radiation or both. The one factor makes Thomson's α too high and the other too low. About all that can be concluded at the present time is that the formula is a simple one and agrees fairly well with certain laboratory experiments and that the true coefficient is probably much more complicated.

10. When the sun is directly overhead, i.e. equinoxial noon at the equator, the y,z curve is assumed to be that of Fig. 2(a), in which the abscissas are the ion densities and the ordinates the heights above the earth. $y=5\times10^9$ from $z=150$ to 180 km, hence $D=1.5\times10^{16}$. The curve below 150 km is calculated from the arbitrary formula

$$1/y-1/5\times10^9=4.54\times10^{-22}(n-1.86\times10^{12}),$$

where 1.86×10^{12} is the value of n at 150 km. With the y,z curve of Fig. 2(a) wireless rays of wave-lengths 5000, 1000, 500 and 400 meters are totally reflected (refracted) at normal incidence at heights of 68, 100, 116 and 120 km, respectively. The absorption of the ionized atmosphere from 100 to 120 km is such that the intensity of 400 meter waves is reduced $1/\epsilon$'th in about 15 km of the medium. These values are in fair accord with observation.[1] The ion curve of Fig. 2(a) gives too low an absorption of waves below 50 meters, and does not account at all for the skip distances and downward refraction of the short wireless waves. This is as it should be, for the curve comprises only a portion of the ionization of the atmosphere. It omits the electrons and to account for the behavior of the short waves the electrons must be considered; this of course has been done in earlier papers.[1]

11. The calculation of the y,z curves at various latitudes falls into two parts, first a calculation of the portion of the y,z curve in the S region and, secondly, of the portion in the D region. The second part is dependent upon the first part with additional factors. Denote the y,z curve of Fig. 2(a) by $\theta = 0°$ and the quantities associated with it by the subscript 1. Referring to the portion of the curve in the S region below 150 km the gravity diffusion term in (10) is found by calculation to be less than the recombination term by an order of magnitude or more. Therefore, at each point on the curve we have approximately from (10) $q_1 = \alpha_1 n_1 y_1^2$, and hence q_1 is known at each height z_1, q_1 being assumed to be due to the solar ultra-violet radiations of appropriate intensities and absorption coefficients which penetrate down to and are completely absorbed in the various levels of the atmosphere. At latitude θ $q = q_1 \cos\theta$, and light, which is totally absorbed (or nearly so, see reference 1, page 1025) in reaching a level z_1 at normal incidence, reaches a corresponding level z at the angle θ, where from (5)

$$z = z_1 - (\log_\epsilon \cos\theta)/p. \tag{11}$$

Thus the y,z curve at angle θ is obtained by calculating y from $y = y_1 (\alpha_1/\alpha)^{1/2}$ and z from (11). This amounts to multiplying the abscissas of the curve for $\theta = 0°$ in the levels below z_c by a constant factor and moving the curve upward a distance $z - z_1$. The values of $z - z_1$ are given in column 5, Table I. The y,z curves for latitudes 40° and 70° on a noon meridian are given in Figs. 2(b) and 2(c), respectively. The method just described for calculating the curves in the S region was also used to continue the curves in the D region from z_c to the level where y becomes constant. Denote this level by z_a; $z_a - 150$ is the same as $z - z_1$ in column 5, Table I. The method, however, gives only a part of the ions in the region from z_c to z_a, namely, those produced there by the ultra-violet solar radiation. In addition there are ions which diffuse down from higher levels; these are considered in section 15.

12. The S region of a y,z curve lies below the z_c level. The electrical conductivity of a 1 cm^3 of the S region is given by the usual expression for an ionized gas, $ye^2\gamma/mu$, and the electrical conductivity σ of a 1 cm^2 vertical column of the S region is

$$\sigma = \int_0^{z_c} (ye^2\gamma/mu)dz. \tag{12}$$

The values of σ in c.g.s.e.m.u. along a noon meridian for various latitudes were calculated from the y,z curves and are given in column 6, Table I.

