OSCILLATIONS IN IONIZED GASES

By Lewi Tonks and Irving Langmuir

Abstract

A simple theory of electronic and ionic oscillations in an ionized gas has been developed. *The electronic oscillations* are so rapid (ca. 10^9 cycles) that the heavier positive ions are unaffected. They have a natural frequency $\nu_e=(ne^2/\pi m)^{1/2}$ and, except for secondary factors, do not transmit energy. *The ionic oscillations* are so slow that the electron density has its equilibrium value at all times. They vary in type according to their wave-length. The oscillations of shorter wave-length are similar to the electron vibrations, approaching the natural frequency $\nu_p=\nu_e(m_e/m_p)^{1/2}$ as upper limit. The oscillations of longer wave-length are similar to sound waves, the velocity approaching the value $v=(kT_e/m_p)^{1/2}$. The transition occurs roughly (i.e. to 5% of limiting values) within a 10-fold wave-length range centering around $2(2)^{1/2}\pi\lambda_D$, λ_D being the "Debye distance." While the theory offers no explanation of the cause of the observed oscillations, the frequency range of the most rapid oscillations, namely from 300 to 1000 megacycles agrees with that predicted for the oscillations of the ultimate electrons. Another observed frequency of 50 to 60 megacycles may correspond to oscillations of the beam electrons. Frequencies from 1.5 megacycles down can be attributed to positive ion oscillations. The correlation between theory and observed oscillations is to be considered tentative until simpler experimental conditions can be attained.

IT HAS been known for some time that if a low pressure mercury arc is maintained using a hot filament as an electron source there is a wide range of conditions under which a large number of the primary electrons rapidly acquire velocites whose voltage equivalent is greater than the total drop across the tube.[1] There is a larger number of electrons which have less than the expected energy, so that as a group the primary electrons have not gained energy. But there is some mechanism which either effects a rapid interchange of energy among the primary electrons or else effects a rapid and random interchange between each primary electron and some other energy store.

Various hypotheses to explain this phenomenon have been advanced,[1] but none has appeared satisfactory. A natural supposition is that electrical oscillations in the arc cause the scattering both by subjecting the electrons to rapidly changing electronic fields and by causing potential changes on the electrodes.

Dittmer[2] obtained evidence pointing in this direction but was unable to detect the oscillations themselves. To explain the scattering it was thought that such oscillations must be of about the same amplitude as the observed scattering, namely 20 volts or more, and these should be easily observable.

Penning[3] has observed oscillations of radio frequencies in low pressure

[1] Langmuir, Phys. Rev. **26**, 585 (1925).

[2] Dittmer, Phys. Rev. **28**, 507 (1926).

[3] Penning, Nature, Aug. 28 (1926) and Physica **6**, 241 (1926).

discharges in mercury vapor under the same conditions as those that lead to electron scattering. We have undertaken a more systematic investigation of these oscillations in order to determine their nature and cause, for although their voltage-amplitude appears to be very small we believe that they play a fundamental role in high current gaseous discharges. We have found that the oscillatory behavior of the arc is extremely complicated in the types of discharge tube which we have been using. Nevertheless certain features stand out as tending to confirm the theory outlined recently by Langmuir.[4] This theory will be treated in greater detail here and some of the more definite experimental results will be given.

I. Theory

Two types of oscillation seem to be theoretically possible, first oscillations of electrons which are too rapid for the ions to follow, and second, oscillations of the ions which are so slow that the electrons continually satisfy the Boltzmann Law. The mathematical difficulties when any dimension of the disturbed region is of the same order of magnitude as the mean distance between electrons or ions are such as to force us to consider only cases where such dimensions are large compared to the inter-ionic distances. Hence "electric field," "density," etc., refer to the average value of these quantities taken throughout a volume which contains many ions and electrons but whose dimensions are considerably smaller than those of the oscillating region.

A rough idea of the magnitudes involved can be obtained from the density of ionization. Taking 10^{10} as a typical low value of electron density, the inter-electron distance is 4.6×10^{-4} cm. If all the electrons throughout a certain region are displaced this distance, the resulting field strength will be 0.028 e.s.u. or 8.3 volts per cm, and the energy density in the field will be 3.0×10^{-5} ergs cm^{-3}.

A. Plasma-electron[5] *oscillations.* When the electrons oscillate, the positive ions behave like a rigid jelly with uniform density of positive charge ne. Imbedded in this jelly and free to move there is an initially uniform electron distribution of charge density, $-ne$. Choosing an orthogonal system of coordinates, consider the portion of the plasma included between two planes each perpendicular to the x-axis. Suppose each electron between these planes to be displaced in the x-direction by a distance ξ which is independent of the y and z coordinates and is zero at each bounding plane. If the displacement ξ is a continuous function of x and $\partial\xi/\partial x$ is small compared to unity, the change in density caused by the electron displacement is

$$\delta n = n\partial\xi/\partial x \tag{1}$$

Originally the net charge was zero, so after the displacement Poisson's equation gives

[4] Langmuir, Proc. Nat. Acad. Sci. **14**, 627 (1926).

[5] The word "plasma" will be used to designate that portion of an arc-type discharge in which the densities of ions and electrons are high but substantially equal. It embraces the whole space not occupied by "sheaths." See Footnote 4.

$$\partial E/\partial x = 4\pi e\delta n \tag{2}$$

E being the electric field strength. Eliminating δn

$$\partial E/\partial x = 4\pi ne\partial\xi/\partial x \tag{3}$$

Integrating, it is found that

$$E = 4\pi ne\xi \tag{4}$$

for the field arising from the electron displacement only, (since the neglected arbitrary constant represents a uniform external field). Eq. (4) shows immediately that the displaced electrons will oscillate about their original positions with simple harmonic motion. The force on each electron is $-Ee = m_e\ddot{\xi}$ giving

$$m_e\ddot{\xi} + 4\pi ne^2\xi = 0 \tag{5A}$$

for the equation of motion. The frequency of oscillation is thus seen to be

$$\nu_e = (ne^2/\pi m_e)^{1/2} = 8980\, n^{1/2}. \tag{6}$$

If a spherical or cylindrical instead of a plane disturbance be postulated, the same natural period of oscillation is found, but in those cases some distortion of the oscillation will occur near the center or axis respectively unless ξ/r remains small in those regions. Oscillations of this type will be called plasma-electron oscillations.

