DEPENDENCE OF ELECTRON EMISSION FROM METALS UPON FIELD STRENGTHS AND TEMPERATURES

By Robert A. Millikan and Charles C. Lauritsen

Abstract

This paper contains a full presentation of the reasons for believing, contrary to results recently obtained elsewhere, that field currents are only independent of temperature up to about 1100°K, and that at that temperature the energy of thermal agitation begins to assist the fields appreciably in causing the escape of electrons from metals. The precise form of function describing this dependence is not accurately determinable experimentally, but the form originally suggested by us fits the facts of observation thus far known satisfactorily, not better, however, than does the theoretical form suggested by Houston.

When we first published the now well established experimental field-current equation

$$i=Ce^{-b/F} \tag{1}$$

we thought it of interest to combine this with an equation of the usual thermionic form, namely

$$i=Ae^{-b/T} \tag{2}$$

for the sake of attempting to describe what happens in field-current work *at each emitting point* under the joint influence of both field and temperature.

For the sake, however, of obtaining as general an expression as we could then suggest, and at the same time following the most approved thermionic form, we wrote the joint, purely empirical, equation as follows:

$$i=A(T+cF)^2e^{-b/(T+cF)} \tag{3}$$

but in the first place we were careful to state that "the constant c of course changes with the condition of the surface," and also that we had "not yet taken sufficient data to know whether, with a given surface, one value of c will correspond to all values of T."[1]

In the second place we had experimentally tested (3) very fully and carefully for the extreme case in which T is very small in comparison with F, i.e., for the case of fields so powerful that (3) reduces to (1), and hence b becomes independent of F, but we had taken only meagre experimental data for the region in which the effect of T begins to be comparable with that of F; so that just how b varied with F for weak fields was unknown to us, though the fact that it was presumably some function of F and T had been discussed at length in the earlier paper by Millikan and Eyring.[2] *In*

[1] Millikan and Lauritsen, Proc. Nat. Acad. Sci. **14,** 15 (1928).

[2] Millikan and Eyring, Phys. Rev. **27,** 51 (1926).

other words, we had from the first regarded both b and c as some sort of functions of F and T, and as a matter of fact at the time of the publication of equation (3) we were initiating new experiments to find something more about the nature of these two functions.

We had, however, made sufficient experiments long before the date of publication of equations (1) and (3) to convince ourselves of the correctness of the qualitative conclusion drawn first by Millikan and Eyring[2] and later stated by us in equation (3) that while field-currents are practically independent of temperature below say 1000°K, *they begin to become definitely dependent upon temperature in the case of tungsten at about 1100°K.* This conclusion is questioned by de Bruyne[3] on new experimental grounds.[4] Also Fowler and Nordheim[5] while deducing (1), following Oppenheimer,[6] from the wave-mechanics, "fail to find any theoretical justification for (3)," though they say that "of course some justification may exist."

This is sufficient to show how important it is to determine if possible, *first*, just how "b" depends upon the field F before, with rising F, it has lost such dependence and equation (1) has taken control of the situation, and, *second*, whether field currents can be definitely shown to depend on T.

The first question we cannot yet answer fully, but we can throw some light upon it. It seemed to be partially answered by Millikan and Eyring's proof that the Schottky theory is not applicable to field currents. This theory assumes that the externally applied field simply acts to neutralize partially the work function b, and so long as these fields are not too strong the Schottky theory demands a linear relation between log i and $F^{1/2}$, a relation unambiguously shown by Millikan and Eyring's paper not to hold in *field-current* phenomena. Recently, however, Phorte[7] and de Bruyne[4] have beautifully verified this equation in the case of *ordinary thermionic* discharge, as Schottky himself had earlier done, the range over which F is varied in de Bruyne's experiments being larger than had been used before, *but still very small in comparison with the actual fields used in field current experiments* (see below).

