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SPACE CHARGE SHEATHS IN POSITIVE RAY ANALYSIS

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Abstract

It is shown that in the usual ionizing chambers used in the positive ray analysis of secondary ion products a positive ion sheath is formed, which concentrates the field applied across the chamber into the portion of the chamber next to the slit, leaving the rest of the space field-free. A modification of the usual apparatus was used to check this fact. The sheath thickness varies with the voltage applied across the chamber, the electron current, and the pressure, and, inasmuch as some ions are formed best in a field-free space, and some best in a field, by varying any of the three variables, we change the relative proportions of the various secondary and net primary ions reaching the analyser collector. Curves are given for the sheath thickness, and for the various ionic currents through the slit for several conditions in the chamber, and these curves are checked with data given by several observers. This analysis shows the complexity of the phenomenon and indicates modifications in the experimental procedure which may help to clarify the interpretation of experimental results.

THE method of positive ray analysis applied to the study of products of ionization was first used by Smyth in 1922 and since that time more than twenty papers have been published giving the results obtained in various gases by Smyth, Hogness and Lunn, Kallmann and others.[1–24] Some difficulty has been found by these workers in distinguishing between the primary and secondary processes which produce atomic, molecular or associated

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[1] Smyth, Proc. Roy. Soc. **102,** 283 (1922), Mercury.
[2] Smyth, Proc. Roy. Soc. **104,** 121 (1923), Nitrogen.
[3] Smyth, Proc. Roy. Soc. **105,** 116 (1924), H_2 and O_2.
[4] Smyth, J. Franklin Inst. **198,** 795 (1924).
[5] Smyth, Phys. Rev. **25,** 452 (1925), Hydrogen.
[6] Kondratjeff and Semenoff, Zeits. f. Physik **22,** 1 (1924), $HgCl_2$.
[7] Kondratjeff, Zeits. f. Physik **32,** 535 (1925), $ZnCl_2$ and NaI.
[8] Hogness and Lunn, Nat. Acad. Sci. **10,** 398 (1924), H_2.
[9] Hogness and Lunn, Phys. Rev. **26,** 44 (1925), H_2.
[10] Hogness and Lunn, Phys. Rev. **26,** 786 (1925), N_2.
[11] Hogness and Lunn, Phys. Rev. **27,** 732 (1926), O_2.
[12] Hogness and Lunn, Phys. Rev. **30,** 26 (1927), NO.
[13] Barton, Phys. Rev. **25,** 469 (1925), Argon.
[14] Barton, Phys. Rev. **30,** 614 (1927), HCl.
[15] Barton and Bartlett, Phys. Rev. **31,** 822 (1927), H_2O.
[16] Harnwell, Phys. Rev. **29,** 683 (1927), Mixtures.
[17] Harnwell, Phys. Rev. **29,** 820 (1927).
[18] Smyth and Brasefield, Nat. Acad. Sci. **12,** 443 (1926), H_2.
[19] Smyth and Stueckelberg, Phys. Rev. **32,** 779 (1928), Mixtures.
[20] Hogness and Harkness, Phys. Rev. **32,** 784 (1928), Iodine.
[21] H. Kallmann and M. A. Bredig, Zeits. f. Physik **34,** 736 (1925).
[22] H. Kallmann and M. A. Bredig, Zeits. f. Physik **43,** 16 (1927).
[23] K. E. Dorsch and H. Kallmann, Zeits. f. Physik **45,** 565 (1927).
[24] Brasefield, Phys. Rev. **31,** 52 (1928).

ions, and in confirming each other's results. This seems to be partly because they have not recognized the importance in their method of space charge sheaths such as were studied by Langmuir.[25] Such effects were not considered in the only attempt at a rigorous analysis of the situation made by Smyth in 1925. The present authors wish to modify and extend his treatment in the light of present knowledge. In particular they wish to emphasize the importance of space charge sheaths and consequently will begin with a qualitative discussion and experimental demonstration of the effect of such sheaths.

