THE RECTIFICATION OF RADIO SIGNALS BY A THERMIONIC TUBE CONTAINING ALKALI METAL VAPOR

By K. H. Kingdon and E. E. Charlton

Abstract

The cathode of a thermionic triode tube is surrounded by a region of minimum potential caused by electron space-charge. If the cathode is of thoriated tungsten, and the tube contains caesium vapor, a few positive ions are formed by ionization of caesium atoms striking the hot cathode. Some of these ions accumulate around the potential minimum, raise the potential at that point, and allow a larger electron current to flow to the anode. The ions have a natural frequency of oscillation about the potential minimum, which is usually several hundred kilocycles. If an alternating voltage is applied to the tube, of a frequency agreeing with this natural frequency, the ions are set in oscillation about the potential minimum. This oscillation of the ions makes the potential minimum more negative, and decreases the anode current. In addition the amplitude of oscillation may build up by a resonance effect to such a large value that the ions are able to discharge to one of the neighboring conductors, leading to a still greater decrease in anode current. This kind of rectification is much greater for small alternating voltages than that resulting from curvature of the triode characteristics.

I. Introduction

SOME years ago one of the writers (E.E.C.) while studying triode thermionic tubes containing caesium vapor, found that under certain conditions these tubes were very sensitive radio detectors.[1] Subsequent experiments have shown that detector action (rectification) by these tubes is due mainly to causes entirely different from those which determine rectification in the ordinary high vacuum triode (curvature of characteristics). In the alkali metal tubes the rectification is caused by the action of the signal on the motion of positive ions formed by ionization of alkali metal atoms coming in contact with the hot filament.[2] These ions change space-charge conditions in the tube, and so change the electron current to the anode. In this article we shall give an account of some fundamental experiments with these tubes, and of the theory to which they lead, leaving to another article a more detailed account of practical alkali metal detector tubes and their characteristics in radio circuits.[3]

[1] The UX 200 A radiotron is a detector of this kind. See also Brown and Knipp Proc. I. R. E. **10,** 451 (1922) and **15,** 49 (1927); Donle, Proc. I. R. E. **11**, 97 (1923) and **12,** 159 (1924).

[2] Langmuir and Kingdon, Phys. Rev. **21,** 380 (1923); Proc. Roy. Soc. **107,** 61 (1925).

[3] Charlton and Hitchcock, to be submitted to Proc. I. R. E. Most of the ideas which we describe in the present article have been under discussion in our laboratory for some time, and all those who have worked with these tubes have made some contribution to them. In this article we try to reduce these ideas to precise form. We therefore wish to express our indebtedness to Messrs. A. L. Samuel, L. P. Smith, and W. J. Hitchcock for their contributions. We are also particularly grateful to Mr. H. M. Mott-Smith, Jr. for his suggestions in connection with the frequency characteristic of these tubes, and the experiments on this point.

II. Experimental Tubes

Most of the experiments to be described here were made with two tubes, *A* and *B*. Both tubes had V-shaped filaments of thoriated tungsten wire 0.00406 cm diameter and about 5 cm total length. The apex of the V was steadied by a flexible wire hook. The thoriated filaments were usually in a well-activated condition. The grids were of the conventional oval wire-wound type, and were surrounded by the usual oval anode. Both tubes had an amplification constant of about 25. In tube *B* portions of grid and anode at one corner were cut away and mounted on separate leads, as shown in Fig. 1. This construction made it possible to study separately phenomena at one end of the filament, usually the negative end. Unless otherwise stated the tube was connected so as to use the whole filament. The tubes were well exhausted, but contained a small amount of metallic caesium, giving a vapor pressure[2] of 1.8×10^{-3} barye when the bulb was at 25°C. This pressure is so low that ordinary ionization by collision phenomena play no appreciable part in the operation of the tube.

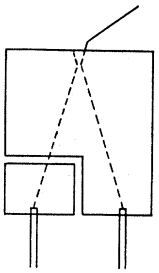

Fig. 1. Construction of grid and anode.

III. Static Rectification Characteristics

The tube was placed in a constant temperature bath, and filament temperature, and grid and anode voltages (E_g, E_p) fixed at the desired values. The multi-range ammeter in the anode circuit had a compensating battery and high resistance connected across it, so that most of the anode current (i_p) could be balanced out, and the residual read on one of the low-current ranges of the ammeter. An alternating current signal of any desired frequency and voltage was then applied between filament and grid, and the resulting change Δi_p in i_p noted. Thus Δi_p is the change in i_p due to an unmodulated signal, and may be said to measure the "static rectification" of the tube.

The a.c. signal was taken from one of two small oscillators, so as to cover the frequency range desired. The coil of a wave-meter was placed near the coil of the oscillator chosen, and a "pick-up coil" was placed on the opposite side of the wave-meter coil. The wave-meter thus served both to measure the frequency and as a filter to free the wave of harmonics. The leads from the pick-up coil went to a variable mutual inductance, the secondary of which was connected from filament to grid of the tube. A thermocouple connected across the secondary served to measure the signal voltage. This voltage was not appreciably affected by the grid current, even when the latter was large.

Fig. 2 shows Δi_p as a function of E_g for tube *B* (whole filament) at filament voltages of 3.1 and 5.0. E_g was measured from the negative end of the filament. The other conditions were—bulb temperature 30°C, $E_p=22.5$, signal voltage 0.050 RMS, signal frequency 600 kilocycles. The filament temperature was about 1700°K at $E_f=3.1$, and 1940°K at $E_f=5.0$. Both curves start at the left with small positive values of Δi_p. This is the same

phenomenon as rectification by curvature of anode characteristic in an ordinary high vacuum triode. Throughout the rest of the curves the values of Δi_p are negative and usually much greater. This is the type of rectification

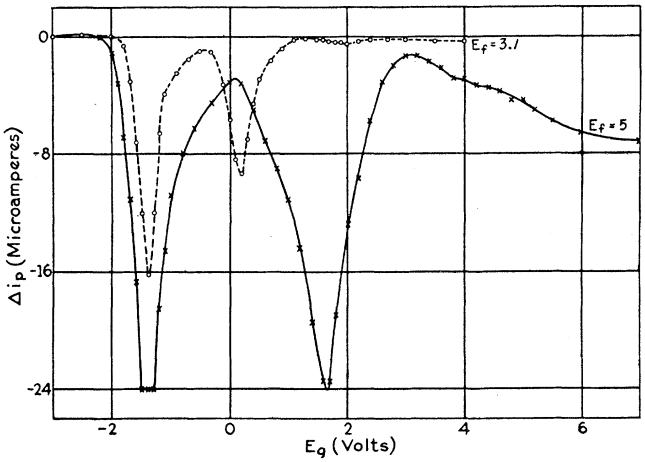

Fig. 2. Δi_p as a function of E_g for tube B at filament voltages of 3.1 and 5.0.

with which we are particularly concerned. Both curves show two main negative peaks, separated by a voltage difference slightly greater than $E_f/2$. For the $E_f=5$ curve the rectification is large even at large positive values

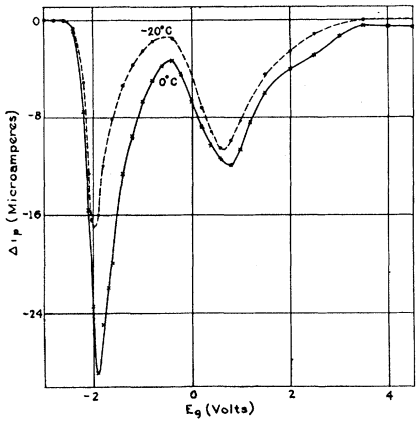

Fig. 3. Δi_p as a function of E_g for tube B at bulb temperatures of 0°C and −20°C.

