# EFFECT OF DIFFRACTION AROUND THE MICROPHONE IN SOUND MEASUREMENTS

By Stuart Ballantine

Abstract

**Proposed method of evaluating the pressure correction made necessary by diffraction.**—The diffraction of sound around the diaphragm of the microphone ordinarily used in the measurement of the instantaneous pressure in a sound wave causes the indicated pressure to vary from equality with the actual pressure in the undisturbed wave at low frequencies, to twice this pressure at high frequencies. Because of the mathematically irregular shape of the conventional microphone and its mounting the effect cannot be calculated. It is proposed to evaluate the correction for diffraction by employing a standard spherical mounting of which the diaphragm occupies a small area about the pole; the increase in pressure for this mounting can be calculated theoretically, and the correction for other mountings can then be obtained by experimental comparison.

**Theory of the diffraction of a sound wave by a rigid sphere.**—The theory of the diffraction of a plane wave of the type exp $i\omega(t-x/V)$ by a rigid sphere is outlined in terms of Hankel's $H^2{}_{n+\frac{1}{2}}$ functions, for which tables exist up to the highest orders required for the computations in practical cases. Numerical computations are carried out in full, giving the vector pressure ratio at the pole facing the source for spheres of various diameters and at various frequencies throughout the acoustic range.

IN MEASURING or recording instantaneous sound-pressure variations with a calibrated condenser microphone it is often assumed that the pressure at the diaphragm is the same as that which would exist in the undisturbed sound wave; also some investigators have assumed that the pressure is *doubled* by reflection, therefore that the apparent values are to be divided by a factor of 2. If the diaphragm were of infinite extent, or part of an infinite wall, the pressure would clearly be doubled at all frequencies since the reflection coefficient at the air-membrane interface is very closely equal to unity because of the stiffness of the tightly stretched membrane. If, on the other hand, the dimensions of the microphone were small in comparison with the wave-length of the sound, we should then have an ordinary problem of the Laplacian flow of air past an irregular obstacle and in these circumstances the pressure at the diaphragm would approach that in the undisturbed sound field. The effect of diffraction around the microphone then is to cause the apparent pressure (the pressure acting upon the diaphragm) to vary from the true pressure in the undisturbed wave, to twice this pressure as the frequency is raised from a low to a high value.

An obvious method of evaluating this effect and rendering useful the ordinary calibrations of the microphone derived from the application to the diaphragm of known alternating pressures produced by the thermophone, piston-phone or by electrostatic means, would consist in the direct comparison with the Rayleigh disk. The sound field at the point to be occupied by the microphone is first measured by means of the Rayleigh disk, the microphone is then substituted and the overall calibration directly obtained in this way will include the effect of reflection. In the previous comparisons between the thermophone calibration of a condenser microphone and the Rayleigh disk calibration certain discrepencies have been noticed which are probably largely due to diffraction around the microphone.

The purpose of the present note is to consider the theory of the effect and to suggest a method for evaluating the appropriate corrections by means of a spherical mounting which can be calculated.

1. *Standard spherical mounting.* The geometrical volume occupied by the condenser transmitter and single-stage amplifier which is usually mounted with it for the purpose of avoiding long high-capacity leads, is of a mathematically irregular shape and not amenable to calculation To facilitate mathematical investigation we may mount the transmitter and amplifier in a substantially rigid spherical shell with the diaphragm as nearly in the surface as possible (Fig. 1). The diffraction of sound by a spherical obstacle is a classical problem[1] and tables now exist which greatly facilitate the actual numerical computations.

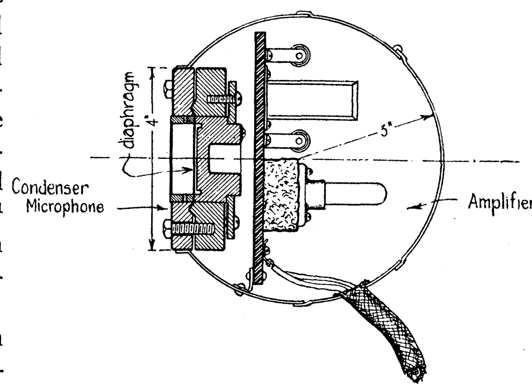

Fig. 1. Standard spherical mounting for condenser-microphone and amplifier stage.

When the relation between the pressure-ratio (actual pressure ÷ pressure in the wave in the absence of the obstacle) and frequency has been calculated for the standard spherical mounting the effect of diffraction with the more usual mountings may be readily evaluated by experimental comparison.

2. *Theory of diffraction of a plane sound wave by a rigid sphere.* With reference to a conventional spherical coordinate system with its polar axis along $x$ we may imagine a plane sound wave of the form $e^{i\omega t}$ propagated from right to left in the direction of the polar axis; we require the pressure $p$ at the pole facing the source or more accurately over a small circular area (representing the diaphragm) surrounding this pole. Since the problem has been discussed by the late Lord Rayleigh it will be sufficient here to indicate the form the solution takes when expressed in terms of Bessel functions whose order is half an odd integer, tables of which have become available since Rayleigh's treatment.