13. In the D region the calculation of the y,z curve is more complex because a new factor enters, namely, an electric field. In this region the positive ions under the action of gravity and the earth's magnetic field move eastward, the negative ions westward, and build up an east-to-west potential gradient E on the day side of the earth, and hence a west-to-east gradient at night. This electric field is in such a direction that, combined with the earth's magnetic field, it causes ions in the D region of both signs to descend during the day, and to rise at night. Because of the westward electric field a current flows westward in the S region in daylight which completes the circuit of the eastward drift current i in the D region. $i=2Dew$, where w is given by (3). For equilibrium conditions a fraction f of the current in the D region flows back in the underlying S region and a fraction 1-f flows elsewhere, as around the night side of the earth. fi is the total current flowing across a vertical section of the S region 1 cm wide. At each latitude we may then write $fi=E\sigma$. It is shown in section 20 that f is about 3/4. Therefore

$$E=3Dew/2\sigma. \tag{13}$$

The velocity v downward in the daytime, due to the fields E and H is equal to $E \sin \phi/H$. Combining this with (2), (4) and (13) gives $v=3ADew/2\sigma H_0$. Introducing (3) and the numerical values of m, g and H_0 yields

$$v=10^{-12}A^2y/\sigma. \tag{14}$$

14. Gravity diffusion is neglected and therefore the same ultra-violet light which gave rise to the straight portion 30 km in length of the y,z curve for $\theta=0°$ causes y,z curves at other latitudes which are also straight lines above the z_a level 30 km in length. In the region above z_a we assume equilibrium and equate the rate of production of the ions to the rate of loss by recombination and to diffusion downward across the z_a level with velocity v. Therefore

$$q' \cos\theta = \int_{z_a}^{z_a+30} \alpha n y^2 dz + vy \tag{15}$$

where q' is the total rate of production of ion pairs in a 1 cm^2 column above z_a. For $\theta=0$ (13), (14) and (15) gave $q'=5.25\times10^{12}$, $v=360$ cm sec.$^{-1}$ and $E=116$ c.g.s.e.m.u. With the value of q' thus determined the y,z curves above z_c were calculated and the values of D, i, v and E were found. These are given in columns 7, 8, 9 and 10, respectively, of Table I. The neglect of the gravity diffusion term was permissible, for N from (9) came out to be less than vy at all points on the curves above z_a.

Table I gives the variation of the respective quantities with latitude along a noon meridian. The variation with longitude along the equator is a little different because H changes with latitude but not with longitude. For θ less than 40° the difference is slight, for θ greater than 40° the difference is perceptible. For example, at 70° longitude from high noon, i.e. 4.7 P.M. on the equator, σ is about 5, D about 0.4 and E about 0.5 times the respective values at 70° latitude.

15. The portions of the y,z curves between the z_c and z_a levels were calculated in section 11 omitting the diffusion terms. The contribution to the ions in this region from those which descend across z_a with velocity v is estimated in the following way. As the ions move downward their density y decreases because of recombination. For equilibrium conditions the increase in y in an element, dz, dz being measured positive upward, is $dy = \alpha ny^2\, dz/v$, where dz/v is the length of time the ions remain in the element. Integrating and setting $y = y_a$ and $n = n_a$ at $z = z_a$ leads to

$$(1/y) - (1/y_a) = \alpha(n - n_a)/pv. \tag{16}$$

This expression gives y, z curves in the region from z_c to z_a which are approximately the same as those of Fig. 2. This means that at each point in this region the ion density y due directly to the ultra-violet light and the density due to the vy diffusion of ions from above are roughly equal, when the two effects are considered separately. When considered together the resulting ion density will be roughly $2^{1/2}y$, since the recombination loss is proportional to y^2. The gravity diffusion term in the region from z_c to z_a is complex (see equation 10) and little more can be said about it beyond the statement that in general it is smaller than the other terms which have been considered.

16. Finally, it is important to point out that although D ions diffuse by gravity across the z_c level into the S region they make only a small contribution to the number of S ions compared to the number produced directly by the sun-light. Their effect is negligible within 3 km below z_c, as calculated from (10) by the approximate method of reference 1, page 1028. There are several other approximations in the foregoing treatment which are, however, hardly worthy of discussion, for they are submerged in the greater approximation involved in the assumption, section 7, that the D region passes at the z_c level abruptly into the S region. On the whole it is concluded that the y,z curves are a fair representation of the hypotheses on which they are based and that the values in Table I are correct within 50%.

