THE EFFICIENCY OF ELECTRON IMPACT LEADING TO RESONANCE IN HELIUM

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Abstract

The efficiency of resonance impact between electrons and helium atoms has been studied as a function of the energies of the impinging electrons. The transition studied is $1^1S \rightarrow 2^3S$ at 19.77 volts. The range of energies is 0.8 volt, i.e. from 19.77 to 20.55 volts. The efficiency with which electrons make inelastic impacts rises to a maximum 0.18 volts beyond the resonance potential of 19.77 volts and then decreases. The maximum average efficiency for electrons of 19.95 volts energy is estimated to be 0.002. The measurements were carried out by comparing the velocity distribution of the electrons leaving an equipotential surface with the drop in current as observed in the inelastic impact method of Franck, and similarly by comparing the velocity distribution of the electrons with the distribution of the positive current caused by them in a Lenard experiment. Both types of measurement yield similar results.

1. *INTRODUCTION.* In order to further our understanding of the mechanics of collisions between electrons and atoms it is important to determine the probabilities of such collisions. For the case of ionization several investigations have been made.[1] It appears that the efficiency of ionization, which is zero below the ionization potential, gradually *increases* from zero to 0.1 to 0.3 at optimum speeds of from 70–160 volts for various gases. The case of resonance efficiency has been studied by Sponer[2] who measured the average efficiency for electron impacts in mercury vapor and by Dymond[3] who studied the first critical potential of helium. Sponer's work would indicate that the efficiency of excitation is large right near the resonance potential and then decreases with increasing energy of the impinging electron. K. T. Compton[1] deduced a similar relation of efficiency and electron energy from the measurements of the photoelectric current in mercury vapor by Franck and Einsporn.[4] Sponer finds an average efficiency of about 0.004 for excitation of mercury atoms by electrons having energies between 5 and 6 volts. Hertz[5] has recalculated her results using a different value for the total number of impacts that an electron makes in crossing the space between filament and plate in Sponer's experiments, and finds for the average efficiency 0.03. Dymond, however, finds that the efficiency of excitation in helium rises to a maximum 0.25 volt above the first resonance potential (19.77 volts), and then decreases to one fourth the maximum value at 0.5 volt above the critical potential. He calculates the maximum efficiency to be 0.001.

[1] Compton and Mohler, Bull. Nat. Research Council, No. 48, 52 (1924); also Compton and Van Voorhis, Phys. Rev. **26**, 436 (1925).

[2] H. Sponer, Zeits. f. Physik **7**, 185 (1921).

[3] G. Dymond, Proc. Roy. Soc. **A107**, 291 (1925).

[4] Franck and Einsporn, Zeits. f. Physik **2**, 18 (1920).

[5] G. Hertz, Zeits. f. Physik **32**, 298 (1925).

From these investigations it is seen that the efficiency of ionization and of resonance are dependent on electron energies in quite a different manner and that their numerical values are of a different order of magnitude. In this paper the probability of excitation of helium has been studied. The transition $1^1S \rightarrow 2^3S$ leading from normal par-helium to metastable ortho-helium has been investigated from 19.77 to 20.55 volts.

2. *Experimental procedure.* The experimental arrangement used has been fully described in a previous paper.[6] In our further discussion we shall call the space between the emitting surface and the first gauze, which is at a distance of one millimeter from the emitter, the first condenser. In this region electrons receive energy from the accelerating field A_1. They will not make many impacts with gas molecules before they enter the space between the two gauzes which is the second condenser. The distance between the two gauzes is 1.1 cm and here the electrons make impacts with the gas molecules. The third condenser is the region between the second gauze and the receiving plate and is of small dimension (one millimeter between second gauze and plate). A small retarding field is applied in the Franck experiment to keep electrons that have lost their energy because of inelastic impact in the second condenser from reaching the plate and a large retarding field is applied in a Lenard experiment to keep all primary electrons from the plate, but to attract positive ions to or remove photo-electrons from the plate.