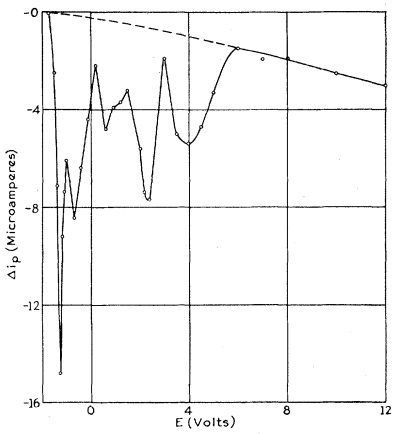

Fig. 4. Δi_p as a function of E, the common potential of grid and anode of tube B.

of E_g. For the $E_f=3.1$ curve at high E_g the sum of anode and grid currents approaches the saturation emission from the filament, and Δi_p becomes small.

Fig. 3 shows Δi_p as a function of E_g for tube B at bulb temperatures of 0°C and −20°C. The other conditions were $E_p=45$, $E_f=5$, signal voltage 0.044, frequency 1050 kilocycles. The state of activation of the thoriated surface of the filament was different from that in Fig. 2. The rectification at −20°C is less than at 0°C and the effect gradually disappears as the vapor pressure of the caesium is reduced to zero.

For the experiment illustrated in Fig. 4 the grids and anodes of tube B were connected together and put at the common potential E. Fig. 4 shows Δi as a function of E. As before, E was measured from the negative end of the filament. The other conditions were—bulb temperature 29°C, $E_f=5$, signal voltage 0.044, frequency 1050 kilocycles. At the left the curve shows large negative peaks, somewhat more jagged than those found when the tube was used as a triode. There is a steady rise in the region of positive E, as in the right-hand part of the $E_f=5$ curve of Fig. 2. Note that in Fig. 4 the right-hand part may reasonably be extrapolated to the left by the dotted line, suggesting that the whole Δi curve is the sum of two different effects, one of which predominates at low values of E, and the other at high. This will be shown to be the case (see below, section 8).

IV. Evidence for Localization of Sensitivity in Limited Region of Filament

The peaks in the left-hand portions of the curves of Figs. 2 and 4 are very striking, and they indicate the presence of effects which vary rapidly with E_g. The signal voltages applied between grid and filament are small, and it is to be expected that they will produce the greatest effect on the motion of electrons or ions between filament and grid when the steady electric field in this region is smallest. In Fig. 2 the tube constants and contact potentials are such that when E_g is about −2, the combined fields of grid and anode are just able to produce a field at the negative end of the filament which accelerates electrons away from the filament, or more correctly, from the potential minimum (due to electron space charge) which surrounds the filament. As E_g is made more positive, this region of zero or small field moves along the cathode towards the positive end. Note that in the $E_f=3.1$ curve of Fig. 2 the peaks extend over a range of 3 volts from $E_g=-2$ to +1, and that in the $E_f=5$ curve they extend over a range of 5 volts from $E_g=-2$ to +3. That is, the "peak" phenomenon extends over a range of E_g wherein the combined values of grid and anode voltages are such as to produce a very small or zero field at some point on the filament. For larger positive fields the phenomenon changes over to the "steady rise" shown at the right. In Fig. 4 the "peak region" extends over a range of E slightly greater than E_f.

This discussion would lead us to expect that if we consider only the current coming from a limited region of the filament, then the contribution from this region to the total rectification will be greatest when the electric field in this region is small. This conclusion is supported by an experiment

made with tube B. The polarity of the filament was such that the small anode (Fig. 1) covered the negative end of the filament. The two sections of the grid were connected together, and the signal applied as before. The anode circuit was arranged so that either the resulting change Δi_p in the total anode current, or the change $\Delta i_p'$ in the current to the small anode only, could be measured. Fig. 5 shows Δi_p and $\Delta i_p'$ as functions of E_g for filament voltages 3.67 and 5.0. The other conditions were: bulb temperature 25°C, $E_p = 45$, signal voltage 0.044, frequency 1050 kilocycles. In the $E_f = 3.67$ curve we note that the left hand negative peak in Δi_p is matched by the negative peak in $\Delta i_p'$ showing that at $E_g = -2$ the greater part of the

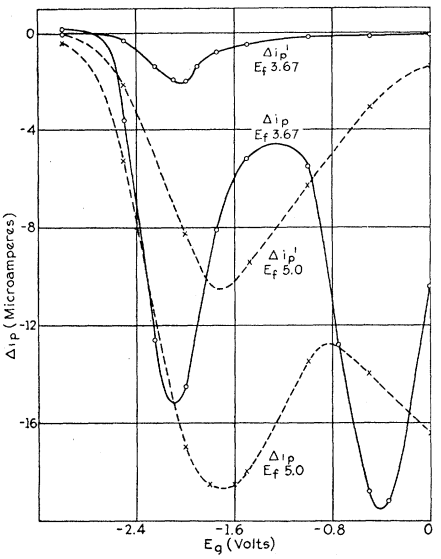

Fig. 5. Δi_p and $\Delta i_p'$ as functions of E_g for filament voltages 3.67 and 5.0.

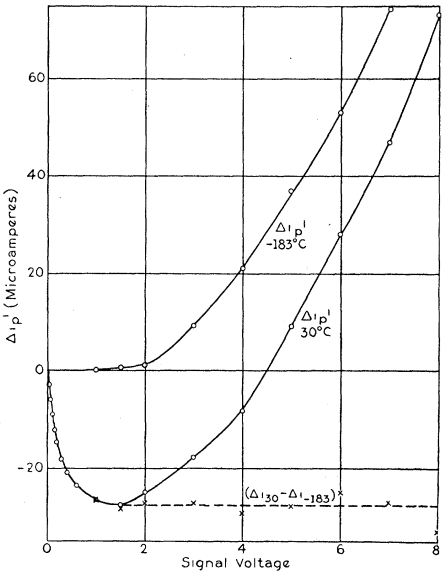

Fig. 6. $\Delta i_p'$ as a function of signal voltage for temperatures of 30°C and −183°C. The difference between the two curves (dotted curve) is a measure of the caesium type rectification.

rectification is taking place near the negative end of the filament. On the other hand there is no trace of a peak in $\Delta i_p'$ corresponding to the peak in Δi_p at $E_g = -0.4$, thus showing conclusively that this peak is due to rectification at a portion of the filament entirely outside the small anode. A similar behavior is evident in the $E_f = 5$ curves.

We are thus led to the idea that in the peak region the rectification is due to a phenomenon taking place chiefly over a limited region of the filament where the field acting on ions or electrons is small. The reason for the rectification being greater at some parts of the filament than at others, thus giving rise to the peaks, is best deferred until later (section IX).

