THE DIELECTRIC CONSTANT OF AIR AT RADIO FREQUENCIES

By A. B. Bryan and I. C. Sanders

Abstract

A radio frequency method has been used to measure the dielectric constant K of dry air free from CO_2, the value obtained being 1.0005893 for standard conditions of temperature and pressure. The probable error in $K-1$ due to accidental variations is 0.34 percent. The method is a modification of the usual heterodyne beat arrangment. Capacity changes produced when the pressure in the test condenser is changed are compensated by a suitable condenser in parallel. The beat note frequency is compared with that of a fork by means of Lissajous' figures. The test condenser is made of invar to avoid temperature effects. Short connecting wires are used to minimize lead-inductance effects. A few preliminary measurements made with a large direct current voltage superimposed on the high frequency voltage across the condenser indicate no change in the dielectric constant of air, hydrogen, or ammonia. There is some indication that a discharge through the gas decreases its dielectric constant but the effect is probably spurious.

Introduction

THE various radio frequency methods for measuring the dielectric constant of a gas are best suited for relative determinations, in which the dielectric constant is obtained in terms of that of some standard gas, usually air. For this reason it is obviously desirable that the dielectric constant K of air be accurately known. The older experiments of Boltzmann[1] and of Klemencic[2] have established the value of K with fair accuracy: their values of 1.000590 and 1.000586 are in good agreement. But there has arisen a possibility that these values, obtained with the older methods, may not be true at radio frequencies because the two radio frequency values 1.000540 and 1.000572, obtained by Fritts[3] and Zahn[4] respectively, are considerably lower. Carman and Hubbard,[5] using a method of the older type, have recently obtained a value 1.000594 which may be considered in agreement with that of Boltzmann. In the present work a new radio frequency measurement is made in an effort to settle the discrepancy between the two types of experiment. There is no theoretical reason why the value of K should be abnormally low at radio frequencies and it has been more generally believed that the radio frequency measurements were inaccurate because of the calibration difficulties involved. This belief is confirmed by the result of the present experiment which is in agreement with the original value of Boltzmann.

[1] L. Boltzmann, Wien. Berichte **69**, Part 2, 795 (1874).
[2] J. Klemencic, Wien. Berichte **91**, 712 (1885).
[3] E. C. Fritts, Phys. Rev. **23**, 345 (1924).
[4] C. T. Zahn, Phys. Rev. **24**, 400 (1924).
[5] A. P. Carman and K. H. Hubbard, Phys. Rev. **29**, 299 (1927).

Apparatus

The experimental arrangement as shown in Fig. 1 is in many ways similar to those previously used. Capacity changes produced when the air pressure in the test condenser C is changed are compensated by a suitable condenser C_c in parallel, so as to keep constant the total capacity and the frequency of the radio frequency oscillator A to which the condensers are connected.

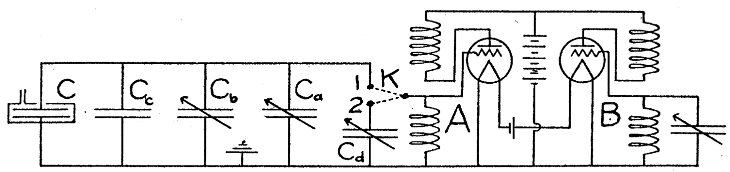

Fig. 1. Arrangement of circuits.

Constancy of frequency is indicated by a constant beat note between A and a second oscillator B. The oscillators use UX201A tubes and have a common plate battery of two 45 volt units and a common 8 volt lead storage filament battery.

The beat note between the two oscillators is obtained by means of a separate arrangement consisting of a small loop antenna, a detector and two stages of amplification connected to a loud speaker unit. This part of the apparatus is at some distance from the oscillators and no reaction on them has been detected. The frequency of the beat note is compared with that of a 300 cycle electrically driven fork by means of Lissajous' figures obtained when a beam of light is reflected by a small mirror attached to the vibrating loud speaker element and then by a second mirror attached to the fork.

A key K is used so that either condenser C_d alone or the system of four condensers C, C_a, C_b, C_c in parallel may be connected to oscillator A. C_a and C_b are similar General Radio Type 222 Precision condensers, each having a maximum capacity of about 1500 mmf. The condenser scale has 25 divisions and a slow motion screw is provided so that readings may be made directly to one hundredth of a division. These condensers are well designed electrically and mechanically. The ebonite insulation is so arranged as to minimize dielectric losses and no flexible leads are used. C_d is a good variable air condenser of another type.