All the positive ray experiments use a negative collecting electrode pierced by a narrow slit. The beam of ionizing electrons is directed straight at the electrode and the conditions are at first sight very different from those of Langmuir.[25] We will, therefore, discuss first a simpler apparatus in which the electron beam was directed parallel to the collector at a distance 2 cm from it. In this case we can apply Langmuir's equation for the thickness of the positive ion sheath on the surface of a plane negative electrode in an ionized gas. It is

$$x^2=0.055V^{3/2}/i(M)^{1/2} \qquad (1)$$

where x is the sheath thickness in cms, V the applied collecting voltage, i the current density in microamperes per cm^2 and M the molecular weight of the positive ions. We see that the formation of a narrow sheath is favored by small collecting voltages and by large positive ion currents.

Now the potential drop is all concentrated in the sheath. Therefore in the particular apparatus we are considering when the collector is covered by a narrow sheath, the ionization will all take place in the field-free space outside, and the positive ions reaching the collector will be limited to the number diffusing per second across the boundary of the sheath. If for example the value of the solid angle ω subtended at the electron beam by the sheath be about 3/4, the fraction of the ions reaching the electrode, $\omega/4\pi$, will be about 1/16. On the other hand, when the whole beam becomes subject to the collecting field all the ions will be drawn to the electrode, so that we should expect the apparent efficiency, as measured by the number of ions collected per microampere of primary current to increase sixteen-fold as the field-free space vanishes. This effect was found in nitrogen and hydrogen in a tube of this type, by varying nothing but the thermionic current, which in turn varied the positive ion current. If the presence of a sheath is the true

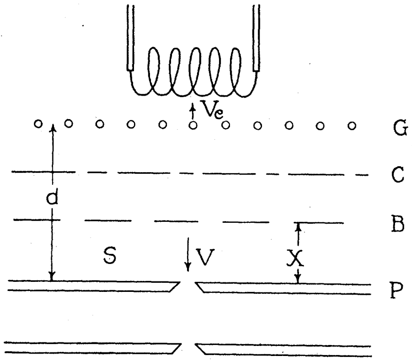

Fig. 1. Arrangement of usual positive ray analysis chamber.

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[25] Langmuir and Mott-Smith, G. E. Rev. **26**, 731 (1923).

cause of the phenomena, then we should expect to obtain a nearly constant efficiency if we work under conditions where there can be no sheath. Using a high collecting voltage—50 volts instead of 4 volts—and low values of the primary current, this was found to be the case.

Usual type of apparatus. Confirmed in our hypothesis by the above results, let us return to the form of apparatus ordinarily used. We will try to obtain a physical picture of the conditions of ionization when the collecting electrode is covered by a sheath. Electrons passing through the grid G produce ions in the space between G and the collecting electrode P. B is the boundary of the sheath S. The electric field in S is a maximum near P and falls almost linearly to zero at B. If the voltage applied to P is small, and the energy of the electrons is considerably greater than the ionization potential of the gas, then the ionization will be approximately uniform. In this case, except for the ions they form, we can neglect the effect of the primary electrons since a positive space charge will be formed almost as if they were absent. This is because the velocity of the electrons used is more than a hundred times that of the positive ions and therefore the density of electrons is in most cases negligible compared to that of the positive ions. We thus have the somewhat paradoxical formation of a sheath of positive space charge over an electrode to which a net negative current is flowing. Thus we can consider the space between G and P as one where there are no primary electrons, but in which I positive ions and I slow secondary electrons are formed per cc per second, and consider the behavior of this simpler case as being practically equivalent to the actual case.

The ions formed in the field-free space F will reach a high concentration, but they recombine so slowly that the loss by recombination will be negligible; at equilibrium then the number of ions diffusing out of the field-free space per second will be equal to the number formed by the electrons per second. We may consider that half of these ions diffuse upwards to the grid, and the other half downwards into the sheath. The latter plus those produced in the sheath form the positive ion current to the electrode. Let d be the distance between G and P, and x the thickness of the sheath. The ions within a distance $=(d-x)/2$ diffuse upwards to the grid and are lost. But the remainder, that is all the ions formed per second within the distance $(d+x)/2$ reach the electrode. The question may be asked: Since these ions reach the electrode whether a sheath is present or not, what is the significance of a sheath? The importance of a sheath is that ions linger in the field-free space for an interval of time about a hundred times as long, and with velocities a hundred times as small as they would if no sheath were present. This is obviously favourable to such processes as the association of H_2^+ and H_2 to form $H_3^+ + H$.