V. Variation of Static Rectification with Signal Voltage

We have seen in the preceding sections that a small radio frequency signal applied between grid and filament causes a decrease in i_p. If however the signal voltage is made much larger (several volts) it produces an increase in i_p. This increase is due to ordinary rectification by curvature of the anode characteristic, which at these large signal voltages over-balances the other type of rectification which we have been discussing. The two types of rectification may be separated in the following way. The experiment to be described was done with the current to the small anode only of tube *B*. The conditions of the experiment where $E_f=5$, $E_p=45$, signal frequency 600 kilocycles. In Fig. 6 the curve marked $\Delta i_p'(30°)$ shows $\Delta i_p'$ as a function of signal voltage, with the bulb at 30°C. The tube was then put in liquid air and a second curve taken $\Delta i_p'(-183°)$. The first curve includes both the caesium type rectification, and the anode characteristic curvature rectification; the second, only the latter. The difference between the two curves is therefore a measure of the caesium type rectifi-

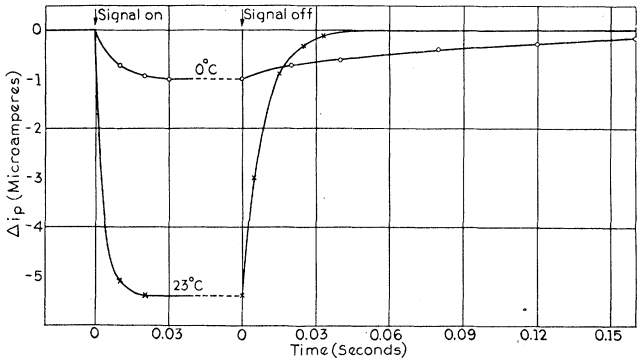

Fig. 7. Time lag effects.

cation, and is shown by the dotted curve. This dotted curve reaches a constant value at a signal voltage of about 1.5, and remains at that value for stronger signals.

This saturation behavior suggests that the caesium ions, whose motion is affected by the signal in such a way as to produce rectification, may all be removed from the vicinity of the cathode by a moderately strong signal, so that stronger signals can produce no greater effect.

VI. Time-lag Effects

Tube *A* was placed in a bath at 23°C, with $E_p=22.5$, $E_g=-1.65$, $I_f=0.194$ amp. (filament temperature about 1700°K). An Einthoven galvanometer was connected in the anode circuit, with a compensating battery and high resistance connected across it, so that only a small residual fraction of the anode current flowed through the galvanometer fiber. An alternating voltage (0.053 volt, 1050 kilocycles) was applied between grid and filament

at the time marked "signal on" Fig. 7, and removed at the time marked "signal off." The curve marked 23°C is a plot of the photographic record obtained in this way. The tube was then placed in ice and water, E_g changed to -1.8, and the record marked 0°C in Fig. 7 taken in a similar way. Both records show that when the signal was applied i_p fell rapidly to a minimum value, and when the signal was removed recovered at a rate which depended markedly on the bulb temperature. Fig. 8 shows semi-logarithmic plots of the rate of recovery of the anode current. Time is measured from the instant of removing the signal, and the ordinates measure the amount by which i_p lies below its final value. If Δi_{p0} is the maximum change in i_p, then both recovery curves are of the form

$$\Delta i_p = \Delta i_{p0} e^{-t/\tau}.$$

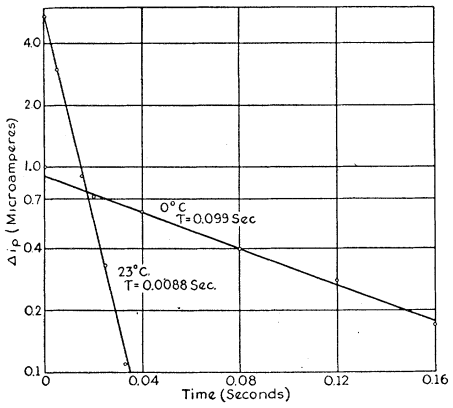

Fig. 8. Semi-logarithmic plot of the rate of recovery of the anode current.

The time constants τ are 0.0088 and 0.099 sec. at 23° and 0°C respectively. The pressures of caesium vapor at these temperatures are respectively 14.7×10^{-4} and 1.12×10^{-4} barye, so that the time constants are approximately proportional to the pressures. (There is some error in τ for the 23° curve, resulting from sluggishness of the Einthoven).

Similar time-lag records were taken for tube B, with the bulb at 0°C, and other conditions varied. Fig. 9 shows two records illustrating the effect

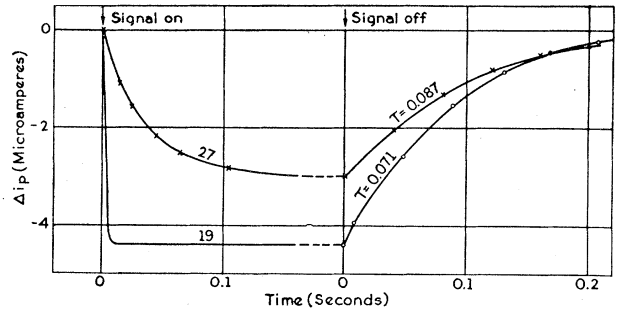

Fig. 9. Two records, tube B, illustrating the effect of strong and weak signals.

of strong and weak signals. For record 19 the conditions were $E_f=3.1$ (1700°K), $E_p=22.5$, $E_g=-1.7$, signal voltage 0.050, frequency 600 kilocycles. The current fell to its minimum value in about 0.01 sec. after applying the signal. The recovery was slow, with a time constant 0.071 sec. For record 27 the conditions were $E_f=3.1$, $E_p=22.5$, $E_g=-1.6$, signal

voltage 0.004, frequency 600 kilocycles. With such a weak signal a time of about 0.15 sec. was required for the current to fall to its minimum value after applying the signal. The rate of recovery was not very different from that of record 19, τ being 0.087 sec.

The decrease in i_p when the signal was applied was due, according to the theory mentioned in section I, to the removal of caesium ions from the region around the filament, with a consequent enhancement of electron space charge limitation of current. The initial parts of the records of Fig. 9 are in agreement with this idea, for it is to be expected that a weak signal (curve 27) would take much longer to sweep away the ions than would a strong signal (curve 19).

The recovery time lags observed at 0°C indicate that a time of several tenths of a second is required for the ion concentration around the filament to build up to its original value. This time is enormous compared with the time required for an ion to move from filament to grid (of the order one microsecond). The positive ion supply around the filament is renewed by ionization of caesium atoms at the surface of the hot filament, and the ion current from the filament depends on the amount of caesium adsorbed on its surface. When the caesium ion concentration around the filament is depleted by the signal, the amount of caesium adsorbed on the filament is also decreased. Hence, when the signal is removed the amount of caesium adsorbed on the filament must first be built up to normal value before the ion concentration around the filament can return to normal. That is, the recovery time lag measures the time required for a certain small fraction of the filament surface to become coated with adsorbed caesium. The initial loss time lags measure the time required for this adsorbed caesium to be removed from the filament (as ions) by the signal.

