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THE SUN'S RADIAL MAGNETIC GRADIENT AND ATMOSPHERE

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ABSTRACT

Theory of the radial magnetic gradient of the sun based on the diamagnetic effect produced by ions spiralling about the impressed magnetic field.—The thermal ionization of the sun's atmosphere, free paths and velocities of thermal agitation of the molecules and ions all have such values that the diamagnetic effect is very large in the regions of the sun's atmosphere where abnomally large magnetic gradients are observed. Extrapolation of the diamagnetic effect into regions where there are no available experimental data shows that the magnetic field at the surface of the sun is several times the generally accepted value.

Ionic densities estimated from magnetic data.—These agree well with data from other sources. The mean atomic weight of the particles in the sun's atmosphere assuming it to be in purely gravitational equilibrium is found to be approximately 3.3 or when a correction is made for the presence of an electric charge on the sun this becomes 5.5.

SPECTROSCOPIC researches at Mt. Wilson Observatory[1] have shown that the sun possesses a general magnetic field which is similar in many respects to that of the earth. The magnetic field in the sun's atmosphere has been studied by means of the Zeeman shift of certain spectral lines which are known to originate at definite altitudes above the photosphere. These researches indicate that the sun's magnetic poles bear the same relation to its rotation as those of the earth. Moreover, at any given layer in the sun's atmosphere the distribution of magnetization is nearly the same as that on the earth and corresponds roughly to the field produced by a uniformly magnetized sphere. It has been possible to determine the magnetic field as a function of the altitude above the photosphere and the available data have been plotted in the form of a smooth curve in Figure 1. This curve will be used in the present paper for the purpose of computation.

The outstanding characteristic of the sun's general magnetic field is the rapid radial diminution of its resultant intensity, which falls from 55 gauss at a level of 250 km above the photosphere to 10 gauss at 450 km. Such a gradient is approximately 7000 times as great as would be expected if the sun were a uniformly magnetized sphere whose polar field strength is 55 gauss. A great difficulty of solar theories has been to explain the general magnetic field and at the same time to account for the rapid radial gradient which occurs in the reversing layer. The results of the present paper show that the radial gradient is a secondary phenomenon associated with the high temperature and ionization of the sun's atmosphere, and the presence of the gradient places no restrictions on the theory of the general field. A

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[1] Hale, Astrophys. J. **38**, 31 (1913), Hale, Seares, van Maanen and Ellerman, Astrophys. J. **47**, 1 (1918).

consistent view of the sun's and earth's permanent magnetic field is therefore possible.

In a recent paper[2] the writer pointed out that, under certain conditions of ionization, temperature, pressure and magnetic field, a true diamagnetic effect exists which is due to the motion of ions or electrons spiralling about the impressed magnetic field. An examination of magnitudes with such spectroscopic data as are now available indicated that conditions on the sun in the region of large magnetic gradient are precisely those most favorable for a large diamagnetic effect.[3] In previous work it was shown that a large diamagnetic effect existed when the free path of the ion is greater than the radius of the helix generated by the ion when acting under the influence of thermal agitation and the magnetic field. This condition exists in the sun at altitudes corresponding to regions of large observed magnetic gradient.

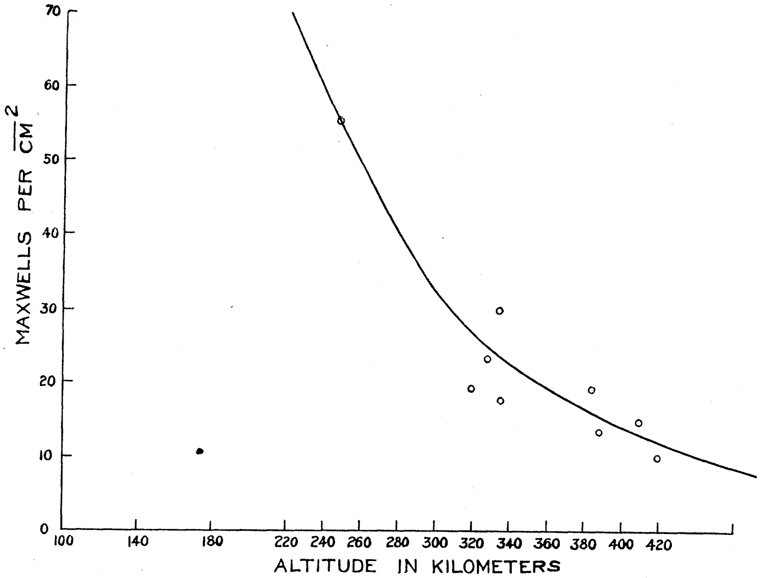

Fig. 1. Magnetic field as a function of altitude above the photosphere. Plot of available data