3. *Types of current-potential curves.* With the experimental arrangement described above it is possible to obtain four types of current-potential curves. The actual experimental curves are shown in Fig. 2 of the previous paper.[6] In the present paper we wish to compare the shapes of these curves with one another and in order to carry out this comparison we have replotted the experimental curves by taking the differences of the ordinates (galvanometer deflections) for each one-tenth volt interval. We then gave to the maximum ordinate of these difference curves of each type the same numerical value by multiplying the ordinates with an appropriate factor and then we averaged them by taking the mean value of the ordinates at the same abscissa and in this way we obtained one *average* current-potential curve representing each of the four types of experimental curves given in the previous paper. The maximum ordinates of our four types of difference curves have all been made 2.68 on an arbitrary scale. The other points vary each about a mean value. The average deviation from the mean ordinate is given in the tables. While the average deviation is large especially near the ends of the distribution curves, it should be noted that this is partly brought about by our method of averaging the curves. We have arbitrarily taken the middle point (the maximum ordinate) the same for all the curves and naturally the error would thereby tend to accumulate near the ends of the curves. By doing so we have given more weight to the greater changes in galvanometer deflection and this procedure seems reasonable because the differences in deflection near the ends of these curves are smaller and therefore less accurate. It will be seen in the course of our discussion that the accuracy of these points is

[6] G. Glockler, Phys. Rev. **27**, 423 (1926).

sufficient, for our argument rests only on the most general shape of these curves and upon the volt values of their maximum ordinates. The individual curves are now to be described in detail.

4. *The electron velocity distribution (Method I).* The distribution of the initial velocity of the electrons emitted by the filament has been obtained by two methods. The current-potential curves giving this distribution were obtained in the first method by the use of a constant accelerating field A_1 in the first condenser and a varying retarding field R_2 in the second condenser. The six experimental curves of Fig. 2-*A* of the previous paper give an average curve and the quantities used to plot this curve are given in Table I, columns 2 and 3. The ordinates G_n are proportional to the decrements of the galvanometer deflections, and the abscissae give the range of the initial velocity and contact potential corrections.

TABLE I. *Electron velocity distribution*

A_1 is the accelerating field. R_2 the retarding field. G_n is the increase or decrease in the galvanometer reading for each 0.1 volt interval.

Interval n	Method I		Method II		Average
	(R_2-A_1)	G_n	A_1	G_n	G_n
1	0.66±0.05	0.08±0.01	0.31±0.05	—	0.08
2	0.76	0.15±0.06	0.41	—	0.15
3	0.86	0.33±0.07	0.51	0.27±0.06	0.30
4	0.96	0.70±0.10	0.61	0.79±0.10	0.75
5	1.06	1.50±0.11	0.71	1.53±0.10	1.52
6	1.16	2.36±0.06	0.81	2.27±0.07	2.31
7	1.26	2.68±0.00	0.91	2.68±0.00	2.68
8	1.36	2.19±0.05	1.01	2.20±0.10	2.20
9	1.46	1.11±0.08	1.11	1.32±0.17	1.21
10	1.56	0.16±0.04	1.21	0.76±0.14	0.76
11	1.66	—	1.31	0.48±0.10	0.48
12	1.76	—	1.41	0.25±0.08	0.25
13	1.86	—	1.51	0.12±0.04	0.12

5. *The electron velocity distribution (Method II).* We have also obtained the velocity distribution of the electrons by measuring the electron stream leaving the filament (with helium in the apparatus) as a function of the accelerating field A_1 applied to the first condenser. The increase in electron current to the first grid was measured as the first accelerating field was increased from 0.5 volt to 1.5 volts while the other fields were zero. The curves of Fig. 2-*F* and 2-*G* of the previous paper show this electron current. We have now averaged these eight curves and we have arbitrarily chosen the ordinates of galvanometer deflections G_n in such a way that the maximum ordinate of the difference curve is 2.68. The abscissae give the range of the initial electron velocity and contact potential corrections. The data are given in Table I (columns 4 and 5).

In our reduction of the experimental curves we have corrected for the constant increase in electron-current to the plate as the first accelerating field is increased. It is well known that saturation is not reached in the usual set-up. The correction has been made by subtracting the constant increment

that appeared in the electron-velocity distribution curve taken in helium. A similar correction was made in the Franck curve. We have studied our data both with and without making this correction and we find that its use does not effect the results of our analysis. The correction has only a slight effect on the shape of the curves.

6. *Comparison of the electron velocity distribution curves.* The data of Table I are averaged in the last column by making the maxima of the curves coincide and the average electron velocity distribution curve is shown in Fig. 1. It is seen that the velocity distribution obtained by the two methods is the same. When it is considered that the conditions under which these two types of curves were obtained are quite different, that however the distribution curves obtained are quite similar, it is evident that any disturbing factors such as random direction of electrons and secondary electron emission are not of sufficient magnitude to vitiate the results. It should be noted that the retarding potential in the first method is applied in the second condenser and it will affect the whole energy of the electron, because of the more or less spherical construction of the electrodes, and not only one component. In Table I column 3 it is noted that the change in current after the ninth interval is less than in Method II (column 5). This lack of agreement may be due to secondary emission. It is evident that the two methods check very well near the maximum point and this section of the distribution curve is the one of greatest interest to us.