Some support is given this view by the fact that when the filament temperature is higher the time lags are smaller, although the actual values of Δi_p are larger. One would expect the amount of caesium adsorbed on the filament to be less at higher filament temperatures. The conditions for record 9 (Table I) were $E_f=5$ (1940°K), $E_p=22.5$, $E_g=-1.6$, signal voltage 0.010, frequency 600 kilocycles. The fall in i_p was complete in about 0.015 sec., and the time constant of the recovery was about 0.028 sec. The recovery time constant is thus about one third of that obtained in record 27, when the filament was at 1700°K.

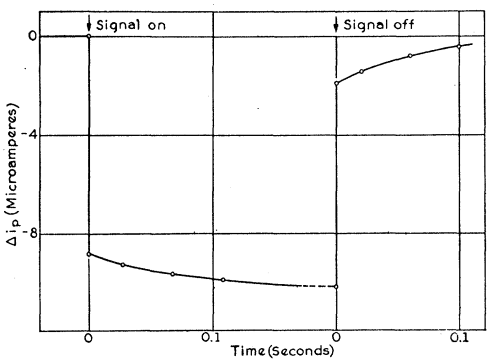

Fig. 10. Time lags (tube *B*) when steady change of potential was applied between grid and filament.

Fig. 10 (record 22) shows the time lags observed when a steady change of potential was applied between grid and filament of tube *B*. The con-

ditions were $E_f=3.1$, $E_p=22.5$, $E_g=-1.7$, signal voltage -0.05, bulb temperature 0°C. When the change in voltage was applied to the grid there was first an extremely rapid decrease in i_p (corresponding to the ordinary mutual conductance change in a high-vacuum tube), followed by a slow decrease taking about 0.15 sec. to reach completion. This slow decrease represents the removal of caesium ions from a portion of the filament which became positive to the grid when E_g was changed. When the grid voltage was restored to its original value, there was first a rapid change in i_p, followed by a slow change as the caesium ion concentration was gradually built up to its normal value. The recovery time constant was 0.068 sec.

A summary of some of the time-lag records is given in Table I.

TABLE I. *Time lags.*

Tube	Record	Bulb °C	E_f	E_p	E_g	Signal *Volts*	k.c.	τ sec.
A	—	23	3	2.25	−1.65	0.053	1050	0.0088
	—	0	3	22.5	−1.8	0.053	1050	0.099
B	8	0	5	−1	−1	0.012	600	≪.01
	9	0	5	22.5	−1.6	0.010	600	0.028
	19	0	3.1	22.5	−1.7	0.050	600	0.071
	20	0	3.1	22.5	−0.15	0.050	600	0.083
	22	0	3.1	22.5	−1.7	−0.050	0	0.068
	24	0	3.1	6	6	0.150	600	≪.01
	25	0	3.1	−0.9	−0.9	0.020	600	0.128
	27	0	3.1	22.5	−1.6	0.004	600	0.087
	29	0	3.1	22.5	−1.6	0.004	250	0.097
	30	0	3.1	22.5	−1.6	0.050	100	0.099

Note that in record 24, where both grid and anode were strongly positive to the most positive part of the filament, there is no appreciable recovery time lag. This must mean that in this type of rectification (the steady-rise part of curves like Fig. 4, but at $E_f=3.1$ instead of $E_f=5$) there is no drainage of caesium ions from the region around the filament. This is to be expected, since the ions could hardly move, under the influence of a small a.c. voltage, to an electrode which was at least 3 volts positive to their point of origin. The rectification in this case must arise from a redistribution of the ions in the space between filament and grid, under the influence of the signal. (See section VIII).

VII. Static Rectification at Various Frequencies

The experiments to be described were made with tube *B*. The values of E_p, E_g and signal voltage were held constant, and the signal frequency was varied from 15 to 1700 kilocycles, Δi_p being noted at each frequency.

Fig. 11 shows two frequency characteristics for which the conditions were $E_p=22.5$, $E_g=-1.6$, signal voltage 0.050, bulb temperature 31°C. For the upper curve the filament temperature was 1700°K, and for the lower 1940°K. The values of Δi_p are largest for frequencies of 250 and 350 kilocycles respectively.

Figure 12 shows frequency characteristics for changes in the current to the small anode only of tube B. The conditions were $E_p = 45$, signal vol-

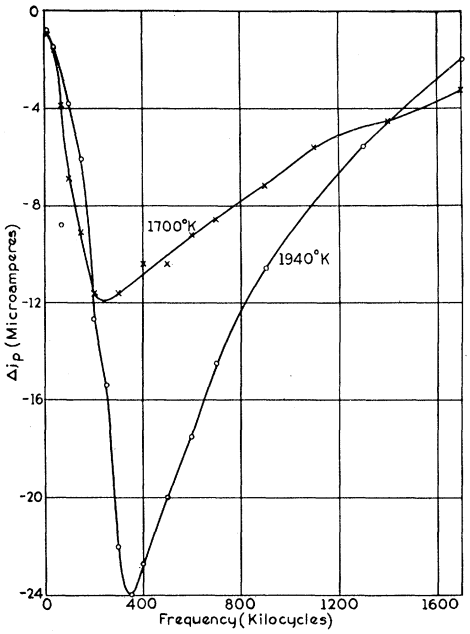

Fig. 11. Two frequency characteristics for tube B.

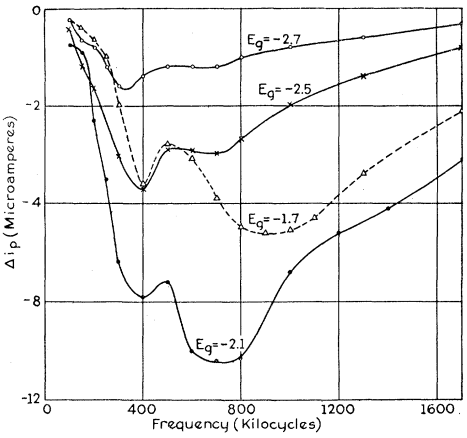

Fig. 12. Frequency characteristics for changes in current to small anode only of tube B.

tage 0.050, bulb temperature 30°C, filament temperature 1940°K, and various values of E_g ranging from -2.7 to -1.7 as marked on the curves. Note that the maximum of rectification moves from 300 to 900 kilocycles as the grid voltage goes from -2.7 to -1.7.

Fig. 13 shows a frequency characteristic for tube B (whole filament) when grid and anode were both positive to the most positive part of the filament. The conditions were $E_p = 10$, (both measured from the negative end of the filament as usual), filament temperature 1940°K ($E_f = 5$) bulb temperature 29.5°C, signal voltage 0.044. The rectification is largest at 670 kilocycles.

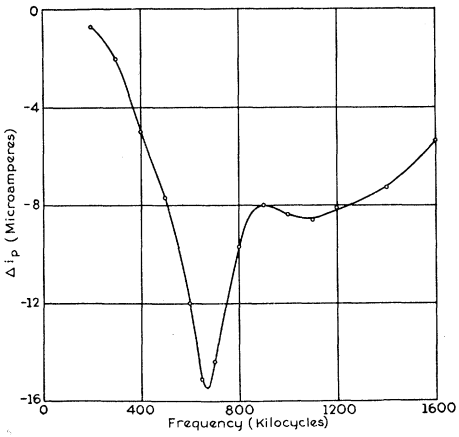

Fig. 13. Frequency characteristic for tube B when grid and anode were both positive to most positive part of filament.

