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THE REFLECTION OF THE Kα LINE OF CARBON FROM GLASS

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ABSTRACT

A vacuum spectrograph was constructed in which a beam of monochromatic radiation in the extremely soft x-ray region could be isolated by means of a ruled grating and then reflected from a mirror. This mirror could be rotated and the intensities of the reflected rays corresponding to various glancing angles of incidence could be compared by a photographic method. Using a glass surface and the $K\alpha$ line of carbon (wave-length = 44.6A) measurements of reflected intensity were made for glancing angles of incidence between 1° and 8°. At 8° the reflected intensity was found to be less than 4 percent of the intensity at 1°. However the curve relating intensity of reflection with glancing angle of incidence shows no abrupt change of slope, such as would occur in the case of hard x-rays. This indicates that absorption effects modify the intensity of the reflected ray sufficiently to obscure any sudden change at the critical angle in the case of the reflection from glass of wave-lengths as long or longer than 44.6A.

THIBAUD[1] has endeavored to measure the critical angle of total reflection from a glass surface by varying the angle of incidence of a beam of x-rays upon a glass grating and determining the angle, for each line in the spectrum, at which the intensity of that line had decreased to one-half value and also the angle at which it had entirely disappeared. For the $K\alpha$ line of carbon he found the angle at which the intensity had decreased to one-half value to be 5×10^{-2} and the angle for complete disappearance to be 5.1×10^{-2}. Expressed in degrees and minutes these angles are $2^\circ\ 52'$ and $2^\circ\ 56'$ respectively. This indicates a rapid decrease in reflected intensity between these angles and would seem to locate the critical glancing angle of total reflection with some definiteness. However, at another place in the same article he gives the angle for complete disappearance of the carbon line as 6.4×10^{-2} which is $3^\circ\ 40'$. Also no statement is made concerning the angle at which 100 percent intensity was obtained for the carbon line.

Henderson and Laird[2] have sought to determine critical angles by measuring the intensity of the reflected radiation when soft x-rays, generated at constant voltage, were caused to fall at various glancing angles of incidence upon glass and iron surfaces. Their curves for a glass surface show that with 194 volts applied to the x-ray tube, the intensity of reflected radiation falls quite uniformly as the glancing angle of incidence is increased from 1° to 20°. The intensity at the larger angle being about 26 percent of that at the smaller. A nearly parallel curve was obtained with 576 volts. In the latter case the intensity of reflected radiation at 20° was about 22 percent of that at

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[1] Thibaud, Comptes rendus **187**, 219 (1928).

[2] Henderson and Laird, Proc. Nat. Acad. Sci. **14**, 773 (1928).

1°. Since the quantum voltage corresponding to the $K\alpha$ line of carbon is about 275 volts the curve relating intensity of reflection of this line with angle of incidence should lie somewhere between the above curves provided that this method is really applicable to the problem. Since an increase in tube voltage merely shifts the position of maximum intensity of radiation toward shorter wave-lengths without eliminating the longer wave-lengths upon which the maximum angle of reflection depends, it appears doubtful that any definite information concerning critical angles of reflection may be thus obtained.

Since neither of these methods makes a direct attack upon the problem a vacuum spectrograph was designed and constructed in which measurements of reflected intensity could be made with the use of strictly monochromatic lines obtained from a ruled grating. The $K\alpha$ line of carbon is one of the most prominent and intense lines in this region and hence was chosen first for investigation.

APPARATUS

The arrangement of the principal parts of the apparatus used in this work is shown in Fig. 1. The spectrograph, mounted upon a steel frame extending upward from the brass base plate B, could be enclosed for evacuation by the brass cylinder C which rested upon the rubber gasket E. This cylinder could be raised by means of a cord and pulley, thus conveniently exposing the apparatus for adjustment and for the changing of photographic plates. The x-ray tube, partly shown at X, has been previously described.[3] The x-ray beam passed upward through two rather wide slits before entering the spectrograph chamber. These somewhat reduced the intensity of the light from the filament. The beam then passed through the collimating slits S_1 and S_2 which were 0.25 mm wide. The plane glass grating G, having 600 lines per mm, was set at an angle of 4° 41′ with respect to this beam. This produced an intense first "inside order" of the $K\alpha$ line of carbon, deviated 6° 43′ from the direction of the incident beam. The arm A carrying the mirror M and the slit S_3 could be turned about an axis passing through the line of incidence of the x-ray beam upon the grating. By taking suitable photographs of the carbon spectrum it was possible to find the angular setting for the arm A which would permit only the central portion of the $K\alpha$ line to pass through the slit S_3 and fall upon the mirror M. The ratchet R could be turned by means of an electromagnet (not shown in the drawing) and the angle of the mirror changed by small known increments from outside the apparatus. The shield and slot S_4 could be caused to move across the path of the reflected beam by means of an electromagnet, likewise controlled from outside. Hence its position could be adjusted so that all parts of the photographic plate were shielded from stray radiation except the part which was being exposed to the beam reflected from the mirror. Another electromagnet gave a step by step translational motion to the photographic plate. Each time the angle of the mirror was changed the shield was moved by such an amount

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[3] Dershem, J.O.S.A. & R.S.I. **18**, 127 (1929).

that the reflected ray would pass through the slit and the photographic plate was moved so that a fresh portion of the plate was exposed. In this way a series of lines could be obtained upon the plate, each being due to the same wave-length but reflected at different angles from the mirror.