The same result in a more general form which shows the oscillations to be independent of any symmetry restriction can be obtained from the fundamental electromagnetic equations applied to an ionized gas. The earlier treatment has been retained, however, because of its simple physical ideas and its application further on to positive ion oscillations. In Gibbs' vector notation the four necessary electromagnetic equations are

$$-(1/c)\dot{\boldsymbol{H}} = \nabla\times\boldsymbol{E} \qquad \boldsymbol{J} = (1/4\pi)\dot{\boldsymbol{E}} - ne\boldsymbol{v}$$

$$(4\pi/c)\boldsymbol{J} = \nabla\times\boldsymbol{H} \qquad -m\dot{\boldsymbol{v}} = e\boldsymbol{E} + (e/c)\boldsymbol{v}\times\boldsymbol{H}$$

For sufficiently small oscillations $\boldsymbol{v}\times\boldsymbol{H}$ is negligible and can be dropped. Eliminating $\boldsymbol{v}$ and $\boldsymbol{J}$ gives

$$\nabla\times\dot{\boldsymbol{H}} = (1/c)\ddot{\boldsymbol{E}} + (4\pi ne^2/mc)\boldsymbol{E}$$

Taking the curl of the first equation, $\boldsymbol{H}$ can be eliminated, giving

$$\ddot{\boldsymbol{E}} + (4\pi ne^2/m)\boldsymbol{E} = -c^2\nabla\times(\nabla\times\boldsymbol{E})$$

When the electric field arises primarily from electric charges, the portion arising from $\dot{\boldsymbol{H}}$ being negligible, $\nabla\times\boldsymbol{E}$ can be put equal to zero and we have

$$\ddot{\boldsymbol{E}} + (4\pi ne^2/m)\boldsymbol{E} = 0 \tag{5B}$$

This result has no symmetry restriction and leads to the frequency given by Eq. (6).

When $\nabla\times\boldsymbol{E}$ is not zero the complete equation in $\boldsymbol{E}$ can be multiplied by $\nabla\cdot$ (i.e. operated on with div.),[6] which causes the right member to vanish. Noting that $\nabla\cdot\boldsymbol{E}=4\pi e\delta n$ we have,

$$\partial^2(\delta n)/\partial t^2+(4\pi ne^2/m)\delta n=0 \tag{5C}$$

Hence, under all conditions, the electron density is capable of simple harmonic variation of small amplitude and of frequency 8980 $n^{1/2}$.

If we omit the $\nabla\cdot$ operation but assume that $\nabla\cdot\boldsymbol{E}=0$ we obtain the equation

$$\ddot{\boldsymbol{E}}+(4\pi ne^2/m)\boldsymbol{E}=c^2\nabla^2\boldsymbol{E}$$

which J. J. Thomson has used[7] in the discussion of the electrodeless discharge. The frequency and wave-length of the corresponding plane waves are related according to the equation

$$\nu^2=ne^2/\pi m+c^2/\lambda^2$$

Thus the lower frequency limit for long waves coincides with the plasma-electron frequency. But there is no natural frequency for these electromagnetic oscillations in an infinite medium. In any actual case, however, reflection at the walls of the discharge tube gives the possibility of the formation of standing waves so that there will be a fundamental resonance frequency and a whole series of overtones. It seems likely, however, that any such oscillation will be very highly damped because of poor reflection at the tube walls.

The density of ions and electrons in the plasma of an arc of the type used experimentally is of the order of 10^{10} cm^{-3}. Using this value for n in Eq. (6) the natural frequency of the plasma-electron oscillation is found to be 9.0×10^8 cycles per sec. The wave-length of this in air is 33 cm, a value to be compared with wave-lengths from 27 cm to 81 cm found experimentally.

To calculate a rough value for the fundamental resonance frequency of the electromagnetic waves we may assume that the diameter of the bulb, 18 cm, is one-half the wave-length. This gives

$$\nu=[(8.9\times10^8)^2+(3\times10^{10}/36)^2]^{1/2}=12.2\times10^8 \text{ cycles},$$

and the wave-length of this in air will be 24.6 cm, a value so near to those found experimentally that the decision as to whether some, at least, of the observed oscillations may be of this type must be reserved.

The absence of space coordinates in Eqs. (5B) and (5C) shows that there is no tendency for oscillations of this type to propagate through the plasma and their group velocity is zero. As a result one can specify the phases of the electron displacement through a certain region in such a way as to give

[6] We are indebted to Sir J. J. Thomson for this method which he outlined in a letter to one of us.

[7] J. J. Thomson, Phys. Soc. Proc. **40**, 82 (1928).

the semblance of a moving wave, a wave, however, which moves continuously through a fixed region without ever progressing beyond, like the familiar rotating barber poles which appear to move steadily upward without rising.

In spite of the unreality of such a wave motion from the energy transfer point of view, it is necessary to suppose the existence of oscillations in that form if it is shown that these oscillations exhibit the Doppler Effect. An analogy will show, we think, that despite its artificiality, this wave motion is one likely to arise. For this purpose the electrons may be likened to the bobs of a set of regularly spaced identical simple pendulums suspended from a rigid ceiling. If the swing amplitudes are small so that the bobs do not collide, these bobs will exhibit the same behavior in a plane that the electrons do in space. Let these pendulums, originally at rest, be set in motion by passing a horizontal bar along beneath them in such a way as just to make contact with the bobs. The bobs so touched will then oscillate with the particular wave motion under discussion. The simplicity of the exciting means, makes it readily conceivable that the electrons are excited in an analogous way.