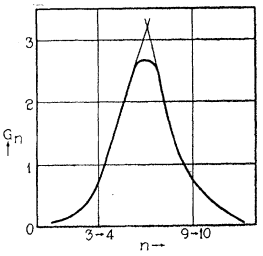

Fig. 1. Average electron velocity distribution curve.

The voltage range in Table I is one-tenth volt. For example, the point 1.26 volts (second column) means that the current to the plate dropped 2.68 units while the voltage changed from 1.21 to 1.31 volts. Similarily the voltage values given in all of the tables always refer to a range of one-tenth volt, the

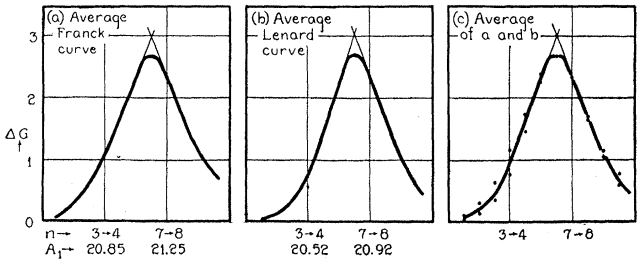

Fig. 2. *a*. Distribution curve of drop in current in Franck experiments. *b*. Distribution curve of rise in current in Lenard expeirments. *c*. Comparison of *a* and *b*.

mean value of the range being given in the table. The volt values in Table I given for the two methods differ by 0.35 volts. This difference is due to a change in contact potential as noted in the previous paper. The distribution curves shown in Figs. 1 and 2 are drawn in such a manner that the fast electrons or the current caused by them is always shown on the left side of

the figure. The intervals "n" given in the first column of the tables are a convenient notation and they give the changes in current for one-tenth volt change in voltage. The numbering of these intervals is accomplished by giving the first interval the number "one" when the galvanometer current (G_n or ΔG_n) rises from zero to the first noticeable reading.

7. *The current drop (Franck Method).* The drop in current near twenty volts obtained by the inelastic impact method of Franck is shown in Fig. 2-*B* and *C* of the previous paper. The current-potential difference curves have been reduced to an arbitrary maximum ordinate of 2.68 and in Fig. 2*a* the average curve of nine experiments is shown. The ordinates are the decrements in the drop in electron current reaching the plate and the abscissae are the increments in the applied accelerating field in the first condenser expressed in volts uncorrected for contact potential. The data for Fig. 2*a* are given in Table II. The correction for the constant increase in current with increase in A_1 has been made.

8. *The current rise (Lenard Method).* In this method the electron current to the plate is zero on account of the large retarding field of thirty volts applied in the third condenser. However, when either secondary or photoelectrons leave the plate or positive ions reach it, a positive current results. This happens near twenty volts. This rise in positive current to the plate is shown in Fig. 2-*H* and 2-*I* of the previous paper. Again the eight difference curves shown there have been united into an average curve with arbitrary ordinates. The maximum ordinate has been given the value 2.68 on an arbitrary scale. The average curve is shown in Fig. 2*b* and the necessary data in Table 2.

9. *Comparison of Franck and Lenard curves.* The Franck curves (Fig. 2*a*) are compared with the Lenard curves (Fig. 2*b*) as is shown in Fig. 2*c*. *The two types of current-potential curves are similar in shape.* The Franck curve gives a picture of the "zero-volt" electrons produced when the electron stream from the filament makes inelastic impacts with helium atoms and the Lenard-curve gives a measure of photoelectrons (or positive ions of an impurity) produced under the same conditions. Both are the effect of the interaction of the primary electron stream with helium atoms, when the primary stream has reached sufficient energy to cause the resonance transition $1^1S \rightarrow 2^3S$. It may then be expected that the two curves should be identical in shape.

We shall assume that the difference which exists between the shapes of the Franck and Lenard curves is due to experimental error. We have therefore averaged the two curves and the averaged data are given in the last column of Table II. The averaged Franck-Lenard curve is shown in Fig. 2*c*.

The question arises what effect secondary electrons would have on the shapes of these curves. If, for instance, the Franck curve includes secondaries then the real ΔG_n values would be larger. If, however, the fraction of secondaries emitted is constant in the small voltage range considered (which is not unreasonable) then the shape of the difference curves described above is not affected, since we always plot them to an arbitrary scale. In the electron velocity curves we deal with very slow electrons which are known not to emit secondaries and in the Lenard curves we have no primaries reaching the

plate. Secondary emission should then have no marked effect on the shapes of our curves although numerical values would be affected.