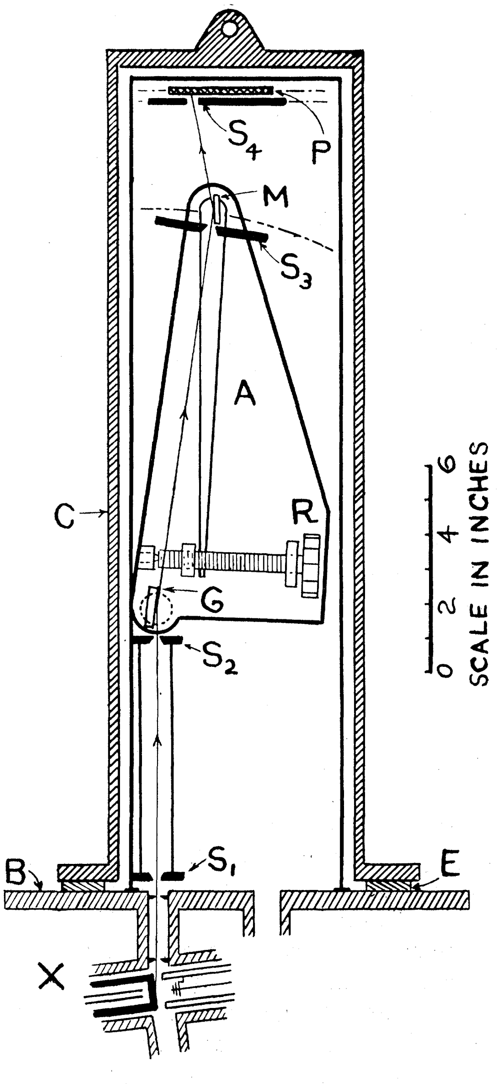

Fig. 1. Sketch of apparatus.

One three stage mercury pump was used to evacuate the spectrograph chamber and another to evacuate the x-ray tube. The pressure in the apparatus ranged between 10^{-4} and 10^{-5} mm Hg. The x-ray tube target was a tungsten button embedded in copper. However the carbon line, due to stopcock grease, always appears. It is possible, by the use of large energy input, to burn off most of the carbon deposited on the target and thus reduce the intensity of the carbon line. In this work 50 ma and 4.2 kv, rms were used and with this power input the intensity of the carbon line usually remained constant but sometimes increased in intensity at a rate of about 5 percent per hr.

For most of the observations taken in this work the mirror was first set at a glancing angle of 2° and an exposure of three or four minutes duration was made. The mirror was then advanced in steps of 15′ each and the time of exposure increased with increase of angle, each exposure being made on a fresh portion of the photographic plate. After several exposures had been made upon a plate, each for a different angle of incidence upon the mirror, the latter was caused to return to the original angle of 2° by means of another electromagnet, also not shown in the drawing. A check exposure was then made upon the photographic plate to determine whether, or not, the intensity of emission of the carbon line had changed since the first exposure. In order that the mirror might intercept the entire beam passing through the slit S_3 the latter was made narrower for angles less than 2° than was desirable at larger angles. However in every case the ray reflected at an angle of 2° was recorded at least once upon each plate. Hence the intensities at all angles could be compared with the intensity at 2° as recorded upon the same plate under as nearly the same conditions as it was possible to obtain.

Results

Eastman x-ray plates were used in this work and it was found that useful comparisons of the densities of lines on these plates could not be obtained with either the Moll or the Hartmann microphotometers. This was partly due to the rather low density of blackening that could be obtained at large angles of reflection in a reasonable time of exposure. Hence recourse was had to a method of direct visual comparison with lines of varying density upon another plate. Such a comparison plate was obtained by keeping the angle of reflection constant while the photographic plate was advanced in steps of 4 mm each. At each step the time of exposure was gradually increased and a series of lines of uniformly increasing density of blackening was thus secured. A number of comparison plates were made in a similar way with the use of other time intervals of exposure.

The relationship between intensity of blackening and time of exposure is not precisely known in this wave-length region and cannot easily be measured. Hence the reciprocity law of blackening is assumed in what follows. Presumably the errors due to the assumption of this law are less than the probable errors of observation.

Assuming the validity of the reciprocity law, it follows that the ratio of the intensities of the rays reflected at two different angles is equal to the product of the ratio of the densities of blackening and the inverse ratio of the times of exposure. The ratio of blackening was obtained in the following way. The photographic plate was placed with its film side in contact with the film side of one of the comparison plates described above. Each line of the plate under test was matched with a line of the comparison plate, estimated interpolations being made in the case of intermediate densities. Since the intensity of radiation was constant for all lines of the comparison plate the ratio of blackening of any pair of lines on the plate under test was given directly by the ratio of the times of exposure of the corresponding lines of the comparison plate.