We wish to point out, however, that contrary to de Bruyne's belief there is no discrepancy at all between his results, Millikan and Eyring's, or our own. The reason that the Schottky equation holds for ordinary thermionic emission but not for field emission is as follows: Field emission, as Millikan and Eyring[2] long ago pointed out only takes place from one or two minute spots—"microscopic mountain peaks"—where the applied potential gradient is a hundred times greater than that computed from the applied potential and the radius of the wire, in other words where the field strength is *enormously* high—so high that no image law, such as underlies Schottky's theory—can possibly apply. On the other hand, in *ordinary*

[3] N. A. de Bruyne, Proc. Cambridge Phil. Soc. **24,** part 4, 518 (1928).
[4] N. A. de Bruyne, Proc. Roy. Soc. **A120,** 423 (1928).
[5] Fowler and Nordheim, Proc. Roy. Soc. **A119,** 173 (1928).
[6] Oppenheimer, Phys. Rev. **31,** 66 (1928).
[7] W. L. Phorte, Zeits. f. Physik **49,** 46 (1928).

thermionic emission, since the electrons are here being boiled out from the *whole surface* of the wire, the applied external field is on the average just that computed from the applied potential and the wire-radius, and is therefore very small (Millikan and Eyring estimated it of the order of one two-hundredths of that existing at a point which is a source of field currents) and hence must follow the image law. *In other words, the image law, and hence the Schottky equation, should hold for the relatively weak fields existing at the smooth surface of a wire, but it is definitely shown by the Millikan-Eyring field-current work not to hold for the very strong fields existing at the emitting points in field-current phenomena, and the validity of (1) proves that for strong enough fields b has become quite independent of F.*

This, then, is the answer as far as it can now be given to the first of the foregoing questions. *For weak enough fields b is to be diminished by a quantity which is proportional to* $F^{1/2}$ *but for strong fields becomes independent of F.*

As for the second question, the quantitative evidence is still very strong that the rise in field-currents observed by Millikan and Eyring at 1100°K is a real effect of temperature upon *the field currents themselves* and not the mere influence of the field in augmenting, in accordance with the Schottky equation, the thermionic current begining to set in at 1100°K. This last interpretation, given by de Bruyne, we think to be incorrect for the following reasons.

First, the thermionic currents setting in at 1100° K from the tungsten wire used in Millikan and Eyring's experiment amount, even when augmented by the Schottky effect due to the highest field employed, to a current of but 10^{-12} amperes, whereas the observed currents at the highest field (see Table VII, Phys. Rev. *27*, 61 1926) amounted to 10^{-4} amperes. This means that the observed *increase* in these field currents (of strength 10^{-4} amperes) of from 10 percent to 20 percent brought about by increasing the temperature from 300°K to 1100°K is not at all the Schottky or field increase in the thermionic currents. It is about a billion times too large to be so interpreted. It must be rather a real temperature effect on the field current itself that begins to set in appreciably at about 1100°K. The interpretation given by deBruyne which makes field currents from tungsten independent of temperature up to about 1900°K is completely irreconcilable with the Millikan and Eyring experimental data given in Table VII of their paper.

Second, the foregoing results have been checked many times in the work herewith presented and with many different wires of different diameters. An altogether typical set of readings on one of these wires is plotted in Fig. 1. The applied potential was here constantly 8000 volts. The wire, of about 0.6 mils diameter, was in the middle of a cylinder of 1.6 cm diameter. The logarithms of the currents corresponding to the various temperature are plotted in the figure. It will be seen that up to 800°K there was no apparent dependence of field currents on temperature: at 800°K the current was 4.6×10^{-8} amperes, while at 1130°K it had risen to 5.6×10^{-8} amperes, an increase of 20 percent due to field alone *at a temperature at which the total thermionic current plus the Schottky effect was not as much as one-thousandth of this* *amount.* The figure gives the complete observed curve of the emission of the whole wire as a function of temperature between 700°K and 2500° K at a constant field strength at the surface of the wire, as computed from the radius and the applied potential of 1,440,000 volts. The extreme left side of the curve is, of course, the usual logarithmic straight line representing the ordinary thermionic current from *the whole surface* of the wire, with its slope decreased about 15 percent by the Schottky effect. The right side of the curve gives the *true field currents* from a single point sharp enough so that the field strength at the point is at least 100,000,000 volts, this being the order of magnitude of the field necessary to pull an electron through a surface having a work function of 4.5 volts. The extension downward of the slope on the left shows at once how completely impossible it is to account by means of the Schottky effect for the increase shown on the right in the field currents with temperature.