On account of the symmetry about the polar axis the wave-equation for the velocity-potential $\phi$ reduces to

[1] Rayleigh, "Theory of Sound," Vol. 2, p. 218 *et seq.* (London 1878): Papers, No. 287, vol. V, p. 112, 1903; No. 292, vol. V, 149, 1904.

Lamb, "Hydrodynamics," 5th Ed., p. 496 (Cambridge, 1924).

$$\frac{\partial^2\phi}{\partial r^2}+\frac{2}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right)+k^2\phi=0, \tag{1}$$

where $k=\omega/V=2\pi\times$*frequency/velocity of sound in air*. The solutions of sufficient generality for our purpose are

$$\phi=\sum_{n=0}^{\infty}A_nP_n(\cos\theta)/r^{1/2}\begin{cases}J_{n+1/2}(kr)\\J_{-n-1/2}(kr)\end{cases}; \tag{2}$$

where $P_n$ ($\cos\theta$) represents the Legendre polynomial of order $n$, $\theta$ being the polar angle (colatitude). We require the expression of exp ($i\omega x/V$) in the form (2); this is given by Rayleigh's expansion:[2]

$$\phi_1=e^{ikx}=\sum_{n=0}^{\infty}i^n(2n+1)P_n(\cos\theta)(\pi/2kr)^{1/2}J_{n+1/2}(kr). \tag{3}$$

The scattering by the sphere can be most simply represented as a diverging spherical wave in terms of the second Hankelian Bessel functions of order $n+{}^1/_2$ thus

$$\phi_2=\sum_{n=0}^{\infty}A_nP_n(\cos\theta)(\pi/2kr)^{1/2}H^2{}_{n+1/2}(kr). \tag{4}$$

The complete solution is then: $\phi=\phi_1+\phi_2=$Eq. (3)+Eq. (4). From the boundary condition at the surface of the sphere that the normal component of velocity ($\mathbf{v}=-\text{grad }\phi$) shall vanish when $r=a$ or $\partial\phi/\partial r(r=a)=0$ we find

$$A_n=-i(2n+1)\frac{J_{n+1/2}(ka)-2kaJ'_{n+1/2}(ka)}{H^2{}_{n+1/2}(ka)-2kaH^{2\prime}{}_{n+1/2}(ka)}. \tag{5}$$

By taking advantage of the Wronskian relation

$$W(J_{n+1/2},J'_{n+1/2})=-(-)^n2/\pi x, \tag{6}$$

the solution represented by (3)+(4) reduces to the simple equivalent forms

$$\phi=\left(\frac{2}{\pi ka}\right)^{1/2}\sum_{n=0}^{\infty}\frac{-(2n+1)i^nP_n(\cos\theta)}{(-)^n(nJ_{-n-1/2}+kaJ_{-n-3/2})-i(nJ_{n+1/2}-kaJ_{n+3/2})}, \tag{7a}$$

$$=\left(\frac{2}{\pi ka}\right)^{1/2}\sum_{n=0}^{\infty}\frac{(2n+1)i^nP_n(\cos\theta)}{(-)^n[(n+1)J_{-n-1/2}+kaJ_{-n+1/2}]-i[(n+1)J_{n+1/2}-kaJ_{n-1/2}]}. \tag{7b}$$

The effect of diffraction may be most conveniently represented by the vector ratio of the pressure as calculated from (7) to that which would exist in the incident plane wave alone. Now the pressure $p=\rho\partial\phi/\partial t=i\omega\rho\phi$, where $\rho$ is the air density, so that the ratio in question $\rho/\rho_0=$eq. (7) $\div e^{ika}$.

[2] Rayleigh, Proc. London Math. Soc. **4,** 253 (1873); H. M. MacDonald, "Electrical and Optical Wave Motion," p. 47 (Cambridge, 1900).

Since the diaphragm covers a finite (circular) area about the pole the variation of the pressure-ratio with $\theta$ should strictly be taken into consideration. Calculation indicates that for large values of $ka$ the variation of the vector ratio is chiefly one of angle, the absolute value remaining substantially constant over a range of 15 degrees; for small $ka$ the variation is chiefly in the amplitude. At $ka=0.3$ the variation of $p/p_0$ from $\theta=0$ to $\theta=15°$ is only about 1 percent. Moreover the pressure variation with $\theta$ must be "weighted," as the effectiveness of a pressure upon any annular element of the membrane falls off rapidly as the radius of the annulus approaches that of the membrane. It seems therefore that no serious error would be committed with a mounting of perhaps 6 in. diameter and a diaphragm of 1.5 in. diameter by regarding the polar pressure ($\theta=0$) as proportionate to the effective average pressure over the diaphragm at all frequencies.

3. *Computations and curves.* The pressure-ratio is a function only of the ratio of the size of the sphere to the wave-length of the sound. For computations from (7) tables are available[3] of $J_{n+1/2}$ for $n$ up to 18, and of $J_{-n-1/2}$ for $n$ up to 6; additional values of $J_{-n-1/2}$ for larger $n$ may be computed from the tables of the related $C_n$ functions given in the *British Association Reports* for 1914 (pp. 88–102) and 1916 (pp. 97–107). Computations made with the aid of these modern tables are given in Table I.