TABLE II. *Comparison of current drop in the Franck experiment and current rise in the Lenard experiment*

Interval n	Franck experiment $A_1 \pm 0.05$	Franck experiment ΔG_n	Lenard experiment $A_1 \pm 0.05$	Lenard experiment ΔG_n	Average ΔG_n
1	20.55	.10 ± .04	20.22	.04 ± .06	.07
2	.65	.32 ± .12	.32	.13 ± .09	.23
3	.75	.66 ± .13	.42	.35 ± .12	.50
4	.85	1.15 ± .09	.52	.77 ± .16	.96
5	.95	1.73 ± .09	.62	1.46 ± .15	1.60
6	21.05	2.38 ± .08	.72	2.26 ± .12	2.32
7	.15	2.68 ± .00	.82	2.68 ± .00	2.68
8	.25	2.34 ± .11	.92	2.34 ± .12	2.34
9	.35	1.68 ± .24	21.02	1.66 ± .14	1.67
10	.45	1.13 ± .30	.12	1.05 ± .15	1.09
11	.55	0.77 ± .28	.22	.58 ± .11	.68

10. *Comparison of electron velocity distribution curve and Franck and Lenard-curves.* The average electron velocity distribution (last column Table I) and the average Franck-Lenard curve (last column Table II) are shown in Fig. 3. The maxima of these curves have been made to coincide. From the values of the abscissae of these maximum ordinates we find the experimental values of the first resonance potential of helium:

TABLE III. *Comparison of values of first critical potential in He as given by Franck and Lenard experiments with the spectroscopic value.*

	Franck expt.	Lenard expt.
Accelerating potential (uncorrected)	21.15 ± 0.06 volts	20.82 ± 0.02 volts
Correction	1.26 ± 0.01	0.91 ± 0.06
First resonance potential of He by electron impact	19.89 ± 0.07	19.91 ± 0.08
Spectroscopic value	19.77	19.77
Difference	0.12 ± 0.07	0.14 ± 0.08

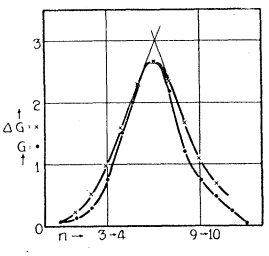

Fig. 3. Comparison of electron velocity distribution and current drop and rise in the Franck and Lenard experiments.

The two types of experiments yield the same result: namely, that *the shapes of the distribution curves are similar and that the maxima of these curves are displaced 0.13 volts.* The reason that the uncorrected values of the critical potentials and the corrections are different in the Franck and Lenard experiments has already been mentioned. The emitter had to be reconditioned after the Franck curves and before the Lenard curves had been taken, and the contact potential had been changed. The corrected values check satisfactorily. The correction of 0.91 volt applied to the Lenard curves had been given in the previous paper (page 428) as 1.00 volt. We have

taken in the present analysis in Method 2 of determining the initial velocity correction, the electron current to the first grid for it is much larger than the plate current and should give more accurate curves. The difference of 0.13 volt between the resonance potential as determined by electron impact and spectroscopy is not very accurate because it is determined as the difference of two large numbers. However, since both types of measurements yield the same average result we may accept it with some confidence.

In our further analysis we shall need to know the total number of impacts that an electron makes while crossing the gas-space in the condenser. This claculation can be made from kinetic theory on the assumption that the electron loses no energy on impact with a helium atom. However, it is known that an electron does lose energy even while making an elastic impact and we shall also calculate the number of impacts an electron makes on the assumption that the impact takes place in accordance with the laws of classical mechanics, while the electron changes its energy from 19.87 to 19.77 volts.

11. *The total number of impacts from kinetic theory.* It can be shown[7] that an electron in crossing the second condenser will make n impacts, where

$$n=(3/2)(a/l)^2 \tag{1}$$

and a is the thickness of the condenser and l is the mean free path of the electron at the gas-pressure p. The mean free path l_1 of helium[8] atoms at pressure p is from kinetic theory:

$$l_1=29.6\times 0.00075/p \text{ cm/mm Hg} \tag{2}$$

As is well known, the mean free path of a particle of small size, like an electron is $4\cdot 2^{1/2}$ times the mean free path of the atom. Furthermore, if we take into account the fact that the mean free path of an electron varies with its speed, as is known from the work of Ramsauer,[9] we must multiply the expression by 0.8, in order to obtain the mean free path for electrons of the velocity used. The thickness of the second condenser was 1.1 cm, but when we take into account the fact that the emitter and first gauze are semispherical in shape, while the second gauze and plate are cylindrical, we find from geometry that the average thickness of the second condenser is 1.3 cm. We have then for the total number of impacts in our apparatus.