There are three factors not yet considered which may result in a transfer of energy by plasma-electron oscillations. First and foremost, of course, the oscillating electrons may themselves be moving in a body through the plasma, an example being the beam of electrons emitted by the hot filament in a mercury arc. Such oscillations will have a lower frequency than the oscillations of the ultimate electrons because of the lower density in the beam. As a typical example we may take the case described later of the 72.6-v. 25-m.a. arc in which the current density of primary electrons at the tube wall under the electrode was probably about 3.1×10^{-4} amps. cm^{-2}. The electron velocities were 5.1×10^{8} cm $sec.^{-1}$ so that the charge density was 6.1×10^{-13} coulombs cm^{-3} corresponding to an electron density of 3.8×10^{6}. By Eq. (6) the natural plasma-electron frequency for this density is 1.8×10^{7} $sec.^{-1}$.

Secondly, the electric field of any unsymmetrical oscillation is unlimited in extent, reaching in varying intensity throughout the discharge tube and extending outside. It may be this factor which is primarily responsible for the detection of the plasma oscillations.

Finally, if a local oscillation be traversed by fast electrons having fairly uniform speeds, the alternate acceleration and retardation or the rythmical deflection of this beam will excite neighboring portions of the plasma into oscillation.

The simple mathematical theory outlined leads to a single frequency for the electron oscillations, but there are two factors which may cause departures from this condition. The Doppler effect has already been mentioned. The other factor is that the electrons are not normally at rest but move with their random thermal velocities. Fast moving electrons completely traverse a local disturbance in a small fraction of a period and do not contribute to the density n which is effective in determining the frequency. Slow moving electrons, on the other hand, remain in the same neighborhood for several complete oscillations and do contribute to n. The division is not definite but can be fixed roughly by assigning an effective wave-length to the disturbance and treating all those electrons which traverse that wave-length in less than one cycle as fast, the others as slow. The slow-moving electrons are then those which maintain the oscillation. Each fast-moving electron in its "impact" with the disturbance either contributes energy to it or draws energy from it.

It is seen, first, that a "fast" electron for a small (short wave-length) disturbance may be a "slow" electron for a large (long wave-length) disturbance, and second, that a "slow" electron for a high-frequency oscillation may be a "fast" electron for a lower frequency oscillation. Thus the greater the wave-length and the higher the frequency the greater the value of the distinguishing velocity, and the greater the number of slow-moving electrons. In this way the effective electron density n depends on the wave-length of the oscillation and this will give rise to a decrease of frequency with decreasing wave-length in accordance with Eq. (6). Since this decrease in frequency of itself decreases the effective density still further, there is a tendency at the smaller wave-lengths toward a kind of cumulative action which may make oscillations below a certain minimum size impossible. That this may actually occur is indicated by some calculations which we have made, but the present state of the investigation hardly warrants attempting a quantitative treatment here.

Interaction of the fast moving primary electrons with oscillations of the ultimates has already been mentioned, and of course there is also the interaction of the ultimate electrons with any oscillation of the primary electrons. In either case electrons of one type stream through an oscillation among electrons of the other type and withdraw energy from it. It is immediately evident that the shorter the wave-length the less will the interaction be. There is one very suggestive possibility that should be mentioned. It has already been pointed out that because of their lower density the beam electrons have a longer period than the ultimates. But if the wave-length and wave velocity of the oscillations have suitable values, then the frequency with which the moving beam waves strike the comparatively stationary ultimates may coincide with the higher natural period of the latter and give rise to resonance.

B. *Plasma-ion oscillations.* We are now ready to discuss the slower ionic oscillations. In this case we shall assume the same type of displacement for the ions as we did for the electrons before. We then have

$$\delta n_p = -n\partial\xi/\partial x \tag{7}$$

for the increase in ion density and (by Boltzmann's Law)

$$\delta n_e = n[\exp(eV/kT_e) - 1] \tag{8}$$

for the increase in density of electrons. Poisson's equation now gives

$$\partial^2 V/\partial x^2 = -4\pi e(\delta n_p - \delta n_e) \tag{9}$$

and for the relation of motion to field we have

$$e\partial V/\partial x = -m_p\ddot{\xi} \tag{10}$$

Substituting (7) and (8) in (9) and dropping terms in V^2 and beyond under the assumption that $eV/kT_e << 1$ we have

$$\partial^2 V/\partial x^2 = 4\pi en(\partial\xi/\partial x + eV/kT_e) \tag{11}$$

Taking the derivative with respect to x twice and substituting from (10)

$$\frac{\partial^2}{\partial x^2}\left(\ddot{\xi} + \frac{4\pi ne^2}{m_p}\xi\right) - \frac{4\pi ne^2}{kT_e}\ddot{\xi} = 0 \tag{12}$$

A qualitative idea of the frequency behavior of these plasma-ion oscillations can be obtained by solving Eq. (12) for the case of an infinite train of plane parallel waves. Thus if we assume the solution $\xi = \exp[2\pi j(\nu t - x/\lambda)]$ we obtain

$$\nu = \left(\frac{ne^2}{\pi m_p + ne^2 m_p \lambda^2/kT_e}\right)^{1/2}$$

for the frequency.