C_c is a specially constructed cylindrical condenser. A length of heavy tube forms the insulated plate and a half-inch rod, running along the axis of the tube, is connected to earth. A long sleeve, fitting the rod closely, can be moved into the tube by a micrometer screw, thus increasing the capacity. A motion of one cm produces a calculated capacity change of 0.254 mmf. The condenser is shielded by an outside cylindrical case, ebonite insulation being used where necessary. All metal parts are accurately made of brass. Readings are taken on the central portion of the scale to avoid end effects.

The test condenser C is made as shown in Fig. 2. The figure is to scale except that for the sake of clearness a condenser with only five plates is shown. There are actually thirteen circular plates, the seven earthed plates being 9.4 cm in diameter and the six insulated plates 4 mm less. The plates are 1 mm thick and the spacing about 1 mm. The capacity is approximately 753 mmf. Each of the two sets of plates is held together by three equally spaced bolts around the edge with metal separators between the plates. Two of these six bolts are shown in the figure. The insulated set of plates is supported by three short sections of quartz tubing, one of which is shown at *Q*. The sides of the condenser are closed by a short length of large thin tubing which fits the earthed plates closely as shown. A half-inch hole in the center of each of the plates except the bottom one allows the air to pass freely into all parts of the condenser. The condenser is made entirely of invar. The outer case is of brass, all joints being made tight with sealing wax. The case is connected to earth through a wire soldered to its side. The small mercury cup connection at *M* prevents any transmission of force to the condenser due to a slight yielding of the top of the case when the air pressure is changed. The screw connector at *A* is joined by a short length of

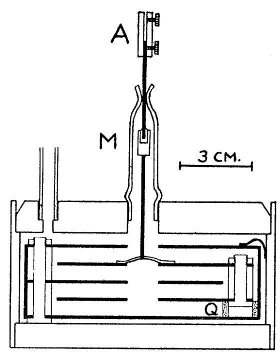

Fig. 2. Design of the test condenser.

wire to a similar connector just above and thence to the other condensers. The test condenser is disconnected when desired by moving this short length a small distance up into the upper connector.

All the condensers are of course shielded by their cases. No other shielding is needed. The several condensers and the key *K* were controlled from a distance by a suitable arrangement of threads.

The air used is passed through a solution of KOH to remove CO_2 and then through a tube of $CaCl_2$ and one of P_2O_5 to remove water and is then stored in a reservoir containing P_2O_5. From this reservoir it passes through a pump into the condenser. This pump consists of a large bulb into which mercury can be forced from the bottom. At the top is a two-way stop cock providing connection to either the condenser or the reservoir. The air can thus be pumped either way and the pressure in the condenser can also be quickly changed back and forth between two definite values by filling or emptying the bulb. Pressures were measured with an open mercury manometer read with a cathetometer. The motion of the mercury in the pump or manometer produces no effect on the oscillators. Temperatures are measured to one twentieth of a degree with a thermometer in contact with the test condenser case.

Experimental Procedure

C_a and C_b are set at any desired value and then, with the key in position (1), the B oscillator is adjusted until a 600 cycle beat note and consequently a 2:1 Lissajous' figure is obtained. The air pressure in the condenser is then changed from its initial value P_1 to a second value P_2 and C_c is changed by an amount δC_1 so that the total capacity is the same as before and the 2:1 figure is restored. Then assuming that $K-1$ is proportional to the density of the gas, δC may be calculated from the relation $\delta C = \delta C_1 (T/273)76/(P_1 - P_2)$ where δC is the value which would be observed for a pressure change of one atmosphere at a temperature of 0°C and T is the absolute temperature at which the measurement is made. $K-1$ is found by dividing this δC by C, where C is that part of the capacity of the test condenser which changes when the air pressure is changed. The accurate determination of C in the same units as δC is a matter of some difficulty and will be considered under "calibration."

There is always a tendency for the beat note to change slowly because of changes in the oscillator batteries or in the temperature and a change of this sort while the gas pressure is being changed can lead to a serious error in δC. This slow drift of the beat-frequency is reduced by the use of common plate and filament batteries and by working in a basement room where the temperature is quite steady. Error due to this effect is further reduced by constant use of the key K and condenser C_d in the following way. C_d is originally set so that its capacity is equal to the sum of the capacities of the other four condensers and the 2:1 figure is obtained when the key is in either position (1) or (2). Later the key is frequently pulled over to position (2) and the 2:1 figure restored if necessary by a slight adjustment of oscillator B. The method may thus be considered as one in which the sum of the capacities of C, C_a, C_b and C_c is kept equal to the capacity of C_d, the oscillators being used merely to indicate when this equality exists. It is realized that this is not a strictly accurate view since switching the key over probably introduces slight changes in the inductance and resistance of the circuit. But even if this is so the accuracy of the result is not seriously affected.