17. Calculation from the y,z curves shows that wireless rays of wavelengths, 5000, 1000, 500 and 400 meters are totally reflected (refracted) at heights of 73, 105, 121 and 125 km at latitude 30° and at heights about 12 km greater at 70°. These heights are about 5 and 17 km greater than the respective heights given by the y, z curve for $\theta = 0$ (see section 10). At latitude 30° the absorption in the levels from 100 to 125 km is such as to reduce the intensity of 400 meter waves to $1/\epsilon$'th in 26 km. The values may also be taken to refer approximately to equatorial longitudes 30° and 70° measured from equatorial noon, i.e. at 2 P.M. (or 10 A.M.) and 4.6 P.M. (or 7.4 A.M.). Because the high atmosphere is cooler at $\theta = 70°$ than at $\theta = 0$ the 100 to 150 km levels at $\theta = 0°$ correspond to levels at $\theta = 70°$ about 20 km lower.

Thus it comes out that the levels where the long wireless waves are turned back to the earth do not undergo wide variations with the latitude and the hour of the day. This is in general accord with the facts for long waves as as far as they are known.

18. To determine the ionization in the night hemisphere we treat the S and D ions separately. In the case of the S ions the rate of decrease of the ion density y at a height z is given by $dy/dt = -\alpha n y^2$, neglecting gravity diffusion; this becomes $(1/y) - (1/y') = \alpha n t$, where y' is the value of y at sun-set. Taking the y, z curve for $\theta = 75°$ to refer approximately to sun-set conditions the equation gives a decrease in y of about an order of magnitude in 9 hours; because of gravity diffusion the decrease will be greater. At higher latitudes the S region is one of greater molecular density and the decrease in y during the night is greater than at the equator. The value of the conductivity σ of the night-time S region was found to be about 1/20 of that of the day S region.

19. The D ions, on the other hand, move upward at night because of the eastward electric field. They pass into regions of low molecular density and their loss from recombination is thereby reduced. Multiplying the values of E of Table I by 0.7 to obtain a rough average over the daylight longitudes gives $E = 81$ and 136 e.m.u. for latitudes 0° and 40°, respectively. Therefore at these latitudes the sun-set meridian is at a potential about 1600 and 2000 volts above that of the sun-rise meridian. The ions rise with a velocity v given approximately by 0.7 times v of Table I. From (5) the molecular density n in an element of ionic density y varies with the time t according to $n = n_0 \epsilon^{-pz+vt}$. Therefore the rate of change of y due to recombination is

$$dy/dt = -\alpha y^2 n_0 \epsilon^{-p(z+vt)}.$$

Integrating,

$$(1/y) - (1/y_2) = \alpha n_2 (1 - \epsilon^{-pvt})/pv, \tag{17}$$

where y_2 and n_2 are the values at sun-set, and the time t is measured from sun-set. We take the y, z curve for $\theta = 75°$ to refer to sun-set conditions; in this curve y has the constant value 1.7×10^7 from 165 to 195 km. Again, as in section 6, we remark that these heights are only parameters, the real heights are probably lower because of the shrinkage of the atmosphere due to cooling. The molecular density n is the important quantity, and in the present calculations whether z represents exactly the distance above the surface of the earth or not is immaterial. In calculations where geometry enters, such as those dealing with the heights reached by wireless waves, the skip distances, etc., the values of z are important.