When $ne^2\lambda^2/\pi kT_e$ is small compared to unity the frequency approaches a limiting value analogous to the frequency of the plasma-electron oscillations. The above condition is equivalent to the requirement that λ be small compared to $2(2\pi)^{1/2}\lambda_D$, λ_D being the distance, calculated by Debye and Hückel,[8] the reciprocal of which is like an absorption coefficient of an ionized fluid for electric forces. Thus the potential in the neighborhood of a sphere charged with q units of electricity is $q/(Kr)$ exp (r/λ_D) in an ionized fluid compared to q/Kr in a non-ionized dielectric. Now

$$\lambda_D = (kT_e/8\pi ne^2)^{1/2} = 4.90(T_e/n)^{1/2}\text{cm}.$$

so that in discharges of the intensity used in our experiments where, roughly, $n = 10^{10}$ and $T_e = 10^4$, $\lambda_D = 0.005$ cm. The distance between ions averages $n^{-1/3} = 4.6\times10^{-4}$ cm. This appears to be enough smaller than $2(2)^{1/2}\pi\lambda_D = 4.4\times10^{-2}$ cm so that without approaching the fine structure limit of the theory too closely there is still room for plasma-ion oscillations near $\nu = (ne^2/\pi m_p)^{1/2}$. For mercury ions this limit is 1/600 that calculated for the electrons. Thus where the frequency of the plasma-electron oscillations is 9.0×10^8 sec.$^{-1}$ (as calculated above), that of the limiting plasma-ion oscillation is 1.5×10^6 sec.$^{-1}$

Most of the considerations applying to plasma-electron oscillations apply also to these short-wave ion oscillations. A factor which may make the latter simpler is the lower "temperature" of the ions, especially as there seem to be good reasons for supposing that the ions do not have so much chaotic as general drift motion through the plasma.

If, on the other hand, λ is considerably larger than $2(2)^{1/2}\pi\lambda_D$, the ion oscillations lose their similarity to the electron oscillations and change over

[8] Debye and Hückel, Phys. Zeits. **24**, 185, 305 (1923).

to electric sound waves. Their group velocity becomes finite and both it and their wave velocity rapidly approach the limiting value

$$v=(kT_e/m_p)^{1/2}=3.9\times 10^5(T_e m_e/m_p)^{1/2}$$

In our experiments then, putting $T_e=10^4$ and $(m_e/m_p)^{1/2}=1/600$ we find $V=6.5\times 10^4$ cm sec.$^{-1}$ when λ is greater than about 10^{-1} cm. Thus frequencies originating from these sound waves may extend from 6.5×10^5 per sec. down, but probably not beyond some point where the wave-length is comparable with the discharge tube dimensions.

The whole transition from the electron type of oscillation to the sound type occurs practically within a 10-fold variation of λ, for when $\lambda=2(2)^{1/2}\pi\lambda_D/(10)^{1/2}$ the value $(ne^2/\pi m_p)^{1/2}$ for ν is only in error by 5 percent and when $\lambda=2(2)^{1/2}\pi\lambda_D(10)^{1/2}$ the value $(kT_e/m_p)^{1/2}$ for v is likewise only 5 percent out.

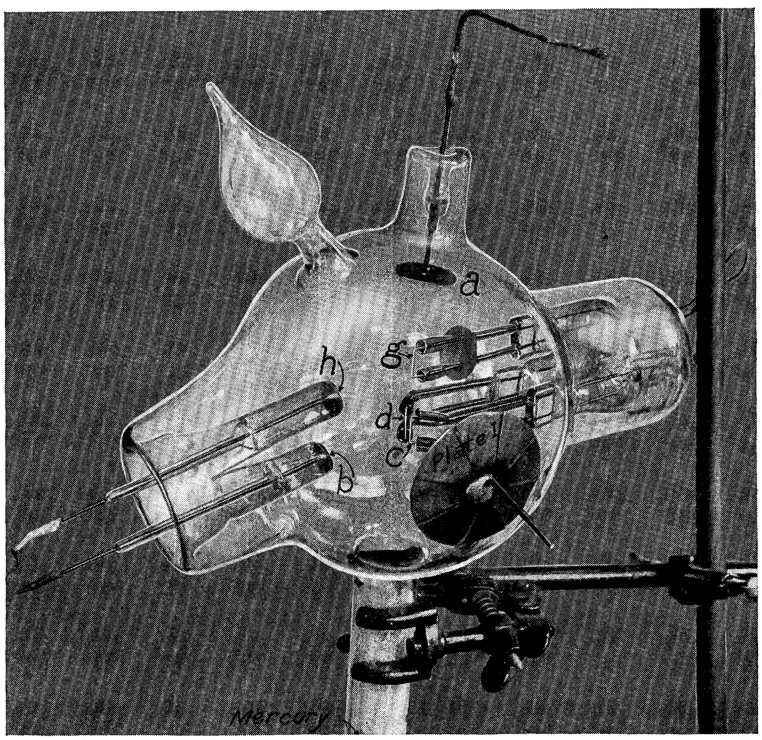

Fig. 1. Experimental tube.

J. S. Webb and L. A. Pardue[9] have described experiments in which they found oscillations that ranged from a few hundred cycles to 240 kc. in a low-pressure air discharge. As no data on the intensity of ionization and electron temperatures in their experiments are given, no quantitative comparison between this theory and their results can be made. The frequency range

[9] J. S. Webb and L. A. Pardue, Phys. Rev. **31,** 1122 (1928) and **32,** 946 (1928).

which they observed, is roughly, the range expected for the sound-like oscillations.

II. Experiments

A. Discharge tubes. On the experimental side we have worked with two tubes, both containing filamentary cathodes used as electron sources, collectors so placed as to receive a portion of the direct beam of primary electrons from a filament, and an anode off to one side to maintain the discharge. The two tungsten filaments in the first tube used were supported near the middle of the 18 cm spherical bulb by long glass-covered leads. Their exposed portions were about 1.1 cm long, parallel and about 0.5 cm apart. At a distance of 4.2 cm from them was the collector, a circular disk backed by mica and 1.1 cm in diameter. It lay in a plane parallel to that of the two filaments. An appendix containing a little mercury extended from the bottom of the bulb and was immersed in a water bath, the temperature of which controlled the mercury pressure.