In this way the intensity of the radiation reflected at each angle was independently compared with that reflected at 2° as recorded upon the same plate. In a similar manner intensity comparisons for the same angles were secured from other plates. In each case estimates were made with the use of several comparison plates. As previously stated a different width of slit was used for angles between 1° and 2°. Hence a different series of plates was used to determine the ratio of the intensities at these angles. With this ratio known the intensity at all angles could be compared with that at 1°.

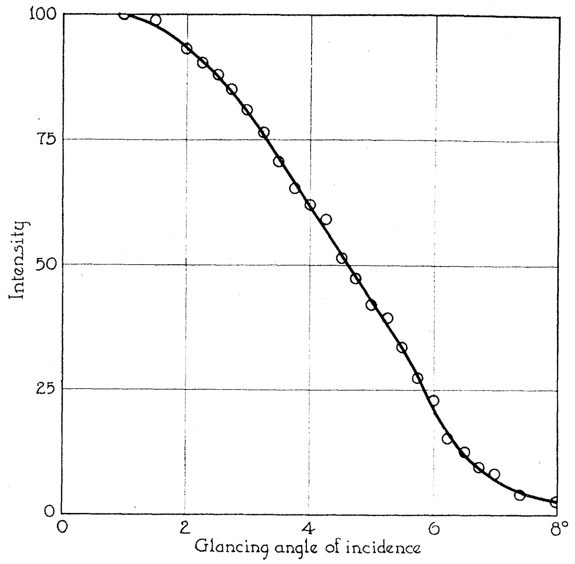

Fig. 2. Intensity of reflection of the $K\alpha$ line of carbon from glass for various glancing angles of incidence

The values thus obtained for the intensity of reflected radiation at a given angle were then averaged and the results are shown in Fig. 2. The accuracy of the method may be judged somewhat by the probable error of observation. For example, the intensity of the radiation reflected at 4° compared to the intensity at 1° was found from 24 observations to be 62.3 ± 0.8 percent. The results of the same number of observations at 5° and 6° was 42.2 ± 0.9 and 23.0 ± 0.4 percent respectively. These computations and similar ones for other angles indicate that the average probable error of observation was not greater than 2 percent. This conclusion is also borne out by the close approach of all the points to a smooth curve. In obtaining these results no estimates were discarded.

The curve of Fig. 2 shows no sudden change of slope which might be taken to indicate the value of the critical glancing angle of total reflection. In this respect it does not confirm the results of Thibaud,[1] the decrease in reflected intensity being much more gradual with increase of angle than indicated by his work. Also the reflected ray is shown to persist with relatively large intensity far beyond the limiting angle which he found. On the other hand these results show that the decrease in reflected intensity with increase of angle is much more rapid than would be inferred on the assumption that the results of Henderson and Laird[2] were really related to this problem.

The sudden change in reflected intensity which occurs at the critical angle in the case of a transparent medium becomes less abrupt in the case of an absorbing one. This effect has been discussed by Prinz.[4] If the coefficient of absorption is sufficiently large the variation in reflected intensity with angle of incidence becomes fairly uniform in the neighborhood of the critical angle with result that the latter may not be definitely located by the presence of any discontinuity in the curve. The results obtained in the present investigation may be explained by such considerations as these. It appears evident that in the case of glass and radiation having a wave-length as long as 44.6A that the coefficient of absorption is sufficiently large to obscure the angle of critical reflection quite completely.

If the index of refraction were known the critical angle could be found by computation. Thibaud[1] has endeavored to do this for the case of the $K\alpha$ line of carbon reflected from glass. He determined the index of refraction by the use of the simplified form of the Drude-Lorentz dispersion formula which is known to give fairly accurate results provided that the frequency of the radiation is much higher than any critical frequency of the elements composing the reflecting substance. In this way he obtained a theoretical value of $6°\,40'$ for the critical angle. It is doubtful that any serious importance may be attached to such computations in this region since the frequency of the $K\alpha$ line is lower than some of the critical frequencies of the principal constituents of glass. The effect of these critical frequencies is to diminish the critical glancing angle. Hence the true value is certainly less than that computed as above. This is borne out by an inspection of the curve of Fig. 2 which shows that the reflected intensity has fallen to a very low value at the above angle.

It is planned to extend this investigation with the use of other wavelengths and other reflecting surfaces. In conclusion I wish to extend my thanks to Professor A. H. Compton and the Ryerson Laboratory Staff for placing the facilities of the laboratory at my disposal for this investigation and for many helpful suggestions during the course of the work. I wish also gratefully to acknowledge the loan by the Department of Physics of the University of California of the x-ray tube used in this investigation.

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[4] Prinz, Zeits. f. Physik **47**, 479 (1928).