Analysis at low pressures.—Two contrasted positive-ion currents must be considered; one is the total number of positive ions reaching the electrode, and the other is the number of positive ions which arrive at the electrode nearly perpendicularly. It is on the former that the thickness of the sheath depends, but it is the latter which determines the number of ions passing through the slits to the electrometer. In the first part of the analysis which we shall now give we are concerned only with the former, our object being to show how to find the thickness of a sheath that will be present under any given conditions of ionization.

The problem of extending Langmuir's analysis to cases where some ionization takes place inside the sheath has already been considered by one of us in collaboration with W. Uyterhoeven.[26] For the case where the ionization was uniform throughout the sheath the approximate solution relating V to x was found to be

$$V=A^{1/3}[(9i/4)^{2/3}x^{4/3}+(aI/2)^{2/3}x^2] \tag{2}$$

where i is the positive-ion current in microamperes per cm^2 diffusing into the sheath across the boundary, I is the number of ions formed per cc per second, and a is a constant of value about 1.5. The constant A is equal to $4\pi(m/2e)$ where m is the mass of the ion. It should be noticed that in this and other expressions in this paper the letters i and I represent quantities which have not the same dimensions, the former being electric charge per second per unit area, and the latter being charge per second per unit volume.

In this expression the first term is the same as Langmuir's formula, and the second term allows for the additional space-charge produced by ionization inside the sheath; it shows that a higher collecting voltage V may be used to produce a sheath of any given thickness x. Near the electrode the ions which have been accelerated through the sheath are travelling with high velocity. On the other hand fresh ions which are formed by electron impact near the electrode all start with zero velocity (or rather random thermal velocities); and being heavy they make a large contribution to the space charge. It has been pointed out above that the ionic current entering the sheath from the field-free space, i is equal to $I(d-x)/2$. Substituting, and putting $(9/8)^{2/3}$ and $(a/2)^{2/3}$ each equal to unity, we have approximately

$$V=A^{1/3}I^{2/3}x^2[(d-1)^{2/3}/x+1]. \tag{3}$$

To make use of this expression we need a convenient method of evaluating I. Compton and Van Voorhis[27] have given curves showing the number of ions produced per cm of path at a pressure of 0.01 mm; let this quantity be N for the velocity of the electrons used. Then if

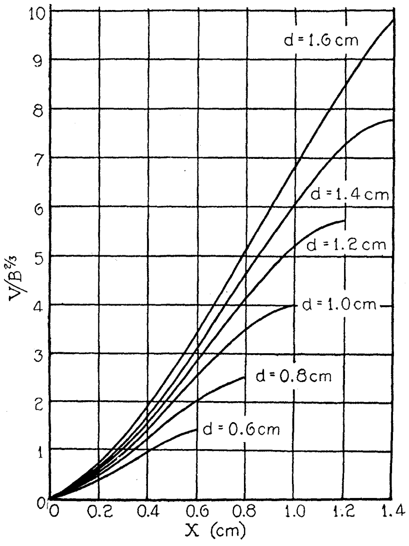

Fig. 2. Sheath thickness for various chambers and various potentials.

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[26] Morse and Uyterhoeven, Phys. Rev. **31**, 827 (1928).

[27] Compton and Van Voorhis, Phys. Rev. **27**, 724 (1926).

p is the gas pressure measured in hundredths of a millimeter, and i_e is the electron-current, we have $i_eNp=I$. The constant A depends on the mass of the ion, so that it is convenient to calculate A for the hydrogen atom and to introduce the molecular weight, M. The value of $A^{1/3}$ for the hydrogen atom is found to be 4.0. Now let $i_eNp(M)^{1/2}=I(M)^{1/2}=B$. Then we have

$$V/B^{2/3}=4x^2[(d/x-1)^{2/3}+1]. \qquad (4)$$

In Fig. 2 are plotted curves from which the thickness of the sheath x may be read off for any conditions of ionization in vessels of various depths d.