20. With the equatorial night value $v = 250$ cm sec.$^{-1}$ and with $y_2 = 1.7 \times 10^7$ the y,z curves were determined from (17) for various values of t. The ion bank was found to decrease in density for about 3 hours and then to remain sensibly constant for the rest of the night, the y,z curve being nearly a straight line with $y = 0.54 \times 10^9$ and 1.5×10^9 at the lower and upper boundaries of the bank, respectively. Similar calculations were made for the various latitudes and the result was reached that after about 3 hours the

total number of ions in the D region at night in a north and south section from pole to pole of unit thickness was about 1/5 of the corresponding number averaged over the daylight hemisphere. This means that the night eastward drift current in the D region is about 1/5 the day eastward drift current. The night conduction current in the S region, also east because it is due to E which is eastward at night, is 1/20 the day eastward current, see section 18. Therefore the total eastward current at night is about $1/5 + 1/20 = 1/4$ the day eastward current of the D ions. This makes f of section 13 equal to 3/4.

21. In the calculations[1] of the electrons in the high atmosphere the effect of the earth's magnetic field upon the diffusion of the electrons was recognized (reference 1, page 1023), but the details brought out in the present theory were not known. The downward daytime diffusion due to the electric field E will modify the earlier electron density curves[1] below their maxima. Instead of turning in sharply to the Z-axis within about 5 km below the maximum the curve approaches the Z-axis at a point about 20 km below the maximum. This result was calculated from the equation for the case of electrons corresponding to (16). There is no need to give the details of the calculation for even when modified in this way the curves do not represent completely the electrons below their maxima; they only include those electrons which diffuse down from over-lying levels and neglect the electrons produced directly by the ultra-violet light in the levels below the maxima. In the electron density equation at night, however, the electrical diffusion term makes a needed modification, without it the equation gave too rapid a decrease in the electrons at night. Neglecting gravity diffusion the equation becomes

$$\log_\epsilon y/y_2 = bn_2'(\epsilon^{-p'vt} - 1)/p'v. \qquad (18)$$

y is the density of the electrons at time t, y_2 their density at sun-set at a height where the density of oxygen molecules is n_2', b is the oxygen electron attachment coefficient,[1] and p' is connected with n' just as p is with n through (5). (18) was derived in the same manner as the ion equation (17). From (18) with equatorial values $y_2 = 1.5 \times 10^5$ and $v = 250$ cm sec.$^{-1}$ y decreases to a constant value 1.1×10^4 in about 3 hours. This is regarded as agreeing well enough with the value 5×10^4 inferred from the skip distances of wireless waves at night.[1] A more exact agreement is hardly to be expected in view of the approximate nature of the calculations. With a velocity $v = 200$ cm sec.$^{-1}$ the upward drift of the electrons and D ions at night is about 60 km in 8 hours. At the same time the ion bank spreads out in a vertical direction because of gravity diffusion, which becomes important at latitudes above 30° where the magnetic field makes an angle less than 45° with the vertical. It is difficult to say how much of the increase during the night of the apparent height of the ionization observed in certain wireless experiments[8] may be a genuine rise and how much may be an apparent rise due to delayed group velocities, or to other causes.

[8] Appleton, Nature **120**, 330 (1927); Hafstad and Tuve, Terr. Mag. and Atmos. Elec. **34**, 39 (1929).

22. The diamagnetism of the ions is proportional to TD. The values of TD along a noon meridian of longitude, taken from columns 2 and 7, Table I, and plotted as abscissas against the latitude θ as ordinates, are shown by the dots of Fig. 3; the smooth curve 1, Fig. 3, gives $\cos\theta$. The values of TD along the equator are a little less than the values of the dots for θ greater than 40°. The temperature of the high atmosphere on the night hemisphere of the earth is about 1/2 the noon day temperatures and, as shown in section 20, the average value of D at night is about 1/5 of the average value of D in the daytime. Therefore the diamagnetism of the atmosphere at night is about 1/10 of the average diamagnetism during the day and 1/15 of the noon maximum value. Thus the ionization of the atmosphere derived on the hypothesis that the ultra-violet light of the sun is the ionizing agency is found to possess the diamagnetism required by Gunn's diamagnetic theory of the solar diurnal variation of the earth's magnetic field.