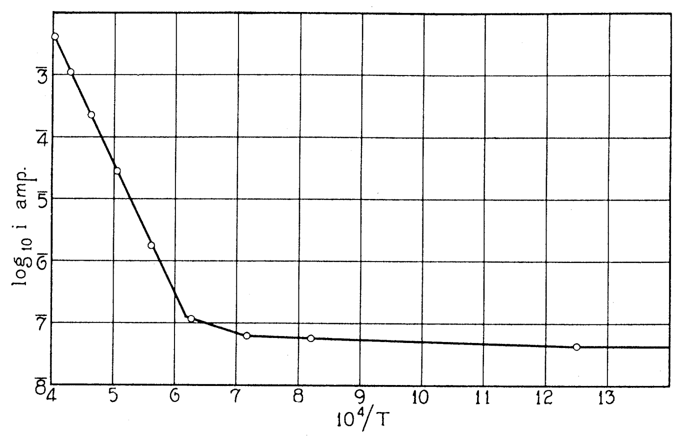

Fig. 1. Typical curve showing the relation between field currents and temperature. Wire #26; 1.56×10^{-3} cm tungsten; $V=8000$ volts.

A third argument is that the Sommerfeld-Houston theory of metallic conduction requires that while at low temperatures there should be, in accordance with the Fermi-Dirac statistics, no appreciable sharing by the electrons of the energy of thermal agitation, as the temperatures increase a condition of equipartition should ultimately be approached. This sharing of thermal energy by the electrons should evidently begin to be appreciable at temperatures at which thermionic emission begins to set in. Dr. Houston[8] has treated this side of the question in a recent theoretical paper, and has found a definite dependence of field emission as such upon temperature.

Wherein then lies the error in de Bruyne's work which leads him to the conclusion that field currents from tungsten are entirely independent of temperature up to 1944°K? His method is to obtain his field currents by subtracting from his observed currents at a given temperature and field-

[8] W. V. Houston, Phys. Rev. **33**, 361 (1929).

strength the computed, or better the extrapolated, thermionic currents as modified by the Schottky effect, and then to plot by means of our equation (1) log i against $1/F$. He thus obtains a series of widely scattering points through which he draws the straight full line shown in Fig. 2, *a line which actually washes out a progression with temperature which appears to us to be shown even by his own data.* For by drawing separate lines through the data taken at each separate temperature, as we have done in our reproduction of de Bruyne's graph in Fig. 2 we think we have shown that his data reveal an increase of field-currents with temperature (see the higher and higher positions of the dotted lines drawn through all the points corresponding to each particular temperature) precisely as do ours and of the same order of magnitude.

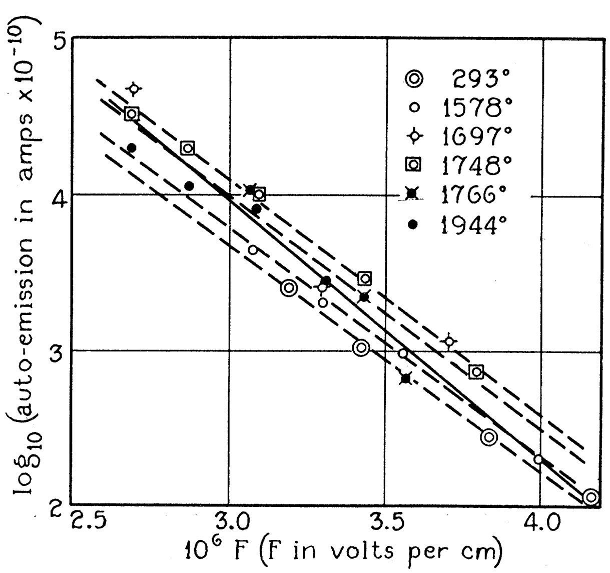

Fig. 2. Curves taken from de Bruyne's work showing relation between field currents and field strength for various temperatures.