The second tube was similar except that it contained three vertical tungsten filaments, g, c, and d in Fig. 1, g above, and c and d about 2.5 cm below it. These two were 0.4 cm apart and all three lay in the same plane. Their diameter was 0.025 cm and their vertical and active portions were 1.1 cm long. Opposite them and about 4 cm away were supported the 1.1 cm disk collectors h and b. The primary electrons are somewhat deflected by the magnetic field of the heating current, and the collectors are inclined as shown in order to give perpendicular incidence of the primaries on them. The glass tubes surrounding the collector leads prevented collection of ions and electrons by the back of the collector and also reduced the electrostatic capacity between leads and ionized gas. It was thought that the latter precaution might prevent a short circuiting of oscillations picked up by the collector and thus increase materially the observed amplitude of oscillation, but no marked effect of this nature is noticeable in a comparison of the results with the two tubes.

B. Detection of oscillations. The oscillations were detected with a zincite-tellurium detector and galvanometer arranged, for most of the work, in a circuit as shown in Fig. 2. The detector was supported by a spring suspension to shield it from mechanical shocks which were found to destroy its sensitivity. The high frequency potential was applied across the two points X and Y, Y being grounded to one side of a filament and also to the metal-screen cage surrounding the apparatus. The inductance L was often only a 10 to 15 cm length of copper wire. The two condensers, which were of 0.0025 μf each, shunted the galvanometer and 60-cycle crystal-calibrating circuits for the high-frequency oscillations, but at the same time allowed known 60-cycle voltages to be conveniently impressed on the crystal for calibrating purposes at frequent intervals. The impressed 60-cycle voltage in millivolts was plotted against the galvanometer deflection on log-log paper, giving a

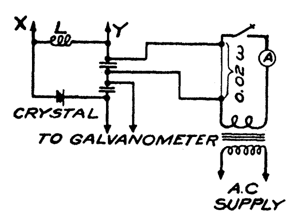

Fig. 2. Crystal detector circuit.

straight line of slope 0.5 which remained practically unchanged over considerable periods of use. This calibration was then used to interpret the galvanometer deflections arising from the gas oscillations as millivolts on the crystal. We found, however, that the same run made at different times always showed the same relative oscillation behavior but in one case the high frequency voltage might appear to be several times that in the other as if a frequency conversion factor for the crystal varied from time to time or with the crystal setting. The comparative amplitudes of a single run thus seem to be reliable, but to make more certain of comparative amplitudes in runs made at various times, high frequency as well as low frequency checking appears to be necessary.

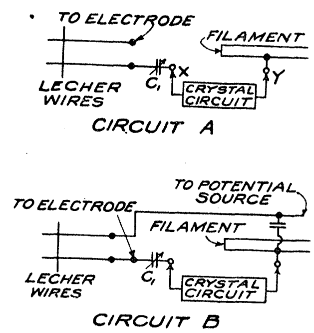

Fig. 3. Oscillation pick-up circuits.

The higher frequencies of the oscillations were determined from their wave-length as measured on a pair of Lecher wires. Circuit *B* of Fig. 3 was used to measure the oscillations of electrodes upon which it was desired to impress a voltage at the same time, while *A* was suited primarily for use with external electrodes.

C. Frequency range of oscillations. Oscillation frequencies as low as 10^6 and as high as 10^9 cycles per second have been observed. Over a large portion of this range it was not found possible to obtain any resonance phenomena, but oscillations were detected throughout so that we lean toward the view that these electric vibrations are irregular, thereby constituting an "electrical noise." Under certain conditions definite frequencies were observed both on the collectors and on external electrodes glued to the tube wall or placed against it.

The frequency range can roughly be divided into three parts, namely from 1 to 100 megacycles, from 100 to 300 megacycles, and from 300 to 1000 megacycles.

D. The range 100 to 300 megacycles. A 5 cm circular electrode (Plate I of Fig. 1) was fitted to the spherical tube wall and fastened with sealing wax so that its center was at a level about midway between the two collectors *h* and *b* and it lay opposite the space between collectors and filaments. Point *X* of the detection circuit was connected to this electrode through a variable condenser C_1 (15 $\mu\mu f$ maximum). With an emission from *d* of 25.0 m.a., an anode voltage of 62.7 and the mercury appendix at 20°C a very critical oscillation was found. Practically the whole effect lay within a range of 1.5 m.a. and 1 v. at this temperature. These oscillations did not occur when the appendix was at less than 19° or more than 21°C, the critical voltage decreasing some 5 v. in this range. The high frequency voltage at the crystal as measured was about 9 mv. The frequency was 263 megacycles at 19°C, 231 megacycles at 20°C, and 250 megacycles at 21°C, these figures probably being correct to within 2 percent.

An attempt was made to investigate the effect of additional ionization generated by emission from *g* and also the effect of the magnetic field generated by a current through *c*, but the results are uncertain because of the difficulty of staying on the oscillation peak. It appears that considerable emission from *g* (somewhere above 2 m.a.) stops the oscillations. Substitution of a smaller pick-up electrode on the tube wall and varying the setting of C_1 failed to change the frequency, but when the Circuit *B* of Fig. 3 was used instead of *A* the 230 megacycle wave disappeared and a 115-to 120-megacycle wave appeared in its place. The 230-megacycle wave was not found on the internal electrode using either circuit *A* or *B*. Reversing the filament heating current stopped the oscillations and they were not found in the similar wall position on the opposite side of the tube either when *c* or *d* was used as cathode.

The 115 megacycle wave was first found on *b* in the search for the 230 megacycle wave on an internal electrode. The galvanometer deflection corresponded to 30 to 50 mv. Decreasing the bath temperature from 20°C to 19.5 or less caused the oscillation peak (as the anode voltage was varied) to decrease sharply in magnitude and the frequency to jump to the neighborhood of 900 megacycles. Unlike the 230 megacycle wave, the 115 megacycle persisted over electron emissions from 22 m.a. to 35, the maximum safe current for the filament. In this range the wave-length varied from 114 to 125 megacycles. This oscillation was also found on *h* which was out of the direct path of the primaries from the filament, but reversing the heating current in *d* caused the oscillations to disappear.