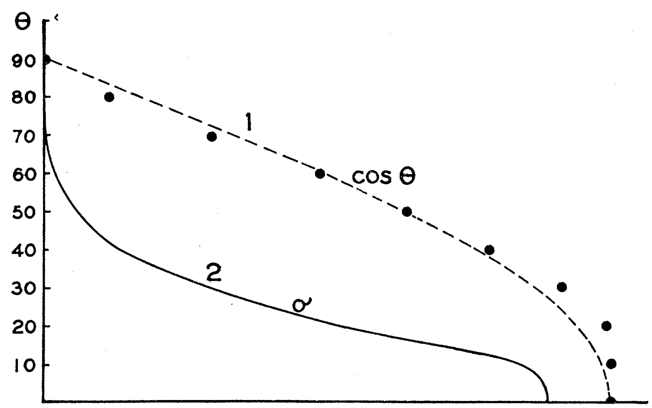

Fig. 3. Curve 1, dots and curve 2 are the values of $\cos\theta$, TD and σ, respectively.

23. The diamagnetic theory requires a maximum ionization $D=1.5\times10^{16}$ at the equator at noon. It is of interest to see what assumptions this involves as to the rate of production of the ions. As a rough beginning we will assume the mechanism of the earlier paper,[1] namely, that the solar ultra-violet radiations cause directly equal numbers of positive ions and electrons and that the electrons become attached to neutral oxygen molecules to form negative oxygen molecular ions. The theory of the skip distances of wireless waves[1] required an electron layer of maximum density 3×10^5 at 190 km. Accordingly we may assume that the electron density y_e is a constant value 2×10^5 in the region from 150 to 180 km; if it exceeded this value the calculated skip distances would be shorter than those observed. The number of oxygen molecular ions which are formed per sec. in the 150 to 180 km region is $\int_{150}^{180} bn'y_e dz=7.8\times10^9$; n' is the density of oxygen molecules, b is the attachment coefficient for a temperature of 500°K, $b=1.1\times10^{-13}$, see equation 11, reference 1. This is the rate of supply of negative ions, and since there is an equal number of positive ions, this is the rate of supply of ion

pairs. Assuming roughly that the density y of the ion pairs in the 150 to 180 km region is a constant and putting the rate of supply equal to the rate of recombination, we have $7.8\times10^9=\int_{150}^{180}\alpha ny^2dz$. This gives $y=2.5\times10^8$ and $D=7.5\times10^{14}$, which is 20 times less than 1.5×10^{16} ion pairs. The conclusion is that the very primitive photochemical reaction which has been assumed is not entirely adequate to account for the 1.5×10^{16} ion pairs. In the reaction negative ions were regarded as being the end product of two processes, first, a splitting up of a neutral particle by light into a positive ion and an electron and, second, the attachment of the electron to an oxygen molecule. It is possible that the light may produce a positive and negative ion directly, for example, by breaking a nitrogen molecule into a positive and a negative nitrogen atom. Further, many photo-chemical effects occur in the atmosphere, such as the formation of ozone, the oxides of nitrogen, etc., which were not taken into account in the foregoing calculation. It is thought that the photo-chemical branch of atomic physics has been developed to a point where it may be used in examining the problem.

24. The eastward drift current sheet in the daylight D region flows along the parallels of latitude and divides into two sheets at roughly the sunrise and sunset longitudes, one sheet flowing westward in the underlying S region and one sheet continuing eastward on around the night side of the earth in the S and D regions. From the values of i in column 8, Table I, the total current in the daylight region from latitude 0° to 40° is 6.8×10^6 amperes. The values of i were, however, for a noon meridian, and to get a rough average over the daylight longitudes 6.8×10^6 is multiplied by 0.7 giving 4.8×10^6 amperes. Similarly the total average daylight current in the D region from latitude 40° to 90° is 1.0×10^6 amperes and from pole to pole is 11.6×10^6 amperes. Since about 1/4 of the current flows around the night hemisphere the values of the westward currents in the daylight S region are 3.6×10^6, 0.75×10^6 and 8.7×10^6 amperes, from latitudes 0° to 40°, 40° to 90° and from pole to pole, respectively. The night current from pole to pole is 2.9×10^6 amperes, of which 2.3×10^6 and 0.6×10^6 amperes are in the D and S regions, respectively. Altogether the drift currents amount to a steady eastward current of 2.9×10^6 amperes flowing along the parallels of latitude around the earth all the time. About 4/5 of this current is between the 40th parallels of latitude north and south, and the distribution of the current density with latitude is given approximately by the values of i in Table I. The value of the magnetic field at the equator due to this current is about 400γ (γ is 10^{-5} gauss). This agrees very closely with the analysis of the permanent field of the earth by Bauer.[9] He showed that a portion of the magnetic field of the earth is of external origin and is northward and upward in the northern hemisphere, nearly horizontal at the equator and northward and downward in the southern hemisphere, the value of the horizontal component at the equator being about 430γ.