The shape of these frequency characteristics suggests that some resonance phenomenon is involved in the mechanism of rectification. It seems

likely that the maximum rectification is obtained when the period of the applied signal agrees with the natural period of oscillation of the positive ions about the potential minimum surrounding the cathode. We shall defer further discussion of this to the next section.

VIII. Theory of the Rectification Effects

We shall make first some calculations of the potential distribution and space-charge density in the region near the cathode. We shall have to make these calculations for the case of parallel plane electrodes, instead of for a wire between two planes, which is the actual geometry of the tubes. However, the calculations will have qualitative, and some quantitative, interest. The necessary equations, with reference to earlier work, have been given by Langmuir[4] in a convenient form for calculation.

The distance from filament to grid in tubes A and B was about 0.1 cm. For calculation involving the field near the cathode we may replace the triode by a diode having an (effective) anode at the position of the triode grid. Accordingly we shall make our calculations for two parallel plates of thoriated tungsten each of area 1 sq. cm and 0.1 cm apart. The cathode is at zero potential, and at a temperature (T) either 1700°K or 1940°K. As we are interested in cases where the potential difference between cathode and (effective) anode is small, we shall suppose the anode potential (V_2) to be $+0.5$ volt. We need the following data

$T=1700$	$T=1940$
$i_0=0.224$ amps.	$i_0=2.79$
$V_2=0.1464(\eta_2-\eta_1)$	$V_2=0.1671(\eta_2-\eta_1)$
$\xi_2=\xi_1+348.5(i)^{1/2}$	$\xi_2=\xi_1+314(i)^{1/2}$

where i_0 is the saturation emission, i is the actual current, and η and ξ are the voltage and space parameters defined in Langmuir's paper. By assuming values of i and finding the corresponding values of V_2, we find by interpolation that when $V_2=0.5$ the values of i, V_m (voltage at the potential minimum) and x_m (distance of potential minimum from cathode in cm) are as given in Table II.

Table II.

T	i (amp)	i/i_0	V_2	V_m	x_m
1700°K	9.56×10^{-4}	0.00427	0.5	−0.790	0.0219
1940	15.3	0.000549	0.5	−1.259	0.0207

Using these values for i/i_0 we may then calculate the potential V_x at any distance x from the cathode. The result of this calculation is shown in the curves of Fig. 14.

We see from these curves that an electron leaving the surface of the cathode is in a retarding field, and only those electrons will escape to the anode whose initial thermal velocities perpendicular to the cathode are

[4] Langmuir, Phys. Rev. **21**, 419 (1923).

greater than $(2eV_m/m)^{1/2}$. On the other hand, a positive ion, formed by surface ionization at the cathode will be attracted towards the potential minimum. If the initial velocity of the ion is less than that corresponding to 0.5 volt it will be unable to reach the anode, and will oscillate out through the potential minimum and back to the cathode. Then it may be reemitted as a positive ion. If the ion loses energy by collision with gas molecules (caesium vapor plus residual gas) it will not return to the cathode, but will execute oscillations of diminishing amplitude about the potential minimum. In this connection it may be noted that the addition of an inert gas at low pressure increases the rectification considerably, presumably because the

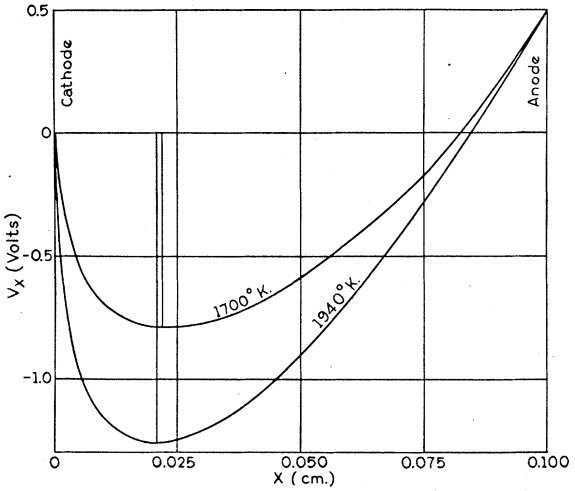

Fig. 14. Theoretical curves for potential distribution between parallel planes of thoriated tungsten.

increased number of collisions leads to the accumulation of more caesium ions at the potential minimum. Now if an a.c. signal is applied between cathode and anode whose period agrees with the period of the ions in this potential "trough," the ions will build up large oscillation amplitudes by a resonance effect, and may finally be able to flow to the cold anode, where they acquire electrons and escape as neutral atoms.

To estimate the natural period of the ions about the potential trough, we try to fit parabolas $x^2=aV$, where a is a constant, to the potential distribution curves on each side of the potential minimum. Then by adding together the half-periods for the two parabolas we obtain the total period. If the potential follows the law $V=x^2/a$ we have that the force on an ion is

$$-edV/dx=-2ex/a=Md^2x/dt^2$$

whence the period τ is

$$\tau=2\pi(Ma/2e)^{1/2}.$$

Taking the mass of the caesium ion (M) to be 2.21×10^{-22} gm, we have

$$\tau=5.23\times10^{-5}(a)^{1/2}$$

where $a=x_m/(V_m)^{1/2}$, x_m and V_m being expressed in cm and volts respectively.

In this way we find the natural periods at 1700°K and 1940°K to be 2.43×10^{-6} and 1.96×10^{-6} sec. respectively. The corresponding frequencies are 411 and 510 kilocycles respectively. These agree as to order of magnitude, and relative value, with the frequencies for maximum rectification in Fig. 11 (250 and 350 kilocycles).

Thus it appears that if the (effective) anode voltage is near that of the cathode, this resonance oscillation phenomenon explains how ions are drained away from the potential minimum region, thus increasing the electron space charge there, making V_m slightly more negative, and reducing the flow of electrons to the anode. On the other hand, when the anode is highly positive to the cathode (as in the right hand part of Figs. 2 and 4, and in record 24, Table I) this explanation cannot be valid.

We have suggested above that if the ions lose energy by collision with neutral atoms, their amplitude of oscillation will decrease, and they will accumulate around the position of the potential minimum at x_m. If we have a fixed total number of ions between cathode and anode, it may be that these will be most effective in raising the potential at x_m (and thus allowing a larger electron current to flow) when they are clustered together at x_m. When the signal is applied the ions will be set in motion and will, on this view, be less effective in raising the potential at x_m, so that the electron current to the anode will decrease. When the signal is removed the ions, as they lose energy by collision with neutral atoms, will again accumulate at x_m and the anode current will rise. There has been no drainage of ions to an electrode in this process, so that time lags of the kind discussed in section VI will be absent. This agrees with record 24 in Table I, where the recovery time lag was too short to be measured.