The radius of the spiral generated by the ion moving in the magnetic field is obtained by equating the centrifugal and magnetic forces, and if we express the velocity of the ion in terms of absolute temperature we have

$$r=(3mkT)^{1/2}/eB \tag{1}$$

where r is the radius of the circle generated by the ion, m the mass of the ion, e its charge, B the magnetic field impressed on the ion, k the Boltzmann constant and T the absolute temperature. The gases of the sun will be

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[2] Gunn, Phys. Rev. **32,** 133 (1928).

[3] Gunn, Note on the radial magnetic gradient of the sun, Science, p. 273 (Sept. 21, 1928).

[4] Saha, Phil. Mag. **40,** 472 (1920).

largely ionized by the high temperatures and if we compute the fractional ionization according to the relations of Saha,[4] we find that to a first approximation we may assume the entire gaseous body to be singly ionized. Moreover, we shall assume that each cm^3 is nearly neutral electrically and that no radial electric field on the sun exceeds that predicted by the theory of Pannekoek[5] which takes account of the gravitational separation of the light electrons from the heavier positive ions. This theory indicates that the sun carries a positive charge of 10^{12} e.s.u. which produces an electric field near the surface of the sun of at least 6×10^{-8} volts/cm.

It has been shown[2] that the diamagnetic intensity of magnetization, I, for long free paths is given by

$$I=-NkT/B \tag{2}$$

where N is the number of ions per cm^3, k the Boltzmann constant, T the absolute temperature and B the magnetic field impressed on the spiralling electrons. This relation has been deduced on the assumption of long free paths and if N is large and the free paths are long the diamagnetism of any region may be large. In order to examine whether the condition of long free paths is satisfied we must determine the kinetic theory values for the length of the free paths. The free path (λ) of an ion is given by

$$\lambda_i=1/2^{1/2}\pi\sigma^2 N \tag{3}$$

and for an electron

$$\lambda_e=4/\pi\sigma^2 N \tag{4}$$

where σ is the kinetic-theory diameter of the molecules or ions. Since the free path of the electron is greater for a given molecular density diamagnetism due to electrons will persist at somewhat higher pressures than for ions. In order to establish the length of free path which certainly yields a strong diamagnetic effect we have assumed arbitrarily that the critical free path was equal to the radius of the helix generated by the ion. It seems probable that this critical free path is nearly a hundred times longer than the shortest allowable path, but the present assumption is certainly safe and will therefore be used.

The molecular density corresponding to the critical free path may then be determined by equating equations (4) and (1), which yields

$$N(\text{critical})=\frac{4eB}{\pi\sigma^2(3mkT)^{1/2}} \tag{5}$$

Thus we have a criterion for the existence of the diamagnetic effect, for if N is less than N(critical) the free paths are long and the resulting effect large. Putting in appropriate values for the constants of Eq. (5) on the sun,

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[5] A. Pannekoek, Astro. Inst. Netherlands, No. 19 (1922).

namely $e=1.57\times10^{-20}$ e.m.u. $\sigma=2\times10^{-8}$ cm; $m=10^{-27}$ grams; $k=1.37\times10^{-16}$; $T=6000°$; we have

$$N\ (\text{critical, sun})=1.2\times10^{15}\cdot B \tag{6}$$

Turning our attention to the experimental data of Fig. 1, we observe by aid of Eq. (6) that an ion density of 10^{16} ions is allowable at an altitude of 425 km and 5×10^{17} ions/cm^3 at 250 km without violating the condition of long free path.