$$n=164p^2 \tag{3}$$

12. *The total number of impacts from mechanics.* The above calculation has been made on the assumption that the impinging electron loses no energy when colliding elastically with a helium atom. However, Hertz[10] has shown that an electron does lose energy even during an elastic impact, and that this loss is the loss that occurs when solid hard spheres of the masses of the electron and helium atom collide. This loss can therefore be calculated from the

[7] G. Glockler, Proc. Nat. Acad. Sci. **12**, 178 (1926).
[8] S. Dushman, High Vacuum, Gen. Elect. Rev. 237 (1922).
[9] C. Ramsauer, Ann. d. Physik **64**, 513 (1921).
[10] G. Hertz, Verh. d. D. Phys. Ges. **19**, 268 (1917).

principles of ordinary mechanics. K. T. Compton[11] gives this calculation. If f is the average fractional loss of energy per impact averaged over all angles, then

$$f=m/M \tag{4}$$

where m is the mass of the electron and M is the mass of the helium atom. If eV_0 be the initial energy of the electron and eV_z its energy after Z impacts then it follows that

$$V_z=V_0(1-f)^z \tag{5}$$

If we consider an electron that has 19.87 volts of energy or 0.1 volt more than it needs to cause resonance in helium we find that it can make 18.3 elastic impacts before its energy is reduced to 19.77 volts. This value for the total number of elastic impacts per 0.1 volt loss in energy is nearly constant in the range between 19.77 and 20.57 volts.

Let us consider a 19.87 volt electron just entering the second condenser. While it will make n impacts in crossing the total distance of the second condenser, we see that after the first 18.3 impacts its energy is reduced to 19.77 volts. During the remaining $n-18.3$ impacts this electron will still make before leaving the second condenser, its energy is less than 19.77 volts. None of these impacts can therefore be inelastic, and they must not be counted in our calculation of efficiency. We shall therefore calculate the total number of impacts from mechanics by means of Eq. (5).[12]

13. *The efficient impacts from the Franck Method.* We next calculate the fractional drop in current (in the Franck method) from the experimental curves (Fig. 2-*B* and *C* of the previous paper) at a point 0.8 volt beyond the beginning of the current drop. A larger range of accelerating voltage cannot be investigated because a second resonance point occurs in helium 0.8 volt higher than the first critical potential. The values obtained for the fractional current drop $\Delta D_8/D_0$ are given in Table IV, column 5. From the variation in individual experiments it is seen that extrapolation of D_0 0.8 volt beyond the beginning of the current-drop is unnecessary.

We have given the values $\Delta D_8/D_0$ obtained from the runs at low pressures where there are no impacts possible in the first and third condensers. If we take into account all the experiments performed at pressures varying from 0.8 to 13 mm, we find a trend with pressure. Sponer[2] has found a similar trend with pressure in mercury. We can eliminate this trend by extrapolating to zero pressure, and then we find for the fractional current drop the value 0.08. We may therefore accept as an approximate value

$$\Delta D_8/D_0=0.1 \tag{6}$$

This ratio will be used to calculate numerical values of the resonance efficiency. We are studying, however, two distinct problems. The first problem is to determine the shape of the efficiency curve as to whether or not the

[11] K. T. Compton, Phys. Rev. **22,** 333 (1923).

[12] H. B. Wahlin, Phys. Rev. **27,** 595 (1926) finds from his experiments on mobilities in helium that λ is 8.6 times greater than the value calculated from mechanics. If we use Wahlin's value for λ the numerical value of the efficiency calculated in this paper would be changed.

efficiency increases to a maximum at some voltage beyond 19.77 volts and then decreases again, or whether the efficiency is constant after the energy of resonance has been reached. These questions can be answered from the similarity in shape of the electron distribution curves and the Franck-Lenard curve and the value of the ratio $\Delta D_8/D_0$ does not enter into these considerations. Only when we wish to give actual numerical values to the efficiencies do we need to use this ratio.

TABLE IV. *Values of* $(\Delta D)_8/D_0$ *for various pressures.*

Pressure (mm)	Retarding volts 3rd condenser	D_0	D_8	$\frac{D_0-D_8}{D_0}$
0.75	0	19.00	18.32	0.036
0.81	0.35	8.30	6.90	.169
0.81	0.35	6.40	5.50	.140
0.83	0.35	5.31	4.70	.115
0.83	0.35	31.45	29.35	.067
1.01	0	25.00	22.42	.103
1.83	0	22.89	19.42	.152
			Average	0.11 ± 0.04

The general scheme of analysis used in the following section is to study all conceivable shapes of efficiency curves, and to apply them to our experimental electron velocity distribution curve (Fig. 1). This procedure will give us calculated Franck-Lenard curves. These theoretical curves will then be compared with the experimental Franck-Lenard curves (Fig. 2). The particular efficiency curve which produced the best fit between experimental and calculated Franck-Lenard curve we will adopt as the true efficiency curve.