We may now consider whether this approximate solution fits the facts sufficiently well for general use at low pressures. The collecting field will retard the electrons entering the sheath, and if sufficiently large will bring them to rest near the electrode. Since they are more numerous than the ions, they will destroy any positive sheath. We must therefore explain why we have so summarily dismissed the effects of electrons. The reason is that the initial energy of the electrons must necessarily be above the ionization potential of the gas. If therefore the collecting field is to diminish their velocities to very low values, it must be a voltage from 10 to 20 volts in the gases used, in which case the space charge sheath will have already completely filled the chamber, except at unusually high pressures. It happens then that for the collecting voltages in which we are interested, around 5 volts, the presence of free electrons may be disregarded.

To sum up, as long as the pressure is small (0.01 to 0.03 mm depending on the gas) and the retarding field V is small compared to the accelerating field V_e, the electrons will traverse the chamber practically unhindered and I will be uniform. In this case the total ionic current to P per unit area will be made up of the ions created in the sheath and also those ions diffusing across S. That is the current measured by the electrometer is

$$K(Ix+i_0)=KI(d+x)/2=KpNi_e(d+x)/2 \qquad (5)$$

where K depends on the size of the slit.

If the pressure were kept constant at a small value and V alone were varied x would change in this equation, or if i_e were varied x and I would change; there would be little scattering of ions as they pass to the slit, nearly all of them would fall perpendicularly through the slit and thus be measured and the analysis would be comparatively simple. If p is varied, however, as has been the experimental practice, the analysis becomes more difficult on account of scattering effects at higher pressures. We will proceed therefore to the more general treatment required for such cases.

Analysis at higher pressures. At higher pressures the effects are complex even in the absence of a sheath; this was the problem attacked by Smyth.[5] He introduced three considerations: (1) Scattering of the ions: the probability that an ion formed at a distance y above the slit is drawn down vertically and suffers no collision with a molecule is e^{-npy}, where n is the number of collisions made by a positive ion per cm of path at unit pressure; (2) Non-uniform ionization by the electrons. When the electrons have not more than about 50 volts energy, as is usually the case, the ionization will be most intense near the grid and will fall off as e^{-npx}, where n is the number of collisions made by an electron per unit path at unit pressure. (3) Loss by ionization between the filament and grid. Since the filament is always placed close to the grid, this is only important at very high pressures, and may be allowed for by a factor e^{-npb}, where b is the distance between the filament and the grid.

In our analysis we retain the first two of these considerations but express them differently, replacing the assumption of a uniform field by that of a sheath and using the experimental value for the probability of ionization, a result not available to Smyth.

First we must correct our expression for I. The electrons make pN ionizing collisions per cm path, and create, say, a ions before they lose their ionizing power. Then the number of electrons losing their energy in traversing a space between y and $y+dy$ will be, $dn=-pNn\,dy/a$ or

$$n=ce^{-pNy/a}+c'.$$

Then if an electron current i_e per cm^2 comes through the grid, there will be available an electron current $i_e e^{pN(y-d)/a}$ at any distance y from the plate P, and the number of ions formed per second per cc will be

$$I'=pNi_e e^{pN(y-d)/a} \tag{6}$$

(the prime will be used hereafter to distinguish a value in which scattering is included from a value computed for the simple case).

Similarly, when the ions are formed, they also will be scattered. In the field free space above the sheath this random motion will not matter, except as the collisions cause secondary ions, for the speed of the ions diffusing into the sheath is so small compared to their speed in the sheath that their direction does not matter. But within the sheath the ion must travel perpendicularly to P in order to get through the system of slits below P. That is, the number counted in the analyzer are those which have been formed directly above the slit, and have fallen to the slit without suffering a deflecting collision. The number suffering two or more collisions such that finally they travel perpendicularly through the slit is negligible. If the average number of collisions per cm path is n_0p, where p is given in units of 0.01 mm, then the probability of reaching the slit from a distance y above it without colliding will be e^{-pn_0y}.