At this time only the 230 megacycle oscillation had been observed on the tube wall and as it was not found on *b* it was thought that it might arise from some interaction between the electrons from *d* and the sheath about *c*, thus being confined to a comparatively narrow region on the wall. To explore the wall, Plate I was removed and a rectangular electrode 2.4 by 4.1 cm was arranged to slide horizontally along the surface of the tube so that its length was approximately parallel to the filaments.

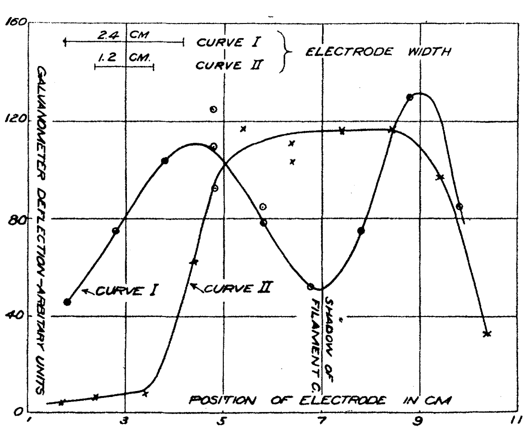

Fig. 4. Oscillations on tube wall.

It was immediately found that the oscillation present was the 115 megacycle one. Curve I of Fig. 4 shows the variation of oscillation amplitude with slider position. Another sliding electrode half as wide (1.2 cm) was then tried. The frequency of oscillation remained unchanged but the distribution on the wall showed a maximum, Curve II of Fig. 4, at the shadow where before it had shown a minimum. The narrower electrode, having the greater resolving power undoubtedly gives the truer picture of the amplitude distribution. The minimum with the larger one must arise from a change of phase centering about the shadow of *c*. As a further test a second narrow electrode was connected in parallel with the first and as symmetrically as possible with respect to the crystal circuit. The curves of Fig. 5 show the effect of moving one while the other was fixed. The minimum followed by a rising branch as the one electrode is moved away from the other indicates that the total phase change is considerably more than 180°.

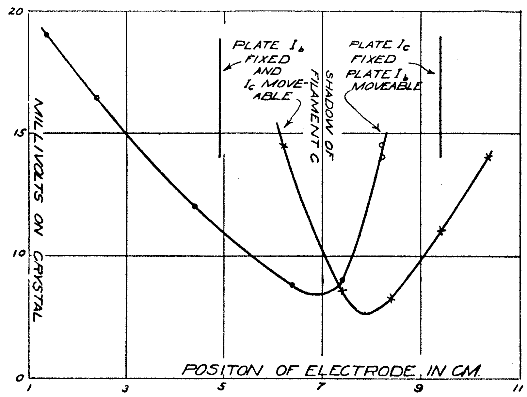

Fig. 5. Interference of oscillations on tube wall.

Apparently the presence of *c* was quite vital to the oscillations so the effect of varying its potential was tried. In the previous work it had been disconnected, its potential then being 2.5 v. positive with respect to the cathode. Fig. 6 shows the effect of varying the voltage on *c* with the 1.2 cm external electrode centered on the filament shadow. Nothing critical appears here, but the potential of *c* had a profound effect upon the localization of the oscillation, Fig. 7.

If the oscillation pattern arose from the action of *c* on the passing electrons, a magnetic field which deflected the electrons should affect the oscillation pattern equally. When tried, however, it was found that the magnetic field which had been calculated as producing a noticeable deflection caused the 115 megacycle wave to disappear and a 730 megacycle wave to appear in its place.

The frequency of 115 megacycles places that wave among the beam oscillations, but the other evidence is not clean cut. The wall pattern tends to support this point of view and the picking up of the oscillations by a collector even when it was repelling all the primaries is not in opposition to it on account of the low capacity impedance offered by the thin ion sheath to frequencies of this order. But the frequency variation with emission is too slow. A 60 percent increase in emission from 22 m.a. should, if the frequency is proportional to the square root of the electron density, cause a 26 percent increase in frequency from 114 megacycles. Actually a 10 percent increase was found, the small simultaneous voltage variation being negligible. The 230 megacycle wave may, of course, be related to the 115 megacycle as harmonic to fundamental.

E. The range above 300 megacycles. These short waves were investigated almost exclusively on the collectors although they were sometimes observed on the external electrode when conditions were not quite right for the longer and more critical oscillations already discussed. The inductance L (Fig. 2) was only that of a short length of wire shunting crystal and condensers and the arrangement of the crystal circuit in conjunction with Lecher wires and electrodes was usually that shown in Circuit B of Fig. 3.

One of the most significant features of these oscillations is that the amplitude and frequency was independent of the potential of the receiving collector over the whole range explored, namely from -80 v. with respect to the cathode to 5 v. above anode potential. Furthermore the frequency was the same whether the primary electrons approached the collector directly, whether they were deflected past by a magnetic field, or whether a collector entirely out of the primary electron stream was used to pick them up. Reversing the heating current in the filament left both frequency and amplitude unchanged. But the situation of the filament used had some effect. With an anode potential of 33 v. and an emission of 30 m.a. a wave-length of 810 megacycles was found when g was used as cathode whereas it was 880 megacycles when d, one of the two filaments close together, was cathode.

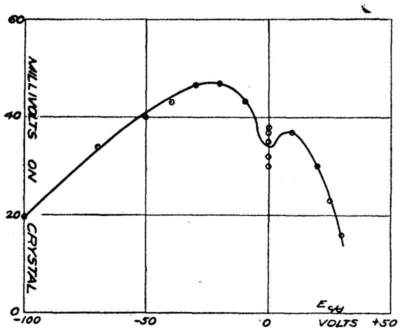

Fig. 6. Effect of potential of c on oscillations on tube wall.