14. *Calculation of resonance efficiency.* If we assume the shape of the resonance efficiency curve and if we apply these assumed values to the electron distribution, we can calculate the Franck and Lenard distribution curves. The following notation is used: ΔD is the current drop or rise in a Franck or Lenard experiment (galvanometer deflection in arbitrary ordinates taken at tenth volt intervals); ΔG the increments or decrements in ΔD(Table II); D the total electron stream leaving the filament (galvanometer deflection in arbitrary ordinates taken every tenth volt); G the increments in D, Table I (For example $D_1=G_1$; $D_2=G_1+G_2$; $D_3=G_1+G_2+G_3$; etc.); E the average efficiency of resonance impact within a one-tenth volt interval. As the accelerating field is increased in steps of one-tenth of a volt, the first group of electrons which will have sufficient energy to cause resonance collision is G_1. They will produce the first current drop or rise in a Franck or Lenard experiment. This group G_1 will make on the average 18.3 impacts before their energy has been reduced below 19.77 volts where they can no longer make inelastic impact. A certain number of these 18.3 impacts will be inelastic because the electrons can cause inelastic impact in this energy range with an efficiency E_1. Therefore

$$\Delta D_1=ZE_1G_1 \tag{7}$$

If we increase the accelerating field another step of one-tenth volt, then the group G_2 of electrons can make inelastic impact in the same manner as the group G_1 per (7). However, the group G_1 can now produce inelastic impact in two ways. It can make 18.3 impact while its energy is reduced from 19.97 to 19.87 volts and if the average efficiency of resonance collision in this voltage interval is E_2 then, the total number of inelastic collisions will produce a change in current

$$\Delta D_2 = Z[G_1E_2 + G_1E_1 + G_2E_1] \tag{8}$$

and similarily

$$\Delta D_3 = Z[G_1E_3 + G_1E_2 + G_1E_1 + G_2E_2 + G_2E_1 + G_3E_1] \tag{9}$$

Subtracting these equations we shall have expressions for the ordinates at one-tenth volt intervals of the distribution curves of the Franck or Lenard type.

$$\Delta G_1 = Z[E_1G_1]\ ;\quad \Delta G_2 = Z[G_1E_2 + G_2E_1]\ ;\quad \Delta G_3 = Z[G_1E_3 + G_2E_2 + G_3E_1]\ ;\quad \text{etc.} \tag{10}$$

We have studied these equations by assuming various values for the efficiency and using the electron velocity distribution (last column Table I) we have constructed the theoretical Franck-Lenard curves. Of the many cases studied the following are of interest:

Case 1. $E_1 \neq 0$; $E_2 = E_3 = E_n = 0$. We assume that electrons can make inelastic impact only in the first one-tenth volt interval after 19.77 volts and that for larger energies the efficiency for resonance is zero. The Eqs. (10) then become:

$$\Delta G_1 = ZE_1G_1\ ;\qquad \Delta G_2 = ZE_1G_2\ ;\qquad \Delta G_3 = ZE_1G_3\ ;\quad \text{etc.} \tag{11}$$

It is seen that *the ΔG-curve would coincide with the G-curve if plotted on an appropriate scale.* If we should then find that the Franck-Lenard difference curves would coincide with the electron-velocity distribution curves, then we would interpret that fact to mean that the efficiency of resonance would have the values assumed above.

Case 2. $E_2 \neq 0$; $E_1 = E_3 = E_n =$ zero. We assume that electrons can only make inelastic impact in the second one-tenth volt interval after 19.77 volts and that in the other intervals the efficiency is zero. Then Eqs. (10) become:

$$\Delta G_1 = 0\ ;\quad \Delta G_2 = ZE_2G_1\ ;\quad \Delta G_3 = ZE_2G_2. \tag{12}$$

Again we find that the ΔG-curve and the G-curve would coincide if plotted on an appropriate scale. However, in order to cause coincidence of these curves, they must be shifted one-tenth volt along the abscissa. From the study of similar cases where always one of the efficiencies had a real value while the others were zero, we derived the following relation:

$$V_m = d + 0.05 \tag{13}$$

where V_m is the abscissa in volts beyond the resonance potential of 19.77 volts where the efficiency is a maximum and d is the displacement in the Franck-Lenard and electron-velocity distribution curves. We therefore interpret the fact that the experimentally determined value of the resonance potential has been found above (Table III) to lie 0.13 volts higher than the

spectroscopic value, to mean that *the maximum efficiency lies 0.18 volts above 19.77 volts.*