Calibration

As previously explained, the capacity C of the test condenser must be measured and the result expressed in scale divisions of the cylindrical condenser C_c. To do this a step-by-step calibration curve of C_b is first obtained, the step used being the interval 22.0 to 23.0 on C_a. C_b is set at 25.0 and C_a at 22.0. Then C_a is increased to 23.0 and C_b decreased until the total capacity is the same as before. This process is repeated, a continuous series of scale readings on C_b being obtained which represent equal capacity intervals. A calibration curve may then be plotted for C_b with the capacities in these arbitrary units as ordinates and the scale readings as abscissae. One of these arbitrary units is about 59.5 mmf.

The next move is to determine the capacity of the test condenser C by comparison with C_b. C is disconnected and C_b increased to restore the frequency to its original value, the capacity of C being equal to the increase in C_b. A series of readings taken in this way is shown in Table I. The scale

TABLE I. *Data for determining capacity of test condenser.*

C_a (scale rdg.)	C_b (scale rdg.)	λ (meters)	C_b' (scale rdg.)	$C_b'-C_b$ (arb. units)
0	6.960	336	19.354	12.734
0	7.986	342	20.301	12.725
0	8.000	342	20.318	12.729
0	9.002	350	21.255	12.723
12.0	6.996	415	19.399	12.746
12.0	7.970	421	20.300	12.741
12.0	8.000	421	20.332	12.744
12.0	8.993	428	21.261	12.739
25.0	7.020	495	19.429	12.753
25.0	7.966	498	20.300	12.745
25.0	7.997	500	20.337	12.752
25.0	9.000	502	21.278	12.751
				mean 12.740

reading of C_a, the initial reading of C_b and the corresponding wave-length are shown in the first three columns. The fourth column gives the second reading of C_b after the test condenser is disconnected. The last column gives the increase in capacity of C_b, expressed in the above mentioned arbitrary units and obtained from the two readings of C_b and its previously obtained calibration curve.

The total capacity of the test condenser is thus 12.740 arbitrary units. A small part of this total capacity does not have the air in the condenser as its dielectric. The capacity through the three quartz ring insulators may be calculated and is 4.5 mmf. The external capacity to the screw connector A (see Fig. 2) and the wire below it is estimated at 1.2 mmf. When this correction of 5.7 mmf or 0.096 arbitrary units is subtracted the corrected value $C=12.644$ arbitrary units is obtained. The error made in estimating this correction is probably not greater than one mmf or 0.13 percent of C

Finally this value of C must be expressed in scale divisions of the cylindrical condenser C_c, that is, the number N of C_c divisions in one arbitrary unit of capacity must be found. This is done by determining the capacity change necessary to produce a certain frequency change when either C_c or C_b is varied by a small amount and then equating the capacity changes in the two cases. The results are shown in Table II. A particular trial, the first in the table, will be described in detail. C_a and C_b are set at 25.0 and 19.6 respectively. It is then found that when C_c is changed from 5.998 to 9.982 and then to 11.312 the corresponding beat notes are 600, 525 and 500 as indicated by the 2:1, 7:4 and 5:3 Lissajous' figures in the three cases. The three condenser settings are repeated several times and the values given are means. Since the capacity and frequency changes are proportional for small changes the above data give $\delta C_c = 5.316$ scale divisions on C_c, where δC_c is the change in C_c for a 100 cycle frequency change, and show incidentally that the scale of C_c is linear within the limits of experimental error. C_c is then set at about the mid-point of the range over which it has been varied and oscillator B is adjusted until a zero beat note is obtained. C_b is then changed, a setting on either side of the original 19.6 being found where the beat note is 2100 cycles, as indicated by a 7:1 figure. The interval between these two settings, corresponding to a 4200 cycle frequency change, is 0.9068 divisions on the scale of C_b, as tabulated under δC_b. In this case the frequency change is so large that it is not quite permissible to assume that the frequency and capacity changes are proportional and it may be shown that the quantity 0.9068 must be divided by a correction factor $(1+3\delta n/2n)$, where $\delta n = 2100$ and n is the frequency at which the trial is made or 533000 in this case. S, in the seventh column, is the slope of the previously obtained calibration curve in condenser divisions per arbitrary unit. The curve has been plotted in a special manner to make possible an accurate determination of S at any point. At the point 19.6 the value of S is 0.948 and δC_b must be divided by this value to convert it to arbitrary units. Finally, it will be seen that N in the last column is given by the relation $N = 42S\delta C_c(1+3\delta n/2n)(1/\delta C_b)$.