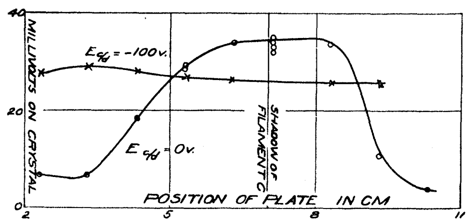

Fig. 7. Effect of potential of c on oscillations on tube wall.

It was natural to seek some correlation between the oscillations and the scattering of primary electrons. Unfortunately the runs meant particularly to elucidate this point are inadequate, first, because compared to the roughness of the present results the range covered was not wide enough; second, because of indeterminateness in the "temperature" of the scattered electrons, and third, because of the absence of high frequency checks on the crystal detector. But some early measurements on the first tube using a 2 m.h. coil for L in the detector circuit before any frequencies had been determined show that with 10 ma. from a 0.0125 cm filament 1 cm long the oscillations disappear in the same range of anode potential that the scattering does, namely between 60 and 75 v.

An unexpected behavior of the discharge was discovered in a certain range of current and voltage. At 20 m.a. emission with 27 v. on the anode the arc was stable in either of two conditions which were distinguished, as far as we could ascertain, only by the volt-ampere characteristic of a collector and the frequency of the oscillations. Fig. 8 shows the collector characteristic in each state. The curves are chiefly distinguished by the degrees of scattering shown. In addition the ion currents at large negative voltages are not equal, the ions in the low scattering state preponderating by about 3 percent in this case. In other cases greater differences in the same direction were found. Transition from one state to the other could be accomplished by manipulating a bar magnet. Lowering the collector voltage caused no switch-over but raising it caused a transition from the high to the low scattering state at about 22 volts. Finally, bringing the anode to 27 v. from a high value initi-

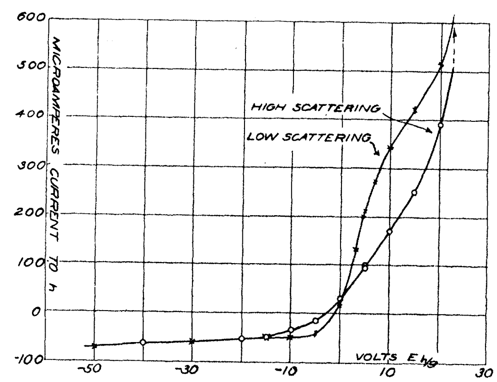

Fig. 8. Volt-ampere characteristics of a collector in the primary electron beam.

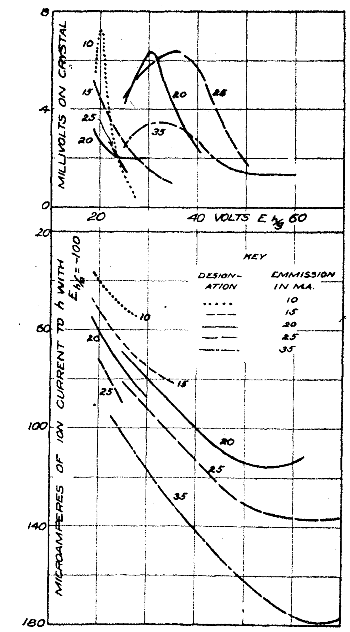

Fig. 9. Oscillation amplitudes and high- and low-scattering states.

ated the high scattering, from a low value the low scattering state. In another run, made with 30 v. on the anode several transitions occurred back and forth between the two states as the collector voltage was varied from -10 to $+20$ v. At the same time the oscillation amplitude changed abruptly, but the gaps in both sets of characteristics could readily be bridged so as to bring out the continuity of each state.

The frequency with the low scattering was almost twice that found with the high. Sometimes the arc showed less tendency to stay in one or the other state so the low scattering measurement was made with 26 v. on the anode, the high scattering with 28 v. The measured frequencies were 1080 and 660 megacycles respectively.

Fig. 9, which shows the variation of oscillation amplitude and positive ion current with anode voltage for different electron emissions, clearly exhibits the transition from the low scattering state at low emissions to the high scattering at high emissions. It is seen that at 10 and 15 m.a. only low electron scattering was found, that at 20 and 25 m.a. both low and high scattering appeared, and that 35 m.a. showed only high scattering.

The frequency of these oscillations places them among the plasma-electron oscillations. Their presence on an internal electrode irrespective of the path of the primary beam and their tendency to appear on the tube wall when other oscillations stop tend to confirm this view. The attempt was made to check them against Eq. (6), but there was no way of finding the absolute value of the ion density in the present tube because of large edge effects on the electrodes. The best we could do was to calculate relative values based on an elaborate series of measurements. Partial failure of some internal checks in these calculations show that too much reliance is not to be placed on them. Fig. 10 shows the grouping of the observed oscillations when plotted with wave-length in air against a scale of the relative ionizations. The simple theory requires that they lie in a line of slope $-1/2$. With the exception of two points out of the fifteen their deviations from the line drawn are within the accuracy of the ionization determinations. The deviations of those two may arise from some of the secondary theoretical factors.

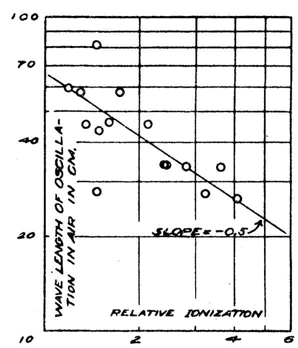

Fig. 10. Oscillation frequency and ionization density.