Since the average Franck-Lenard curve and the electron-velocity distribution curve do not exactly coincide as is seen from Fig. 3, we have studied various other cases with the efficiency maximum at 0.18 volt above 19.77 volts. The following case represents the experimental Franck-Lenard curves the best:

Case 3. $E_1=0.0011$; $E_2=0.0022$; $E_3=0.0022$; $E_4=0.0007$; $E_5=E_6=E_n=0$. The shape of Franck-Lenard curve obtained by applying this set of efficiencies to our electron-velocity distribution curve does not depend upon these particular values of the efficiencies. Any set of values which, however, must have relative magnitude as indicated will of course produce the desired shape in the Franck-Lenard curve, since we can always plot to the same scale or reduce calculated values by the use of a multiplying factor to the arbitrary chosen maximum (2.68). Table V shows the experimental and calculated Franck-Lenard curves. According to the result of section 13, the drop in

TABLE V. *Comparison of experimental and calculated Franck-Lenard curves*

Interval n	Velocity Distribution ΔG_n	Franck-Lenard Curve ΔG_n (exp.)	Franck-Lenard Curve ΔG_n (calc. case 3)
1	0.08	0.07	0.07
2	0.15	0.23	0.23
3	0.30	0.50	0.45
4	0.75	0.96	0.92
5	1.52	1.60	1.70
6	2.31	2.32	2.42
7	2.68	2.68	2.68
8	2.20	2.34	2.37
9	1.21	1.67	1.68
10	0.76	1.09	1.06
11	0.48	0.68	0.65
12	0.25	—	0.35
13	0.12	—	—

current in a Franck curve 0.8 volt beyond the beginning of the drop should be 10 percent of the total electron stream. The first eight intervals of the electron stream (${}^{8}_{1}\sum G_n$) amount to ten units of current on the arbitrary scale chosen. The first eight intervals of the Franck curve (${}^{8}_{1}\sum \Delta G_n$) should count unit current on the same scale. We need only reduce the ordinate actually used to plot the Franck-Lenard curve by a factor of ten to conform to the result of section 13.

The values of the efficiencies used in Case 3 above and shown in Table V give a Franck-Lenard curve whose ordinates are one-tenth as large as the electron-velocity-distribution curve. The efficiency curve which is used to obtain the best fit of the Franck-Lenard curve is shown in Fig. 4.

Case 4. $E_1=0.001$; $E_2=0.002$; $E_3=0.002$; $E_4=0.001$; $E_5=0.0006$; $E_6=E_7=E_n=0.0004$. This case differs from the preceding case only in that the efficiencies from E_5 to E_n have been given values different from zero. It is

seen that the resulting Franck-Lenard curve does not fit the experimental electron velocity distribution curve as well as in Case 3. We therefore believe that the efficiency of resonance *decreases rapidly beyond the maximum to a very small value.*

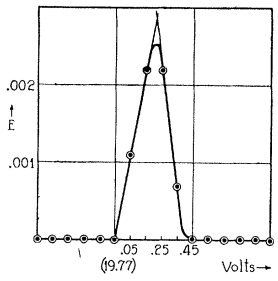

Fig. 4. Efficiency of resonance as a function of electron energy.

Case 5. $E_1=E_2=E_n=$constant. The types of efficiency curve studied in the cases cited so far had a maximum at some point beyond 19.77 volts and they decreased rapidly beyond the maximum. We investigated the possibility that the efficiency might not be small beyond the maximum. The resultant Franck-Lenard curves do not fit the experimental distribution curve at all. The extreme case studied is the case of constant efficiency throughout the voltage range. The Eqs. (10) then become:

$$\Delta G_1=ZED_1\,;\quad \Delta G_2=ZED_2\,;\quad \Delta G_3=ZED_3 \tag{14}$$

The Franck-Lenard curve resulting from the assumption of constant efficiency shows no maximum and it is evident that our experimental data do not fit such a case at all.