Therefore the effect of increasing the pressure will be to reduce the number of ions measured which come from above the sheath, by a factor e^{-pn_0x}. The number formed above the sheath per second and going to the sheath is half the total number formed there

$$i_0''=(1/2)\int_x^d I dy=(pNi_e/2)\int_x^d e^{pN(y-d)/a}dy=(ai_e/2)(1-e^{pN(y-d)/a})$$

and the number of these going through the slit undeflected will be

$$Ki_0' = K(ai_e/2)e^{-pn_0x}(1 - e^{pN(y-d)/a})$$

The number measured which are formed within the sheath will be

$$Ki_s' = K\int_0^x Ie^{-pn_0y}dy = KpNi_e\int_0^x e^{-p(n_0-N/a)y-pNd/a}dy$$

$$= \frac{KNi_e}{(n_0 - N/a)}\left[e^{-pNd/a} - e^{-p(n_0-N/a)x-pNd/a}\right].$$

Then the total undeflected primary ion current which is measured will be

$$Ki_t' = Ki_e e^{-pn_0x}\left[\frac{a}{2} - \left(\frac{a}{2} + \frac{N}{n_0 - N/a}\right)e^{pN(x-d)/a} + \frac{N}{n_0 - N/a}e^{pn_0x-pNd/a}\right]. \quad (7)$$

Fig. 3 shows such a curve for neon, $N=0.01$, $a=2$, $n_o=1$, $V_e=50$v, $V=5$v, $i_e=300$ microamps/cm^2 and $d=1$cm. This corresponds fairly closely to the experimental curve obtained by Harnwell.[16] The sheath thickness x is also given, to show that it is present throughout most of the pressure range. All these scattering corrections do not affect materially the determination of x, for the current determining the space charge is not affected by the deflection of the individual ions. However, impurities in the gas, which are easily ionizable, and of large molecular mass, will affect the sheath thickness, though they will not directly affect the measured ionic currents.

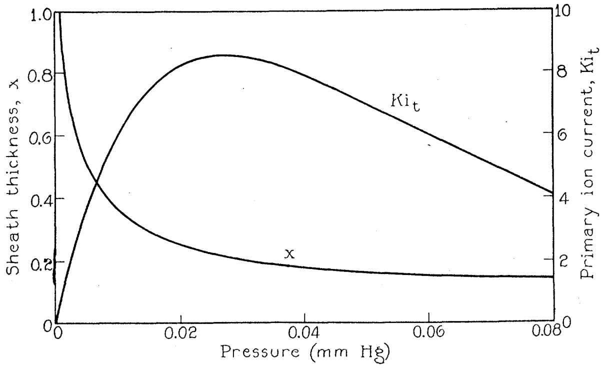

Fig. 3. Primary ion current and sheath thickness in neon at various pressures.

Secondary ions: Eqs. (5) and (7) give the primary ion current only when no secondary ions are formed. When these are created the measured primary ion current is less than this, and will depend on the character of the secondary ion formation.

There are probably two general types of secondary ions; those which require low ionic velocities for formation, and which will form most easily in the field free space above the sheath; and those requiring high velocity collisions for formation and which form most easily within the sheath.

Type I. So far we have not needed to consider the actual concentration of ions at various points in the chamber, but it can easily be seen that inasmuch as the drift velocity in the space *GS* is due only to diffusion and therefor is much smaller than that inside the sheath, the concentration in *GS* will be correspondingly greater, perhaps fifty to a hundred times as large as in the sheath. Therefore nearly all of type one ions will be created in this space.

The usual diffusion equations will hold here. We neglect diffusion to the side wall since the side wall is partly insulator. Then for equilibrium the number diffusing away from any element of volume per second must equal the number formed minus the number changing to secondary ions.