That the ions are most effective in raising the potential at x_m when they are grouped at x_m, may be shown by a simple argument suggested to the writers by Dr. L. Tonks. In Fig. 15 the two points A and B are held at the same potential. If we place a positive charge ρ at C, the potential will rise linearly along AD and fall linearly along DB, the slopes of these lines being fixed by ρ. If instead of concentrating ρ at C we distribute it between A and B, the potential distribution will follow some curve such as $AD'B$. Since the total ρ is the same as before, it follows from Gauss' theorem that AD and DB will be the tangents to the curve $AD'B$ at A and B. Hence D' must lie below D. This shows then that the ions will be most effective in raising the potential at x_m when they are grouped together at x_m.

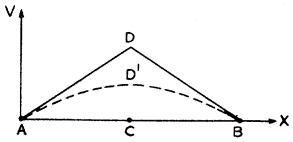

Fig. 15.

In the discussion in this section it has been assumed that the electron density between the electrodes is so small that an ion may travel back

and forth across the potential minimum many times before being neutralized by an electron. We shall therefore make a calculation of space-charge density to see if this is actually the case. We follow a method used by Dr. Langmuir in unpublished work, and start from Eq. (11) of reference 4.

$$\xi = \int_0^\eta [\epsilon^\eta - 1 \pm \epsilon^\eta P(\eta^{1/2}) \mp 2(\eta/\pi)^{1/2}]^{-1/2} d\eta \tag{1}$$

where $\eta = e(V - V_m)/kT$ and $\xi \propto (x - x_m)$. P denotes the Gauss probability function, and $\epsilon = 2.718.....$ The upper signs are for $x < x_m$ and the lower for $x > x_m$ (Fig. 14). When $x > x_m$ we find from Eq. (1)

$$(d\eta/d\xi)^2 = \epsilon^\eta \text{ erf } \eta^{1/2} - 1 + 2(\eta/\pi)^{1/2} \tag{2}$$

$$d^2\eta/d\xi^2 = \tfrac{1}{2}\epsilon^\eta \text{ erf } \eta^{1/2} \tag{3}$$

where erf $\eta^{1/2} = 1 - P(\eta^{1/2})$.
and when $x < x_m$

$$(d\eta/d\xi)^2 = 2\epsilon^\eta - 1 - \epsilon^\eta \text{ erf } \eta^{1/2} - 2(\eta/\pi)^{1/2} \tag{4}$$

$$d^2\eta/d\xi^2 = \epsilon^\eta(1 - \tfrac{1}{2} \text{ erf } \eta^{1/2}) \tag{5}$$

If ρ_m is the charge density at x_m we find from Poisson's equation and Eqs. (3), (5).

$$\rho = \rho_m \epsilon^\eta \text{ erf } \eta^{1/2} \text{ for } x > x_m \tag{6}$$

$$\rho = 2\rho_m \epsilon^\eta (1 - \tfrac{1}{2} \text{ erf } \eta^{1/2}) \text{ for } x < x_m \tag{7}$$

Now the density ρ_1 at the cathode surface is connected with the temperature and the saturation emission i_0 by the relation

$$\rho_1 = i_0(2\pi m/kT)^{1/2} = 6.43 \times 10^{-6} i_0/T^{1/2} \tag{8}$$

when i_0 is measured in amperes.
Hence from Eq. (7) knowing i_0 and ρ_1 we can find ρ_m. For the case of the thoriated tungsten plates, cathode at 1700°K, we have $i_0 = 0.224$ amp, whence by Eq. (8), $\rho_1 = 2.2 \times 10^{11}$ electrons/cm^3. We found previously (Table II) $V_m = -0.790$, whence η_1, the value of η at the cathode is found to be 5.4. From Eq. (7) we find $\rho_m = 4.7 \times 10^8$ electron /cm^3.

From the equations above we can find the electric field at any point between the electrodes. Thus for $x > x_m$

$$(dV/dx)^2 = -8\pi\rho_m(kT/e)(\epsilon^\eta \text{ erf } \eta^{1/2} - 1 + 2(\eta/\pi)^{1/2} \tag{9}$$

and for $x < x_m$

$$(dV/dx)^2 = -8\pi\rho_m(kT/e)(2\epsilon^\eta - 1 - \epsilon^\eta \text{ erf } \eta^{1/2} - 2(\eta/\pi)^{1/2} \tag{10}$$

Now dV/dx is zero at x_m, so that from Gauss' theorem and the values of dV/dx at anode and cathode, we may find from Eq. (9) and Eq. (10) the total charge between x_m and the anode, and between the cathode and x_m.

Adding the results we find for the total charge between the plates (1 sq. cm in area) in the case we are considering

$$N_e = 1.75 \times 10^8 \text{ electrons}.$$

Since the plates are 0.1 cm apart, this corresponds to an average electron density $n_e = 1.75 \times 10^9$ per cm^3.

We wish to calculate the average time which will elapse before an ion immersed in an electron atmosphere of this density will recombine with an electron. There appears at present to be no accurate method[5] for making this calculation, but we can estimate a lower limit for the life of an ion by calculating the rate at which electrons come close enough to the ion to have their paths deflected through $\pi/2$. It seems reasonable that a much closer encounter would be required for capture. We use Maxwell's formula as quoted by Richardson or Jeans.[6] For Coulomb's law of force this formula reduces to

$$\sin\theta = 1/(1+\alpha^2)^{1/2}$$

where θ is half the angle through which the path of an electron is turned,

$$\alpha = -bmv^2/e^2 \quad ,$$

and b is the perpendicular from the ion to the path of the electron when at a great distance from the ion. m, v, and e are the mass, initial velocity and charge of the electron. When $\theta = \pi/4$, $\alpha = 1$. For an electron having the average Maxwellian velocity for 1700°K, $mv^2/e^2 = 3.08 \times 10^6 \text{cm}^{-1}$, so that $b = 3.25 \times 10^{-7}$cm. The total number of electrons per second whose paths are deflected through $\pi/2$ or more will be given by the number of electrons flowing per second through the surface of a sphere of radius b. This number is

$$4\pi b^2 n_e (kT/2\pi m)^{1/2} = 14500.$$

Hence our minimum estimate of the life of an ion is 10^{-4} sec. Since the time required for an ion to cross between the electrodes is about 10^{-6} sec, it follows that the ion may oscillate back and forth many times without recombination. This is the condition we have postulated in the theory.

The probability of such a long life for a positive ion raises a question as to whether the time-lag effects of section VI might not be due to an accumulation of caesium ions around the potential minimum region, rather than to an accumulation of caesium on the filament. Quantitative deductions from the theory of the parallel plane case make this seem improbable. We have found the number of electrons between the plates to be 1.75×10^8 in the case considered above. The presence of caesium vapor does not change the anode

[5] Oppenheimer's formulae for hydrogen recombination (Phys. Rev. **31**, 349 (1928)) cannot be applied directly to the case in point.

[6] Richardson: Electron Theory of Matter (1914) p. 417 Jeans: Dynamical Theory of Gases (1916) p. 231.

current a great deal, so that the number of caesium ions between the electrodes must be less than 10^8;—if it were greater, there would be complete neutralization of electron space charge, and large currents would flow. We now wish to calculate the time required to supply 10^8 ions to the space between the electrodes, by ionization at the surface of the cathode. From unpublished data it appears that 0.02 of the caesium atoms striking a fully thoriated tungsten surface at 1700°K leave it as ions. With the bulb at 0°C ($p=1.12\times10^{-4}$ barye) the number of atoms striking a sq. cm surface per sec is 1.55×10^{13}. Hence the number of ions emitted per sec. is $3.1\times10.^{11}$ This would supply the quota of 10^8 for the space between the electrodes in 3×10^{-4} sec, so that no time lags of tenths of a second should be found. It thus seems likely that accumulations of caesium on the filament (section VI) is the correct explanation of the time lags. The time lags observed at 0°C correspond roughly to the time required for enough caesium atoms to strike the filament to cover 1/1000 of its surface.