F. The range below 100 megacycles. While investigating the effect of Hg pressure on the 230 megacycle wave the appendix temperature was lowered to 15°C. With the emission still at 25 m.a. the oscillation maximum had apparently moved to 72.6 v. and was much less critical. Further tests showed, however, that the frequency was considerably lower than 230 megacycles and using a calibrated series tuned circuit in series with the crystal circuit it was located between 50 and 60 megacycles. At first sight it would appear possible to ascribe this frequency to the computed 18 megacycle oscillation for beam electrons together with the Doppler Effect. The frequency, ν_d, arising from the Doppler Effect is

$$\nu_d = (v_w + v_e)/\lambda$$

where v_w is the phase velocity of the waves with respect to the oscillating medium (primary electrons) and v_e is the velocity of the medium with respect to the observer. Since the beam electrons oscillate only at their natural frequency ν_e, we can replace v_w/λ by this quantity, and solving for λ find

$$\lambda = v_e/(\nu_d - \nu_e)$$

Using the values 55 and 18 megacycles for ν_d and ν_e respectively, and 5.1×10^8 cm sec.$^{-1}$ for v_e, we find λ to be 14 cm. This does not seem to be unreasonable, for it might in some way be connected with the 18 cm bulb diameter. Imagining ourselves to be travelling with the primary electrons, however, we see that each plasma electron takes $14/(5.1 \times 10^8) = 2.8 \times 10^{-8}$ sec. to traverse a wave-length. The oscillation period is only 5.5×10^{-8} sec., which taken in conjunction with the enormous preponderance of plasma electrons makes it impossible to think that these electrons do not modify the beam oscillations tremendously. In fact, a theory of infinite trains of plane oscillations as worked out by Mr. H. M. Mott-Smith, Jr. indicates that for a stationary observer the plasma-electron frequency ν_e is the minimum to be expected, the lower frequency of the beam oscillations (as seen from the beam) being more than compensated in the Doppler effect by the shortness of wave-length required to avoid interaction with plasma electrons. Thus it seems to be impossible to identify the 55 with the theoretical 18 megacycle oscillation.

Some of the early work with an external electrode on the first tube had, we thought, indicated the presence of 15 or 20 megacycle oscillations. In searching for these Plate I was used as pick-up, C_1 was taken out of the detection circuit, Fig. 3, and a 2 mh. coil used for L. An oscillation peak of some 90 mv. was located at 25 m.a. emission and 63 v. on the anode—practically the condition for the 230 or 115 megacycle oscillation. Tuned circuits covering successive ranges from 100 megacycles down were tried in various ways in the attempt to locate the frequency. Resonance was finally established at 1.38 megacycles although considerable voltage appeared across the crystal throughout the whole range.

Between 26 and 42 v. on the anode the oscillations also reach the neighborhood of 90 mv. but the attempts to find the wave-lengths met with no particular success.

We attempted to check the 1.38 megacycle wave by using a tuned radio amplifier. For this purpose a Radiola 17 having the range 550 to 1450 kc. was set up in the cage and an antenna wire was bought near Plate I. Unfortunately the 1380 kc. wave had become so elusive that even the previous measurements of its wave-length could not be repeated so we were unable to check this. At this time the amplifier gave a noise maximum in the middle of its scale. This was not sharp, the noise extending over most of the range. Slight changes in the tube conditions caused large differences in the way the amplifier responded, sometimes several sharp noise peaks appearing, at others a single peak, and at still others only a very dull maximum. This, it seems to us, shows conclusively that there is often "a continuous spectrum" of electric oscillations in this long wave range.

The strong critical oscillation at 1380 kc. lies in the neighborhood of the theoretical short-wave limit at 1500 kc. calculated above. Whether this was actually the short-wave limit is not known, for frequencies higher than this were not sought with an amplifier.

G. Future experiments. The experiments described are in general accord with the theory outlined. A closer checking and further development of the theory, particularly with respect to the cause and critical nature of some of the oscillations, await experiments under essentially simpler conditions. With this in view we are continuing the experiments with a cylindrical tube containing a cylindrical collector at the tube wall and an axial filament. The ionization can be controlled independently of these electrodes by a hot cathode and an anode at the two ends of the tube.

In addition to Sir J. J. Thomson's letter which has already been mentioned, we gratefully acknowledge the aid and suggestions of Mr. H. M. Mott-Smith, Jr. in the development of the theory and of Mr. S. Sweetser in the experimental work.

Research Laboratory,
General Electric Company,
Schenectady, N. Y.
November 20, 1928.

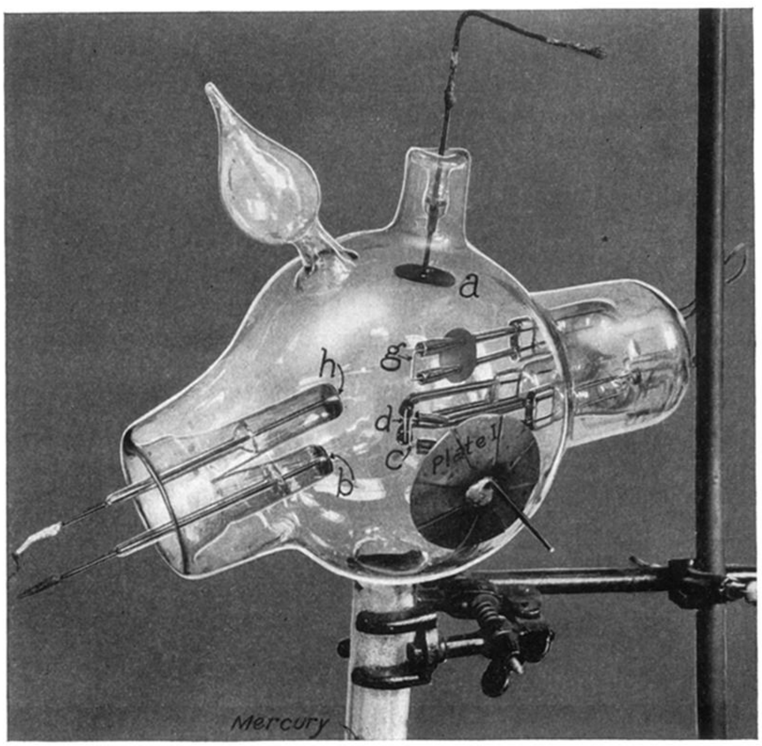

Fig. 1. Experimental tube.