$$-Dd^2n/dz^2 = Npi_e/e - m_s$$

where z is the distance from C, D is kT/e times the ionic mobility ($D = D_0/p = kT\mu_0/ep$, where μ_0 is the mobility at 0.01 mm pressure), n is the concentration of positive ions and m_s the number changing to secondary ions per second. If the probability of change to a secondary ion at each collision is P_1, and the number of collisions a primary ion makes per second is pn_1, then $m_s = npn_1 P_1$, and

$$n = \frac{Ni_e}{en_1P_1}\left[1 - \frac{\cosh\left((N_1P_1/D_0)^{1/2}P^2\right)}{\cosh\left(n_1P_1/D_2\right)^{1/2}P(d-x)/2)}\right]$$

if we consider n to be practically equal to zero at both grid and sheath boundary. The total measured current of secondary ions formed per second which go to the sheath will then be

$$Ki_1 = eK\int_0^{d-x/2} m_s dz$$

$$= KpNi_e\left[\frac{d-x}{2} - \left(\frac{D_0}{n_1P_1p^2}\right)^{1/2}\tanh\left(\frac{n_1P_1}{D_0}\right)^{1/2}P\left(\frac{d-x}{2}\right)\right].$$

When (n_1P_1p/D_0) is small this can be expanded and becomes

$$K\frac{n_1P_1p^3Ni_e}{3D}\left(\frac{d-x}{3}\right)^3$$

to the first approximation.

The current of unchanged primary ions reaching the sheath will be the difference

$$i_0 - i_1 = i_{0p} = Ni_e\left(\frac{D_0}{n_1P_1}\right)^{1/2}\tanh\left[\left(\frac{n_1P_1}{D_0}\right)^{1/2}p\left(\frac{d-x}{2}\right)\right] \tag{8}$$

which reduces to i_0 if p is small.

If only secondary ions of this type are formed, $K(i_{0p}+I)$ will be the primary ionic current measured. These results, of course, have disregarded the effects of scattering.

When the pressure is high the analysis is somewhat changed in that the diffusion equation now is

$$-D\frac{d^2n}{dz^2}=\frac{Npi_e}{e}e^{-(N/a)P[z+(d-x)/2]}-nn_1pP_1$$

which gives for the measured current of secondary ions

$$Ki_1'=\frac{Kan_1P_1i_ee^{-n_0px-Np(d-x)/a}}{n_1P_1-DpN^2/a^2}\Bigg[(e^{Np(d-x)/2a}-1)$$

$$\frac{N}{a}\left(\frac{D_0}{n_1P_1}\right)^{1/2}\tanh\left(\frac{n_1P_1}{D_0}\right)^{1/2}p\left(\frac{d-x}{2}\right)\Bigg] \tag{9}$$

and for the resulting primary ion current to the sheath boundary

$$i_{0p}'=ai_e\left(1-\frac{n_dP_1}{n_dP_1-D_pN^2/a^2}\right)(e^{-Np(d-x)/2a}-e^{Np(d-x)/a})$$

$$+\frac{Ni_en_1P_1}{n_1P_1-DpN^2/a^2}\left(\frac{D_0}{n_1P_1}\right)^{1/2}\tanh\left(\frac{n_1P_1}{D_0}\right)^{1/2}p\left(\frac{d-x}{2}\right) \tag{10}$$

If no secondary ions are formed in the sheath, the measured primary ion current will be $Ki_{0p}'e^{-n_0px}$.

Type II. In this case the formula $m_s=npn_dP_2$ is the same as for type one ions, but the various quantities must be interpreted differently, for here all the ions are assumed to have a definite drift velocity perpendicular to P, instead of having only a random motion. n_d is then equal to n_2u^+ where u^+ is the velocity of the ion due to the electric field, and n_2 the number of collisions an ion makes per cm path when in an electric field. n is i_y/u^+, where i_y is the current of primary ions passing a unit area parallel to P and a distance y from P. Then the number of secondary ions formed per sec per cc will be $n_2pP_2i_y$, and for equilibrium

$$di_y/dy=-I+pn_2P_2i_y=-pN+n_2pP_2i_y$$

or

$$i_y=(i_0-Ni_e/n_2P_2)e^{pn_2P_2(y-x)}+Ni_e/n_2P_2$$

and the total measured current of this type ion is

$$Ki_2=KpP_2n_2\int_0^x i_ydy$$

$$=K[(i_0-Nie/n_2P_2)(1-e^{-n_2P_2px}+pNi_ex) \tag{11}$$

The total current of primary ions which pass through the slit and is measured is Ki_y evaluated at $y=0$

$$Ki_p=K(i_0-Ni_e/n_2P_2)e^{-pP_2n_2x}+Ni_e/P_2n_2 \tag{12}$$

When secondary ions of type one are also formed, i_{0p} from Eq. (12) is used instead of i_0 in these formulas.