The determination of the ionic densities from spectroscopic pressure shift, absorption, etc., in the region under consideration by St. John and Babcock,[6] Russell,[7] Stewart,[8] and Fowler and Milne,[9] all point to an ion density of about 10^{16} or 10^{17} which increases to 10,[19] perhaps, directly at the photosphere. These determinations are very difficult and are subject to wide variations and can therefore only be considered tentative.

If we accept the values above which are seen to satisfy the requirement regarding free paths and compute by aid of Eq. (2) the intensity of magnetization substituting in known values for B and T; it turns out that I is absurdly large compared to the resultant observed field. The assumed ionic densities must therefore be too large for N is the only quantity which is not observed directly. We are led to the conclusion that the lower lying layers of the sun's atmosphere are strongly diamagnetic and shield the upper layers by their magnetic action. This shielding effect makes it very difficult to specify the magnitude of the impressed field at any given point.

Certain energy relations give a clue to the value of intensity of magnetization in terms of the resultant field B. Consider a region which impresses a constant magnetic field H upon rings of electrons having a total magnetic moment M. The energy U necessary to line up the electron rings with the impressed field is given by

$$U=-M\cdot H \tag{7}$$

or the energy per unit volume u

$$u=-I\cdot H \tag{8}$$

In addition to this relation we have the usual relation which gives the energy resident in the medium.

$$u'=BH/8\pi \tag{9}$$

With the exception of the radial variation, the field about any given region may be taken to be symmetrical and therefore the energy resident in the medium may be taken as somewhat less than the energy producing the magnetic effect. That is $u>u'$ or

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[6] St. John and Babcock, Astrophys. J. **60,** 323 (1924).

[7] Russell, Astrophys. J. **55,** 135 (1922).

[8] Stewart, Phys. Rev. **22,** 324 (1923).

[9] Fowler and Milne, Royal Astron. Soc. Monthly Notices, **83,** 402 (1923).

$$I\cdot H \geqq BH/8\pi. \tag{10}$$

Substituting for the value of I we have by aid of (5)

$$\frac{4eB}{\pi\sigma^2(3mkT)^{1/2}} \geqq N \geqq \frac{B^2}{8\pi kT}\cdot \tag{11}$$

Numerical considerations have lead to the conclusion that the facts would be best represented by assuming that $B=4\pi I$ or

$$N \doteq B^2/4\pi kT \tag{12}$$

which satisfies (11) and permits at once an estimate of the ion density of the diamagnetic atmosphere of the sun.

Since B is known from observational data the ionic densities may be computed by aid of Fig. 1 and equation (12). The results of these calculations assuming T constant at 6000° are plotted in Fig. 2. The curve gives the approximate ionic densities which exist at various altitudes above the sun's photosphere based on the assumption that the observed effects are due to diamagnetism. Within the limit of experimental error the curve of Fig. 2 is logarithmic and it has been possible to determine the effective mean atomic weight of the particles in the reversing layer. We shall assume the atmosphere to be in purely gravitational equilibrium and may therefore express the ionic density at any level by