The cases of especial interest are shown in Fig. 5. The type of efficiency curve studied is shown in the upper portion of the figure and the calculated Franck-Lenard curves obtained by applying the respective efficiency curves

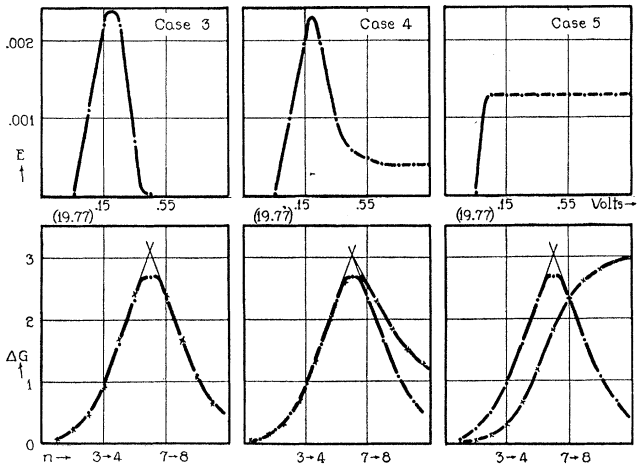

Fig. 5. Assumed efficiency curves and the resulting Franck-Lenard curves.

to the experimental electron velocity distribution curve are shown in the lower position of the figure. The calculated Franck-Lenard curves have been displaced 0.13 volt so that their maxima coincides with the maximum of the experimental Franck-Lenard curve. This coincidence could of course not be brought about in Case 5. It is evident that the efficiency curve of Case 3 shown in Fig. 4 gives the best fit to the experimental curves.

15. *Discussion of Results.* We have established the experimental fact that the electron velocity distribution curve and the Franck-Lenard curve have very similar shapes. The efficiency of resonance as a function of electron speed has been deduced. It is shown that the maximum of the efficiency curve lies 0.18 volts beyond 19.77 volts. The efficiency curve must have a maximum as the assumption of constant efficiency beyond 19.77 volts leads to Franck-Lenard curves which do not fit the electron velocity distribution curve.

It may be thought that the displacement of 0.13 volts of the curves is not known with sufficient accuracy to warrant any deduction in regard to the efficiency. However, our measurements of critical potential have an average deviation of only 0.3 percent and it will not be an easy matter to increase the accuracy of these measurements. Our potentiometer batteries were of the small lead storage cell type and their voltages were measured in terms of a standard cell before and after each run and they never changed more than a few hundredths of a volt during a run. We considered the possibility that the current flowing across the first condenser might vitiate the potentiometer principle but from the known sensitivity of our galvanometer we found that the current involved could have no effect on the voltage as calculated from the resistances of the potentiometer.

Even if the details of our efficiency curve are not accepted, certainly our experiments do support the statement: *electrons of energy somewhat larger than 19.77 volts make inelastic impact with greater efficiency than do electrons with larger velocities.*

If we accept the situation that the maximum of the efficiency curve lies about 0.2 volt beyond 19.77 volts and if we consider that we have found (previous paper) that the difference between the first and second transition in helium is 0.74 volts it would seem to follow that the second transition must have its maximum efficiency about 0.15 volts beyond 20.55 volts. It may seem surprising that critical potential measurements made so far have checked with spectroscopic data but the accuracy of the former measurements has never been sufficiently high to take into account or discover differences of the order of one-tenth of a volt. There exists no *a priori* reason to believe that the two transitions studied should have the maxima of their efficiency curves displaced the same volt amount above their respective critical potentials and we must at present simply accept the situation as an empirical fact.

It is of interest to summarize the arguments that support a relation between efficiency and electron energy of the type shown in Fig. 4. (1) The relative intensity of the ortho and para-helium spectrum as observed by Runge and Paschen[13] depends on the relation between field strength and pressure. At high pressures the ortho-helium spectrum predominates since the resonance potential for the transition to ortho-helium is less than the first resonance potential of para-helium. At low pressures the para-helium spectrum is the more intense. (2) One single correction of the volt scale brings all the critical points of a current-potential curve into agreement with

[13] Runge and Paschen, K. Preuss. Akad. Wiss. Berlin 323, 377 (1895).

spectroscopy.[14] (3) The slope of a photoelectric current potential curve is largest at the beginning of the curve and then decreases.[1] (4) The distribution curve of the current drop in a Franck experiment and of the current rise in a Lenard experiment follows the distribution curve of the impinging electrons, (our own experiments outlined in this paper).

If we compare our results with the results of Dymond[3] we see that in both sets of experiments the efficiency shows a maximum very near the resonance potential. It is evident from the foregoing considerations that the complications in the analysis are all due to the fact that electrons from the filament have a velocity distribution and in order to obtain better agreement in such experiments by different methods it will be necessary for more exact measurements to use a velocity filter in order to produce a homogeneous electron stream.

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[14] Franck and Jordan, Handbuck der Physik. Julius Springer, Berlin 1926. Page 723.