[9] Bauer, Terr. Mag. and Atmos. Elec. **28,** 1 (1923).

25. The foregoing values furnish a quantitative basis for the suggestion in the ultra-violet light theory[4] of magnetic storms that the first phase of the world-wide storm was due to a pulse of current in an equatorial belt around the earth and that the current pulse in the night hemisphere flowed in the S region. According to the theory a flare of solar ultra-violet light created rather suddenly an additional 10^{16} ion pairs in a vertical column of the D region in the case of an average magnetic storm. This is about the same as the number of D ions which were used in the present paper for an average quiet day. Therefore the first effect of the storm is to double the daylight ionization and hence E, which doubles the current in the night S region making it $2\times 0.6\times 10^6$ amperes. No immediate change in the night D current occurs. The value of the current which girdles the earth is increased from 2.9×10^6 to $(2.9+0.6)\times 10^6$ amperes. This produces a storm magnetic field of the observed order of magnitude $10^2\gamma$.

26. The drift current theory[5] of the solar diurnal magnetic variation made no quantitative suggestion as to the manner in which the gravitational drift currents would give rise to the current system of Fig. 1. According to the present calculations such a system might be produced if the westward current in the daylight S region which arises from the eastward current in the daylight D region between latitudes 0° to 40° should not flow entirely in the S region between these parallels but should spread into higher latitudes. To estimate the spreading of the S current the conductivity σ of the S region, given in column 6, Table I, is plotted in curve 2, Fig. 3. The curve shows that σ becomes small at high latitudes, the total conductivity from 60° to 90° being 0.47%, and from 70° to 90° 0.03%, of the total conductivity from 0° to 90°. The S current from $\theta=0°$ to 40° is 3.6×10^6 amperes and the spread of this current into the latitudes from 60° to 90° is $0.47\%\times 3.6\times 10^6=17{,}000$ amperes, whereas the chart of Fig. 1 calls for 45,000 amperes by day and $-22{,}000$ amperes by night or an excess of day over night of 67,000 amperes. Similarly from $\theta=70°$ to 90° the current spread is $0.03\%\times 3.6\times 10^6=1000$ amperes and the chart requires 45,000 amperes. The current sheet is therefore quite different from that required by the drift current theory of the diurnal magnetic variation.

27. The values of Table I were obtained by calculating the equilibrium conditions for each slice of the atmosphere along parallels of latitude independently of the influence of adjacent slices. Such a procedure is not entirely correct, for the separate slices influence each other because the S current of one slice spreads into the S regions of the other slices. For example, a greater E at, say 60° than at the equator will cause a current system opposite to that of Fig. 1 which will reduce the spreading of the S current into high latitudes below that which was estimated in section 26 from the σ,θ curve of Fig. 3. We may conclude that a calculation taking into account this effect would give less spreading than was found and would result in lower values of E at the higher latitudes than those of Table I.

28. The conclusion that the electric currents due to the ions in the atmosphere produced by solar ultra-violet radiations do not cause the solar

diurnal variation in the earth's magnetism remains true if the ions, instead of being nitrogen atomic ions, were ions of other types, as helium ions, molecular ions, electrons, etc. The conclusion does not depend upon the exact form of the y,z curve assumed at $\theta = 0°$. For the conclusion depends essentially on the form of the σ, θ curve of Fig. 3 and in all these cases the curve would remain practically unchanged. In order for a current system similar to that of Fig. 1 to exist the conductivity of the S regions at high latitudes relative to that at low latitudes must be greater than the values of σ in Table I. To have this would conceivably require the hypothesis of another type of ionizing agency than ultra-violet light, or some other hypothesis. It does not seem probable that a rearrangement of the S ions would be brought about under normal conditions by winds blowing in the high atmosphere from warmer to colder latitudes. Winds of this kind in all probability exist, but the recombination of the ions is so rapid that no appreciable transport of the ions over distances of a thousand kilometers or more seems likely. One can not say with certainty whether high flying ions[4] have any effect or not during quiet conditions. Under unusual circumstances, during a magnetic or an ordinary storm, the ions might be blown about and give rise to electric currents and magnetic disturbances.