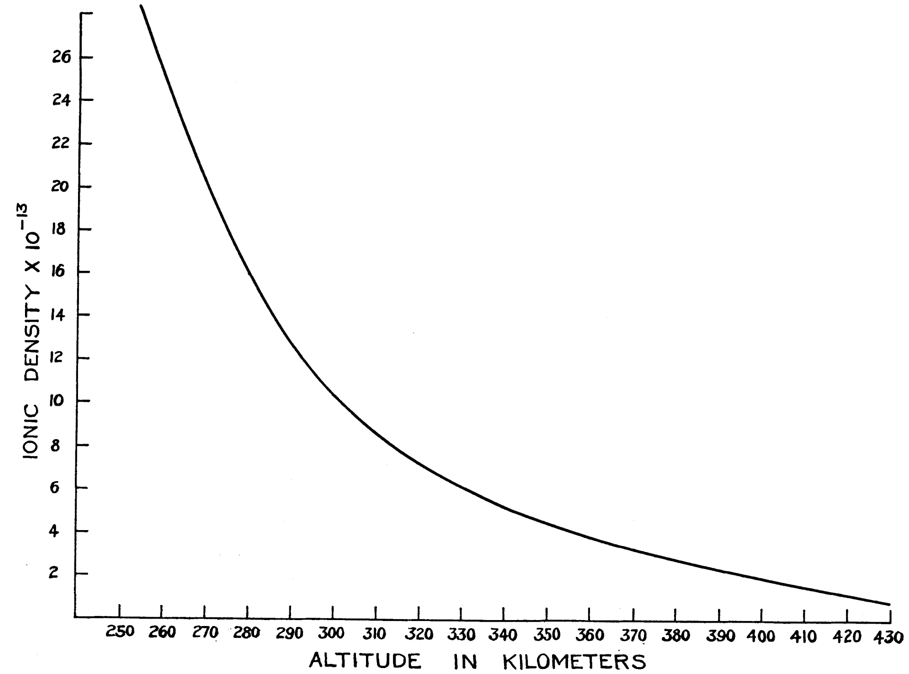

Fig. 2. Computed values of ionic density as function of altitude above the photosphere.

$$N=N_0 \exp\left(-Zm_Hgh/RT\right) \tag{13}$$

where N_0 is the number of ions at the photosphere, N the number of ions per cm^3 at a distance h above the photosphere, Z the apparent atomic weight, M_H the mass of a hydrogen atom, R the gas constant, T the absolute temperature and g the acceleration due to gravity. We find on applying the above relation to the data of Fig. 2, that the relation is satisfied throughout the region of observed data for $Z=3.3$ and $N_0=2\times10^{17}$ ions/cm.[3] Eddington concludes that in the interior of a star the most probable value for Z is about 2.2. In regions on the surface of a star marked deviations are to be expected and indeed the value we find is too small rather than too large. Pannekoek's work[5] indicates that there must be a slight radial electric field on the sun and if it exists it will be in such a direction that the electric field will tend to support the positive ions while the electrons will be subject to electric forces which will urge them downward. Since the mass of an electron is negligible compared to the mass of an ion, the contribution to the density is overwhelmingly due to the ions and if we take the value of the field computed by Pannekoek; namely, 6.0×10^{-8} volts/cm, and correct for the electrical forces this introduces on each ion, we secure a value for Z of 5.5. It is perhaps well to point out that since Z is the mean atomic weight of the particles and since on the average there are nearly as many electrons as ions the average atomic weight of the positive ions alone will be 6.6 and 11.0 respectively according to forces assumed to act on the ions.

An estimate of the magnetic field intensity at the photosphere has been made assuming that diamagnetism plays an important part down to the photosphere proper as, indeed, is indicated by the present discussion. The substitution of N_0 in equation (12) gives a value of B at the surface of 400 maxwells/cm.[2] The estimates of ion densities derived in this paper are in substantial agreement with those determined spectroscopically and lends considerable support to the view that the diamagnetism of ions and electrons plays an important part in magnetic phenomena associated with the sun and earth. A volume distribution of diamagnetism of the type which has just been considered accounts definitely for the observed magnetic gradient of the sun and seems to indicate that its magnetic moment is several times greater than the commonly accepted value.

ADDENDUM—February 2, 1929. A paper on the above subject by S. Chapman[10] has just appeared in which he attributes the limitation of the sun's field to eastward gravitational currents flowing in the ionized layer. A consideration of this phenomenon has led Chapman to nearly the same relation between magnetic field strength and ion density as has been found in the present paper. (Compare his equation 31 with 11 above.) The two views, therefore, predict the same reduction in field strength and if eastward currents actually exist the net effect must be considered as being due to both causes. A new electromagnetic effect arising from the drift motion imposed on ions spiralling about in homogeneous magnetic fields has been found by the

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[10] S. Chapman, Monthly Notices. Roy. Astro. Soc., Nov., 1928.

writer and this effect predicts a westward current of the same order of magnitude as the eastward current studied by Chapman. It is yet too early to deny the existence of an eastward current since the data now available regarding the constitution and distribution of the sun's atmosphere are incomplete. In his discussion, Chapman has taken unusual liberty with the observed data for he has assumed that the major portion of the sun's field has vanished in a layer depth of 19 km whereas measurements at Mount Wilson Observatory indicate that this change takes place in 190 km. The numerical calculations in the present paper are based on Mount Wilson data. A discussion of the new effect that produces the westward current referred to above and its application to the magnetic fields associated with sunspots, the sun and earth, has been prepared and should appear shortly.

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