When the case is considered for high pressures the only change necessary is to multiply I by $e^{-Np(d-y)/a}$ and to introduce the scattering factor e^{-n_0py} the secondary current becomes

$$Ki_2' = K\left(i_{0p}' - \frac{Ni_e}{n_2P_2 - N/a}\right)\left(\frac{P_2}{1-P_2}\right)(e^{-n_2P_2px} - e^{-n_1px}) + \frac{Kai_e}{1-N/aP_2n_2}(e^{-Npx/a} - e^{-n_0px}) \quad (13)$$

and the final positive ion current measured is

$$Ki_p' = K\left(i_{0p}' - \frac{Ni_e e^{-Np(d-x)/a}}{n_2(P_2+1) - N/a}\right)e^{-n_2p(P_2+1)x} + \frac{Ni_e e^{-Np(d-x)/a}}{n_2(P_2+1) - N/a}\,. \quad (14)$$

Fig. 4 gives curves for x, i_p', i_1' and i_2', for hydrogen, $n_0 = 3$, $i_e = 300$, $V = 9$v, $d = 0.8$cm, $N = 0.03$, $V_e = 25$v, $a = 1$, $D_0/n_1P_1 = 1/2$, $P_2n_2 = 5$. The curves show considerable similarity to those of Smyth.[5] They should not be expected to correspond exactly however, for from Smyth's data, i_e the electronic current density going through the grid, cannot be determined with any degree of precision, and this factor has considerable effect on x, and therefore on the position of the peak in i_p'. Also there was present an appreciable amount of water vapor, which would reduce x, and bring these maxima to smaller pressures. The similarity between the curves for H^+ and the i_2' curve, and the curve for H_3^+ and the i_1' curve suggests that H^+ is formed within the sheath, by high velocity collisions, and that H_3^+ is formed above the sheath by low velocity collisions. Of course some H_3^+ will be formed in the sheath, and some allowance has been made for this effect in drawing the dotted i_1'' curve.

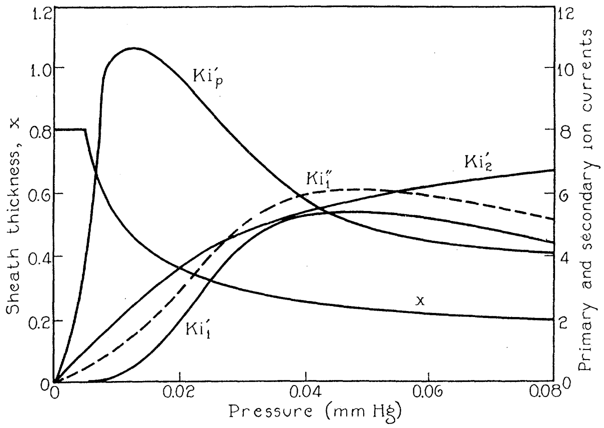

Fig. 4. Primary and secondary ion currents and sheath thickness in hydrogen at various pressures.

Discussion: Thus the curves obtained for the case where no secondary ions were formed, and those obtained where both types were formed all resemble the experimental curves. This check is not conclusive, however, for the values of many of the variables were not measured or regulated when the experimental curves were taken.

It should be possible to determine quantitatively the probabilities of formation of the various secondary ions, and measurements are being made in this laboratory to see whether the above analysis is adequate for such a determination. The above analysis has shown that in making any further measurements of this kind it should be borne in mind that the voltage across the chamber and the electronic current are nearly as important factors in determining the proportion of the various ionic products as is the pressure and that perhaps the method which is easiest to analyze quantitatively is that of keeping the pressure constant at a low value, and varying either V or i_e.

But even if further data show that the phenomenon is still more complicated than we have assumed, it is hoped that the above analysis may suggest modifications in experimental procedure and help to clarify the interpretation of experimental results.

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