TABLE I. *Values of the vector ratio ($|p/p_0|e^{i\psi}$) of pressure at pole of rigid sphere to pressure in incident plane wave at same point for various values of $ka=2\pi\times$frequency$\times$radius of sphere $\div$velocity of sound in air.*

| $ka$ | $p$ Equation (7) | $\left\lvert \frac{p}{p_0} \right\rvert$ | Angle $\psi$ (radians) |
|---|---|---|---|
| 0.1 | | 1.0005 | |
| 0.2 | | 1.0027 | |
| 0.3 | 0.909+0.458i | 1.019 | 0.168 |
| 0.5 | 1.043+0.299i | *1.085 | 0.279 |
| 0.7 | 0.597+1.040i | 1.198 | 0.345 |
| 0.85 | 0.468+0.374i | 1.325 | 0.359 |
| 1.0 | 0.310+1.370i | 1.406 | 0.340 |
| 2.0 | 1.593+0.468i | *1.660 | 0.286 |
| 3.0 | −1.762−0.165i | 1.772 | 0.235 |
| 4.0 | −0.910−1.595i | 1.835 | 0.194 |
| 6.0 | 1.895−0.292i | 1.913 | 0.128 |
| 10.0 | 1.540+1.207i | *1.958 | 0.665 (?) |

* Values calculated by the late Lord Rayleigh.[2]

The angle $\psi$ of the vector ratio is given as well as the absolute value. This is of interest in estimating the dispersion which must be corrected by means of phase-equalizers in the electric circuits when an exact recording of the wave forms of the sound is desired. For many purposes, however, the absolute value alone will be sufficient.

Fig. 2 contains a set of curves based upon these computations which represent the effect of diffraction for spherical mountings of various sizes over the audio range of frequencies. The form of the correction curves for other

[3] G. N. Watson, "Bessel Functions," pp. 740–743 (Cambridge, 1922).

practical mountings may now be obtained experimentally by comparison with the standard mounting This experimental program is under way and will be reported in a subsequent paper. It is necessary to investigate the functional relation for a single size of each shape; the position of the curve for any other size may be determined by the principle of similitude.

In the case where the condenser transmitter is mounted integrally with the apparatus comprising the first amplifier stage and occupies a volume of reasonable (say cubical) shape it is often sufficient to estimate an "equivalent sphere" on the basis of equal volume to represent the irregular actual volume. The accuracy of this seemingly crude assumption is somewhat surprising.

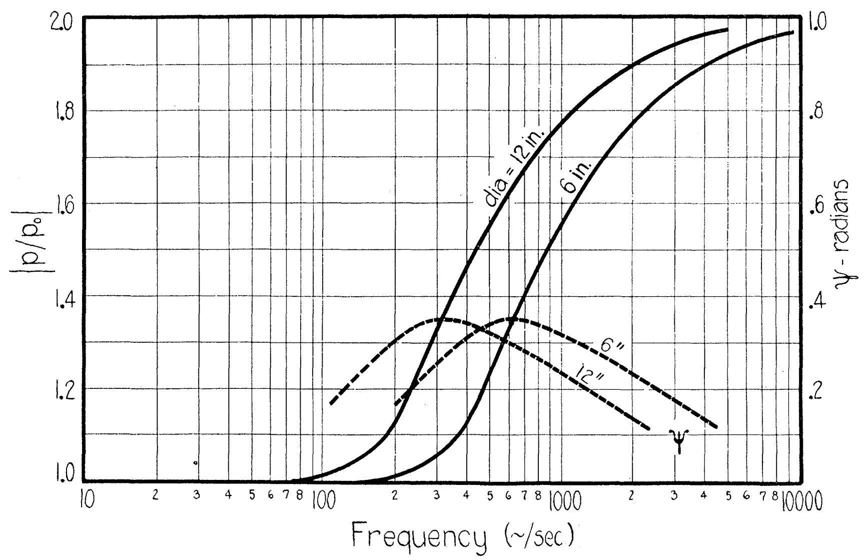


Fig. 2. Effect of diffraction of plane wave with spherical mountings: absolute value $|p/p_0|$ and angle $\psi$ of ratio of pressure at pole of sphere to pressure in undisturbed wave.

Although the condenser microphone has been mentioned particularly in this discussion, it is obvious that it applies equally well to other pick-up instruments of the exposed diaphragm type, such as for example, the double carbon button microphone used extensively in radio broadcasting and public address systems. When the microphone is used for technical purposes and sound-wave recording it is convenient to compensate the diffraction effect either in the design of the microphone (which is feasible with air-damped types) or by equalization in the electrical circuits.

In conclusion it may be noted that by the reciprocity theorem equation (7) is equally appropriate for the representation of the pressure at a large distance in the sound radiated by a small piston located in the surface of a sphere. This throws some light on the action of baffles for loud-speakers.

Radio Frequency Laboratories, Inc.,
Boonton, N. J.
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