29. The drift current theory of the diurnal magnetic variation encounters a difficulty due to dynamo action. Maris' calculations[3] of the heating of the upper atmosphere show that a daytime expansion takes place, the atmosphere rising in the morning and descending in the afternoon. His tables show that the atmosphere at 150 km at summer noon descends to 120 km at summer midnight, a vertical movement of 30 km in 12 hours. This amounts to a rough average velocity $v' = 70$ cm sec^{-1} of the S conduction region in tropical latitudes vertically upward during the morning and downward during the afternoon. The motion is a real one and not a fictitious one as would occur if the upward, or downward, displacement of the S region came about by removal of the underneath ions by recombination and a production of ions on the upper side. For at 150 km at the equator the number of recombinations cm^{-3} sec.$^{-1}$ is 3.2×10^6 whereas the number of collisions cm^{-3} sec^{-1} is 7.3×10^{12}. There are thus many collisions which do not result in recombination and therefore the ions are forced by molecular impacts to move up or down with the atmosphere of which they form a part. On the other hand the motion of the daytime ionization around the earth with the sun is a fictitious one, the ions being formed at dawn and blotted out at evening by recombination.

30. The movement of the S region across the magnetic field $H = 0.32$ gauss gives rise to an e.m.f. cm^{-1} E' which at the equator is westward in the morning and eastward in the afternoon. $E' = v'H = 22$ e.m.u. cm^{-1}, a value about 1/6 of the temperate and equatorial values of E of Table I. The e.m.f. E' gives rise to a current system roughly opposite to that of Fig. 1 in the morning and in the same direction in the afternoon. Thus if the high latitude values of σ are such that E causes the current system of Fig. 1 the resultant current system will be that obtained by the superposition of

the systems due to E and E', i.e. $E-E'$ in the morning and $E+E'$ in the afternoon. This would displace the maximum of the diurnal magnetic variation an hour or two later than noon, which is contrary to the fact that the maximum occurs at 11 A.M.

31. Several small magnetic effects of the drift currents remain to be discussed. The equivalent vertical distance between the S and D current sheets in the day hemisphere is not greater than 30 km. These current sheets will cause a magnetic variation apart from, and opposed to, that due to the diamagnetism of the D ions, which, however, is found by calculation to be less than 5% of the diamagnetic effect at the equator, and of a less amount at the higher latitudes. The dynamo effect causes a 50% variation in this small magnetic effect due to the current sheets. Certain other currents not mentioned here-to-fore, such as the westward gravitational drift current in the S region, the drift currents due to temperature gradients and to magnetic gradients[10] were found to give magnetic effects negligibly small compared to those of diamagnetism. The general conclusion is that the main features of the diamagnetic theory are undisturbed by drift currents.

32. For the purposes of this paper tables of the ion distribution, i.e. the y,z curves, at all points on the earth were worked out as described in sections 10 to 16. Glimpses of the tables are given in the curves of Fig. 2 and in Table I, but the tables themselves are not printed. I think that the explicit publication of such tables would be premature. The physics of the upper atmosphere has developed so rapidly during recent years that the tables can at best be regarded as an early attempt, to be modified rather than to be used by future writers. Additional electromagnetic effects, old in the literature of physics but novel in their application to the atmosphere, continue to be recognized and new experimental discoveries, such as those dealing with wireless waves, with ozone, etc., continue to be made. One can not be sure at the present time that all the important facts have been brought to light, nor is it at all certain that the various facts have been given their proper emphasis in the theory.

[10] Gunn, Phys. Rev. **34,** 335 (1929).