IX. The Flow of Ions Along the Potential Minimum

In the preceding sections we have confined the discussion to the flow of ions perpendicular to the filament surface. Since there is a drop of potential along the filament, there will be a flow of ions parallel to the filament. It is necessary to take this into account in order to understand the tube characteristics fully.

The potential at a point just outside the filament surface in one of our tubes is indicated semi-quantitatively by the full line in Fig. 16 for $E_f=3.1$. Starting at the cold negative terminal where the filament is coated with caesium, as we go to the right the potential first falls as the caesium layer is driven off the hot filament. This drop is about -1.3 volts, the contact potential difference[7] between an adsorbed caesium layer and thoriated tungsten. The potential then rises as we move to the right, the rise being caused by the voltage drop in the filament. At the center the filament is cooled by the anchor wire (Fig. 1) caesium deposits on it, and causes a little hump in the potential. The potential curve proceeds in a similar way up to the positive terminal.

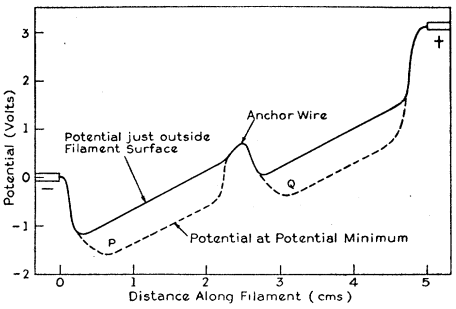

Fig. 16. Distribution of potential along filament.

At the ends and center of the filament, where there is very little electron emission, the filament surface itself will be the "potential minimum." In the regions where there is large electron emission the potential of the "po-

[7] See an article on contact potential measurements by Langmuir and Kingdon to be submitted to the Physical Review.

tential minimum" may be lower than that just outside the filament surface by an amount of the order of 1 volt (Table II). The course of the potential minimum is indicated by the dotted curve in Fig. 16.

Ions will therefore flow along the potential minimum trough from the more positive parts of the filament towards P and Q. There seems little doubt (section 8) that the life of the ions is great enough for this flow to increase appreciably the concentration of ions at P and Q. This accounts for the two peaks usually observed in the static rectification curves (Fig. 2). Contamination of parts of the filament surface or incomplete formation of the thoriated surface might produce several regions like P and Q, leading to several peaks, as in Fig. 4. In this connection it may be noted that the characteristics of Fig. 2 were taken some days after Fig. 4, and that the filament had been given a severe thermal treatment between.

X. Comparison of Static Rectification with the Rectification of Modulated Radio Frequency Signals

The static rectification curves of section III represent the response of the tube to radio frequency signals of zero frequency of modulation. The occurrence of the time lags described in section VI indicates that if a modulated signal is used the ion concentration around the cathode will not have time to return to its normal maximum value in the interval between the peaks of

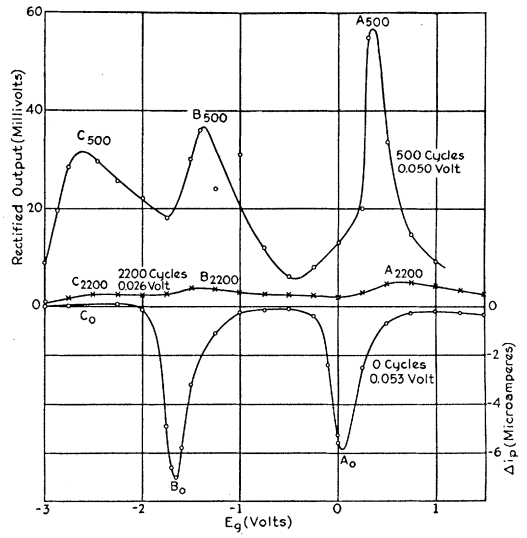

Fig. 17. Comparison of rectification for modulated and unmodulated signals.

the modulating wave. The rectified currents will therefore be smaller than those observed in the static rectification tests, and will decrease as the frequency of modulation is increased.

The first point may be illustrated by an experiment made with tube A. The conditions were $E_f = 3.3$ (1700°K), $E_p = 22.5$, bulb at room temperature,

signal frequency 1000 kilocycles. In the first test the signal was modulated at 500 cycles, signal voltage 0.050, and the 500 cycle voltage generated across a 50,000 ohm output impedance in the anode circuit was measured. This voltage in RMS millivolts for various settings of E_g is indicated in the curve of Fig. 17 marked 500 cycles. The tube was then placed in another test set and a similar run made, this time using 2200 cycle modulation, 0.026 volts signal, and a 690,000 ohm output impedance. The curve obtained is that marked 2200 cycles in Fig. 17. We are indebted to Messrs. Darlington and Hitchcock for these measurements. Finally, a static rectification curve was taken using a signal voltage 0.053. The values of Δi_p are plotted in Fig. 17, using the ordinate scale on the right (microamperes.)

The two negative maxima A_0, B_0 and the positive maximum C_0 of the static rectification curve, are reproduced at approximately the same grid voltages in the curves for modulated signals. It is interesting to note that although C_0 is insignificant compared with A_0 and B_0, yet C_{500} and C_{2200} are about as large as their companion maxima. This is because the C maxima represent rectification by curvature of the anode characteristic, and involve no appreciable time lag (electron flow only) whereas the A and B maxima involve relatively large time lags (section VI). At the normal filament temperature, 1940°K, A_{500} and B_{500} would be much greater than C_{500}.

It is instructive to compare the maximum changes in anode current for the B_0 and B_{2200} maxima. The double amplitude of the current through the output impedance at the B_{2200} maximum, when the RMS voltage across it was 4×10^{-3} must have been 1.64×10^{-8} amp. The internal resistance of the tube anode circuit at $E_p=22.5$, $E_g=-1.4$ was 91,000 ohms, so that if this had been the only resistance in the circuit (as in the static rectification test) the maximum change in anode current would have been

$$1.64\times10^{-8}\,\frac{690000+91000}{91000}=1.4\times10^{-7}\text{ amp.}$$

The B_0 maximum was 7μa for a 0.053 volt signal, or since a nearly linear law is followed at very low voltages (Fig. 6), the maximum would have been about 3.4μa for 0.026 volt. This current change is 24 times as great as that at the B_{2200} maximum, the difference being due to a time lag of the order of that given in the 23°C curve of Fig. 8.

The variation of rectified output with modulation frequency is illustrated by Fig. 18, compiled from a report by Mr. Oakley. This was taken with another tube. The filament temperature was 1940°K. The signal voltage was 0.1, carrier frequency 1000 kilocycles, modulated at various frequencies from 20 to 10,000 cycles. The modulation frequency is plotted as abscissas, while the ordinates measure volts generated across a 100,000 ohm resistance in the anode circuit. Curves are shown for the bulb at 40°C and at 0°C. The variation in output with frequency is quite pronounced, and is caused by the time lag effects discussed in section VI.