When the previously obtained corrected value of C in arbitrary units is multiplied by the mean value of N from Table II it is found as the final result of the calibration that $C = 2957.5$ divisions on the scale of the cylindrical condenser C_c.

TABLE II. *Data for determining the capacity of the test condenser in terms of C_c.*

C_a	C_b	δC_c	δC_b	n	$(1+3\delta n/2n)$	S	N
25.0	19.6	5.316	.9068	533000	1.006	.948	234.8
25.0	22.0	5.628	.9639	522000	1.006	.948	233.9
25.0	18.0	5.008	.8687	540000	1.006	.988	234.6
0	7.5	1.359	.2496	862000	1.004	1.021	234.4
0	8.5	1.443	.2628	845000	1.004	1.011	234.1
*0	20.6	1.342	.2279	862000	1.004	.948	235.4
*0	21.6	1.431	.2441	845000	1.004	.948	234.3
12.0	7.5	2.337	.4321	704000	1.005	1.021	233.0
*12.0	20.6	2.321	.3987	704000	1.005	.948	232.8
12.0	8.0	2.392	.4387	700000	1.005	1.015	233.5
*12.0	21.0	2.368	.4057	700000	1.005	.948	233.4
12.0	7.0	2.283	.4223	714000	1.004	1.023	233.3
12.0	7.5	2.341	.4309	704000	1.005	1.021	234.0
12.0	8.0	2.392	.4380	700000	1.005	1.015	233.9
12.0	8.5	2.441	.4468	698000	1.005	1.011	233.0
							mean 233.9

* Inidicates test condenser disconnected.

It may appear that a more direct determination of N could be made by merely increasing C_b and decreasing C_c so as to keep the frequency constant. But this involves measuring a very small motion of C_b because the range of C_c is very small and trials have shown that the required accuracy cannot be obtained because of some very slight looseness of the condenser bearings.

A direct comparison of this sort would require a condenser for which the capacity at a given setting is reproducible with almost impossible accuracy.

A mirror mounted directly on the moving plates and a telescope and scale have been used to test the driving mechanism of C_b. It has been found that the angular rotation for a complete division is accurately the same at all points on the scale but that there is a slight irregularity in the readings between divisions due to some slight defect in the slow motion screw. And so in the above work, where extreme accuracy is required, the values of δC_b are measured with the teslescope and scale.

Results

In Table III are shown the results of eight consecutive trials for air made at various times during one week. δC_1 is the observed change in C_c for a pressure change (P_1-P_2) and an absolute temperature T and δC the change for a pressure change of one atmosphere at 0°C, calculated from δC_1 as previously explained. The values are in scale divisions of condenser C_c. Each trial is obtained by changing the pressure from P_1 to P_2 and back again to P_1 several times and observing the corresponding readings of C_c so that

Table III. *Change in capacity of test condenser when pressure of air is changed.*

T	P_1-P_2	δC_1	δC
297.9	57.40	1.205	1.741
297.9	57.40	1.200	1.734
292.7	57.35	1.228	1.745
292.7	57.29	1.235	1.757
292.7	56.51	1.206	1.739
291.0	57.50	1.240	1.747
291.4	57.58	1.235	1.740
291.8	57.67	1.237	1.742
			mean 1.743

each trial is really the mean of several. For the third, fifth and seventh trials C_a is set at 12.0 on its scale and the wave-length is 421 meters. For the others C_a is set at zero and the wave-length is 342 meters. C_b is set at 8.0 for all trials. As a matter of convenience the two pressures used are approximately the same in all cases, P_1 and P_2 being about 76 cm and 18 cm respectively. Preliminary results have shown that the value of δC is not dependent on the particular pressures used.

When the mean value of δC from Table III is divided by the value $C = 2957.5$ C_c divisions previously found, the value $K-1 = 0.0005893$ for dry air free from CO_2 at a pressure of one atmosphere and a temperature of 0°C is obtained as the final result.