But it is after all only his observations taken at the four lowest temperatures that have any significance whatever, for the others are obtained by subtracting from an observed reading a computed or extrapolated reading which differs from the observed by as little as one percent, which is much less than the uncertainty in most field-current measurements. Such observations cannot usually be relied upon to one percent, for the reason that the emitting point does not in general retain its properties unchanged for long periods. Only by keeping fields constant, as was done in the experiments represented in Table VII of reference 2, changing temperatures rapidly, and going back and forth between say 300° and 1100°K, can these fluctuations be avoided. *In a word, we do not think that de Bruyne's work is actually at variance with any of our published conclusions.*

As to the best equational form in which to present the results obtained to date in the domain of field currents de Bruyne suggests the form

$$i = AT^2 e^{-\{\phi - (e^3F)^{1/2}\}/kT} + Ce^{-K/F} \tag{4}$$

The first term is simply the thermionic equation *from the whole surface* as modified by Schottky; the last is our field current equation without indicating its dependence upon temperature. This failure to indicate such dependence we regard as an error, since the last term must, we think, contain T if it is to represent the facts thus far brought to light. Up to the present we have not found a form which is more satisfactory for representing the dependence of field currents upon both F and T than that we originally suggested, namely, $i = A(T+cF)^2 e^{-b/(T+cF)}$, though since we can test this only when temperature is *just beginning* to influence the field current, other forms of dependence upon temperature would doubtless fit the limited experimental data as well. How well this particular form reproduces our own field-current work as well as that of de Bruyne is shown in Fig. 3 in which our own results are plotted as circles, de Bruyne's as crosses[9].

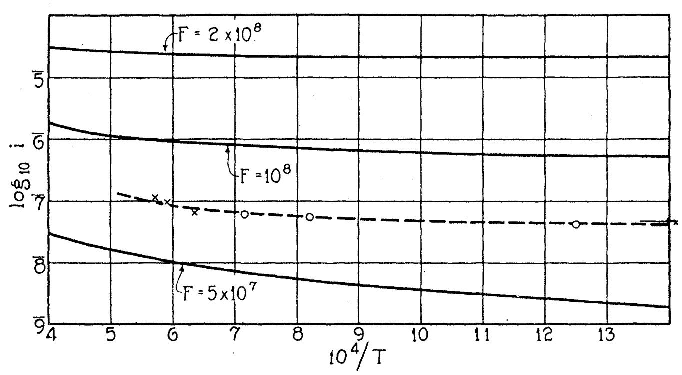

Fig. 3. Curves of variation of $\log_{10} i$ as function of $10^4/T$ as calculated from Eq. (3) for $A = 60$, $b = 52600$, $c = 10^{-4}$, area $= 10^{-14}$. The crosses represent data by de Bruyne.

Equation (3) obviously makes the increase of current with temperature greater for low fields than for high (see Fig. 3), and this result is found to be in agreement with the data in Table VII of reference 2, though the accuracy is perhaps not sufficient to make this experimental conclusion unquestionable.

It must be clearly understood that our equation (3) represents here, and has always been intended to represent, *merely the emission from a particular point under the joint influence, at that point, of field and temperature.* For such a point the Schottky equation cannot be expected to hold, since the image force at such a point has no meaning except for distances that are large compared to the dimensions of the "microscopic mountain peak", i.e., for *very weak* fields.

[9] These points are obtained by taking a vertical section through the dotted lines in Fig. 2 at a suitably chosen value of F.

Our equation (3) makes no attempt to include the thermionic emission from the whole surface of the wire. It is this emission alone to which the Schottky equation applies. This is a phenomenon entirely distinct from field currents and one correctly represented by the first term of de Bruyne's equation (4).

Our equation (3) is merely one way of depicting the fact that energy of agitation begins at some temperature to assist the field in extracting electrons from the point. Dr. Houston has obtained a theoretical expression of this fact which, however, leaves out T from the exponent of (3) but inserts it in the coefficient in much the same way in which it appears in (3). This may be a form more satisfactory than (3), and we shall be glad to use it in preference to (3) if it has better credentials. At present we see no *experimental* way of differentiating between Houston's form and that of (3) since we can only *begin*, as we have done above, to bring to light the effect of temperature in assisting the field electrons to escape. After the thermionic emission from the whole surface sets in it obviously masks the effect of temperature on the point. This thermionic emission from the whole surface, graphically shown by the steep slope on the left of Fig. 1, is something of an entirely different order of magnitude from the effect of temperature on field-currents, represented by the very small slope on the right side of Fig. 1.

Norman Bridge Laboratory of Physics,
California Institute, Pasadena,
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