Thus there are two frequency effects,–(1) a high-frequency effect due to the natural period of oscillation of the caesium ions about the potential minimum, and (2) a low-frequency effect due to the time taken for the concentration of caesium ions around the filament to be built up to normal value when the signal is removed (or falls to a minimum in modulation).

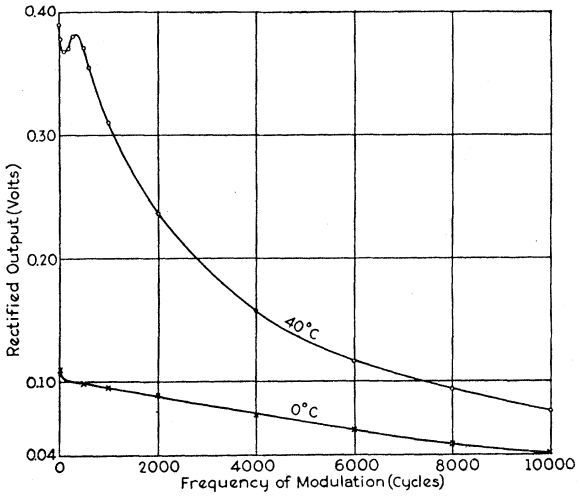

Fig. 18. Variation of rectified output with modulation frequency.

XI. Static Rectification at Low Frequencies

We saw in Fig. 10 that a d.c. change in grid voltage produces both an "instantaneous" and a slow change in anode current, the latter being due to a change in the density of caesium ions around the filament. Since this density changes as we move along the filament, we have corresponding changes in the mutual conductance curve. This is illustrated by Fig. 19 (Tube A). The ordinates measure the change in i_p when the grid was made 0.05 volt more negative; the abscissas are E_g. The negative peak at -1.8 volts is due to the caesium ion effect mentioned above, and corresponds roughly in position to the B_0 peak of Fig. 17.

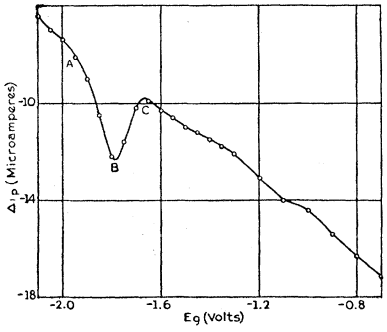

Fig. 19. $\Delta 4_p$ for a steady change in grid potential.

Proceeding now to low frequency alternating voltages applied between grid and filament, it is obvious from section VII that at frequencies below a few thousand cycles the natural period of the positive ions can play little part in the rectification. At very low frequencies the concentration of caesium ions around the cathode

can keep pace very nearly with the fluctuations in signal voltage, so that rectification due to the drainage away of these ions is small. Fig. 20 shows some static rectification curves at various frequencies taken by Mr. A. L. Samuel, using another tube. The 60 cycle curve has a marked positive peak at $E_g = -2.6$. This is "rectification by curvature," and corresponds to a region of the $i_p E_g$ characteristic where the curvature has been enhanced by the effect shown in the AB portion of Fig. 19. At higher frequencies this peak becomes much smaller, since the ion concentrations depart more and more from the steady value. Following the positive peak on the 60 cycle curve is a small negative peak. This corresponds to the region BC of Fig. 19, where the curvature of the $i_p E_g$ characteristic is small, and the "ion drainage" rectification is able to overbalance the "curvature" rectification. At more positive E_g the rectification is small, and of uncertain sign, as might be expected from the conflict between the two types of rectification, each of them giving only small effects. The curve for a 970 kilocycle signal is included in Fig. 20. Here the ion drainage rectification far outbalances the curvature type except at very negative E_g.

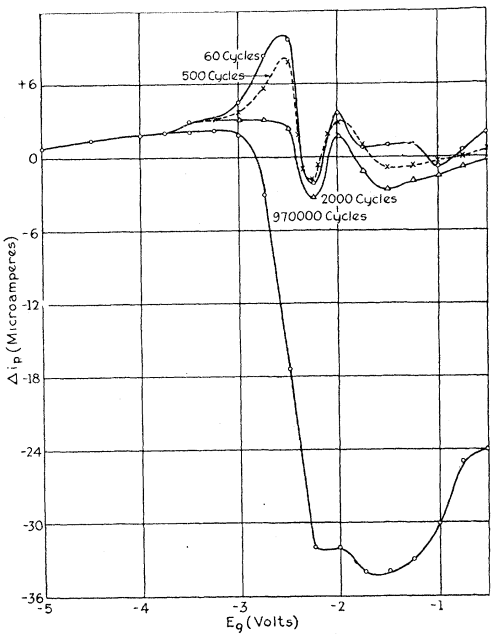

Fig. 20. Static rectification curves.

XII. Effect of State of Activation of Thoriated Cathode

The static rectification of tube A was measured when the thoriated cathode was activated to different degrees. The curves are shown in Fig. 21, the emissions being marked on them. The other conditions were $E_f = 3.25$ (1700°K), $E_p = 22.5$, signal voltage 0.053, frequency 1050 kilocycles, bulb at room temperature.

XIII. Comparison of Rectification with and without Grid Leak

The ordinary high vacuum tube is about 4 or 5 times as sensitive a detector when used with grid leak and condenser as when used without. This is not true of the alkali vapor tubes, the sensitivity being about equal in the two cases. The grid leak serves to keep the grid current small, and hence in the alkali vapor tubes fixes the point of operation at about the B maximum in Fig. 17. Since the use of a grid leak does not increase the rectification, the

change in grid current when a signal is applied must be relatively much smaller than the change in anode current. This may be accounted for as follows. The grid current is small, so the potential of the grid must be about the same as that (V_m) of the potential minimum region at the negative end of the filament. When the signal is applied and caesium ions drained off,

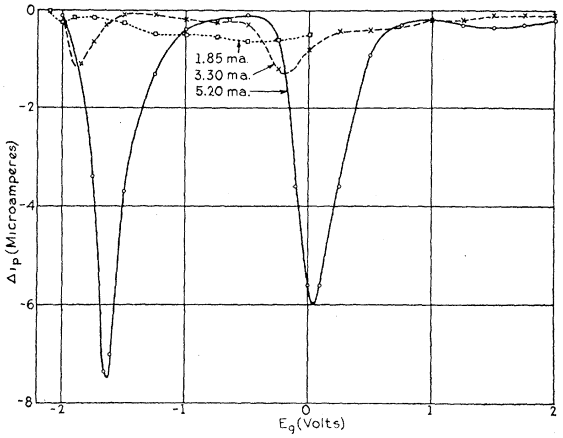

Fig. 21. Static rectification curves for tube A with thorium cathode activated to different degrees.

V_m becomes more negative. This reduces the total number of electrons escaping through the potential minimum, but at the same time will increase the fraction of the escaping electrons which goes to the grid since the grid is now on the average appreciably more positive to V_m. This action reduces fluctuations in the grid current, and thus reduces grid leak rectification.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.
February 18, 1929.