Sources of Error and Experimental Difficulties

The value of $K-1$ has been calculated from the average values of the three quantities tabulated in the last columns of the three tables and the probable error due to the accidental variations in these quantities is 0.34 percent. There is a possible constant error due to the somewhat uncertain correction which has been used to allow for the test condenser capacity which does not have the gas as its dielectric. This error has been minimized by making the total capacity of the test condenser large and has already been estimated as not greater than 0.13 percent.

Two of the experimental difficulties encountered may be mentioned. The first was that in some cases the initial rise in capacity when air was admitted was followed by a slight continued increase so that the capacity did not reach a steady value until several minutes after the pressure had become constant. Simultaneous measurements with a thermocouple showed a similar rise of the temperature of the condenser plates when air expanded into the condenser and this may have been the cause of the disturbing effect. However, the temperature rise was apparently too small to be entirely responsible and some yielding of the condenser was probably also involved. At any rate the trouble was overcome when, after several other designs had been found defective, the invar condenser which has been described was made.

The second difficulty is due to the inductance of the wires connecting the condensers. When the condenser is mounted in a liquid bath to permit temperature variations, as was done at one time in the present work, the leads to it must have considerable length and inductance. Measurements made under these conditions gave values of $K-1$ which showed a definite increase with frequency in spite of the fact that an apparently suitable correction for the lead inductance was applied. It is believed that corrections of this sort, made with the usual simplifying assumptions of localized inductances and capacities and negligible resistances, are somewhat doubtful unless subject to direct experimental test. Of course it should be possible to overcome the difficultly if all possible inductances, capacities and resistances were taken into account but apparently a simpler and more direct result is obtained by using short leads of negligible inductance, and this has been done. The fact that the results which have been given show no dependence on the frequency indicates that the lead inductances are negligible.

This question of lead inductance has also been considered by Watson[6] in a paper which has recently come to the writers' attention. Watson uses a radio frequency method to measure the dielectric constants of several gases, including air, and devotes speacial attention to obtaining accurate absolute values. He applies corrections for lead inductance which are apparently satisfactory in that his results do not in general show any dependence on the frequency, although he does report a very slight increase with frequency in the value of K for CO_2. Watson's method of correcting for lead inductance is essentially the same as that which has been found inadequate in the present work and the fact that he finds it satisfactory may possibly be due to his use of a test condenser of rather small capacity (133.3 mmf). It is obvious that at a given frequency the impedence due to the inductance in the connecting wires to a condenser is a smaller part of

[6] H. E. Watson, Proc. Roy. Soc. **117A,** 43 (1927).

the total impedence when the capacity is small and any error in estimating this impedence is less serious. For this reason the use of a small test condenser is an advantage, although it is a decided disadvantage for other reasons. Watson's work has been carefully done and the fact that his value 1.000601 for air is apparently a little too large, as compared with previous results, serves to emphasize the difficulties of this sort of experiment.

Watson also points out that a change in the amplitude of the radio frequency current may change the capacity of a condenser because of the changing electrostatic attraction between the plates. The method used and the results obtained in the present work indicate that this effect is not a source of error. The experiments described below also show that the effect of electrostatic forces is so small that no appreciable effect would be produced by such amplitude changes as might occur.

Effect of a Superimposed Direct Current Voltage

A 0.3 mf condenser was put in the lead to the test condenser and the section of the circuit between these two condensers was connected through a 17 megohm resistance and a micro-ammeter to the positive side of a high voltage direct current generator, the negative side of which was grounded. With this arrangement a direct current voltage could be superimposed on the high frequency voltage across the test condenser. The results of a few preliminary measurements may be stated briefly. When 900 volts measured at the generator, is applied the capacity of the evacuated test condenser increases by about 0.04 mmf. The effect is the same when the condenser contains air at a pressure large enough to prevent a discharge. When the pressure is reduced to 60 cm or less a discharge through the gas begins and under these conditions the capacity increase is definitely less than the value given above. Similar results are obtained with hydrogen and ammonia.

These results indicate that 900 volts or approximately 18,000 volts per cm produces a very small increase in the capacity of the condenser and no change in the dielectric constants of the gases used. The smaller effect observed when a discharge is passing through the gas is apparently not due to the slightly smaller voltage across the condenser in this case. It may be a genuine decrease in the dielectric constant produced by the discharge but is more probably a temperature effect. However, Seth[7] has recently reported an apparent decrease in the refractive index of air due to a discharge and a more careful investigation of the above effect might be worth while.

Department of Physics,
The Rice Institute,
April 28, 1928.

[7] J. B. Seth, Nature **120**, 880 (1927).