THE DIAMAGNETIC LAYER OF THE EARTH'S ATMOSPHERE AND ITS RELATION TO THE DIURNAL VARIATION OF TERRESTRIAL MAGNETISM*

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ABSTRACT

Anisotropic conductivity of upper atmosphere due to presence of earth's magnetic field.—An investigation of the motion of ions and electrons in the region of long free paths shows that the electrical conductivity in the direction of the earth's magnetic field is that predicted by simple theory. The conductivity in a direction at right angles to the magnetic field and in the direction of the applied electric field is zero while the pseudo conductivity at right angles to both is finite and the mechanism such that ions of both kinds are swept in the same direction. As a consequence of this situation it appears that large circulating currents in the upper atmosphere of the type assumed in present theories of the diurnal terrestrial magnetic variations are hardly possible.

Theory of the diurnal magnetic variation based on diamagnetic effect of ions in upper atmosphere.—A theory of the diurnal magnetic variation is worked out which explains quantitatively the major phenomena in terms of the diamagnetic effect produced by ions spiralling about the earth's magnetic field. The diamagnetic intensity of magnetization of the upper atmosphere in the region of long free paths is found to depend upon the total number of ions of all kinds per cubic centimeter; their mean kinetic energy and the impressed magnetic field. The maximum calculated diurnal magnetic variation as a function of latitude was found to agree in form with observation. The average maximum number of ions of all kinds per cm^3 in the upper atmosphere was then computed from observed magnetic data and found to be approximately 5×10^{10}, a number not inconsistent with the ionic density inferred from data derived from radio phenomena.

IT HAS been long recognized that the solar and lunar components of the diurnal magnetic variation of the earth are intimately related with the radiant energy of the sun. The maximum magnetic effects usually occur near mid-day local time and are more pronounced during the summer months. Radio and other data have led to the assumption of a highly ionized conducting layer at some distance above the earth's surface which, it has been assumed, is due to ionization by the ultra-violet light of the sun, high speed corpuscles, etc.

A theory of the diurnal magnetic variation of the earth's field originally proposed by Balfour Stewart and examined in great detail by Schuster[1] and Chapman[2,3] depends on the circulation of electric currents in a non-uniform conducting layer situated somewhere in the upper atmosphere. The electric currents are assumed to be due to electromotive forces induced in the con-

* Released for publication by Navy Department.

[1] Schuster, Phil. Trans. Roy. Soc. **A208,** 163 (1908).

[2] Chapman, The Solar and Lunar Diurnal Variation of Terrestrial Magnetism, Phil. Trans. Royal Soc. **A218,** 1 (1919).

[3] Chapman, Ionization in the upper atmosphere, Roy. Meterol. Soc. **52,** 225 (1926).

ducting layer by the action of tidal and thermo-tidal motions of the atmosphere. Chapman[3] has made a calculation of the conductivity necessary to account for the observed effect assuming that the current flows in the direction of the induced electric field. This view is undoubtedly correct at low altitudes where the free path of the ion is short but is open to serious objection in regions where the free path is long. Radio data seem to indicate that the ion and electron banks are located in the region of long free paths. Moreover, Hulburt's[4] calculations on the upper atmosphere show that the number of electrons or ions available in the region of short free paths is entirely inadequate to explain the currents necessary to account for the observed magnetic variations.

A critical examination of the properties of the upper ionized or Kennelly-Heaviside layer shows that it differs from a simple conducting layer in two very important respects. First, the electrical conductivity is anisotropic. Second, the layer is strongly diamagnetic. The presence of these two effects modifies profoundly present theories of the diurnal magnetic variations of the earth's field.

A consideration of the conditions existing in the upper atmosphere shows that on a winter day at altitudes as low as 140 km any electron in thermal equilibrium with the layer will spiral around the earth's magnetic field some 3000 times before colliding with a molecule while even the heaviest ion will make two revolutions before colliding. The direction of rotation of the ions or electrons is always such as to reduce the impressed field, irrespective of the sign of the charge on the ion. If there are a great number of these ions their demagnetizing effect may be considerable.

In the presence of an electric field at right angles to the magnetic field there is superimposed on this spiral motion a quasi-periodic motion by which the ion advances along a path at right angles to both the electric and magnetic fields but has no net motion in the direction of the electric field.

The conductivity of the ionized layer may be computed by considering the equations of motion of an ion under the influence of an electric and a magnetic field. In a system of rectangular coordinates let the direction of the magnetic field H be along the Z axis and let an electric field be impressed in such a manner that it has components E_x, 0, E_z, then if m is the mass of the ion, e its charge, the equations of motion are:

$$md^2x/dt^2 = E_x e - Hedy/dt \tag{1}$$

$$md^2y/dt^2 = Hedx/dt \tag{2}$$

$$md^2z/dt^2 = E_z e \tag{3}$$

The velocities of the ion are then determined by assuming the initial velocity components in the x, y and z directions to be U_0, V_0 and W_0, respectively. The velocity components of the ions on integration are as follows:

$$V_z = (E_z e/m)t + W_0 \tag{4}$$

$$V_y = E_x/H - U_0 \sin \omega t + V_0 \cos \omega t - (E_x/H) \cos \omega t. \tag{5}$$

$$V_x = U_0 \cos \omega t + V_0 \sin \omega t + (E_x/H) \sin \omega t \tag{6}$$

[4] E. O. Hulburt, Phys. Rev. **31**, 1018 (1928).

where

$$\omega = -eH/m \tag{7}$$

For infinitely long free paths the average velocities become:

$$\overline{V}_z = \infty \; ; \; \overline{V}_y = E_x/H \; ; \; \overline{V}_x = 0 \tag{8}$$

Thus remembering that the conductivity on the simplest basis is given by:

$$\sigma = Ne\overline{V}/E \tag{9}$$

where N is the number of ions, e their charge, $\overline{V}$ their average migration velocity and E the impressed electric field. The expressions for the conductivity become approximately the following, remembering that the free path is not strictly infinite:

$$\sigma_z \doteq Ne\lambda/2mu \; ; \; \sigma_y \doteq Ne/H \; ; \; \sigma_x \doteq 0 \tag{10}$$

A restricted definition of σ_y must be made since it is the conductivity in a direction at right angles to the impressed electric field.

The most important consequence of the above considerations is that the conductivity in the direction of the impressed electric field which itself is perpendicular to the magnetic field, approaches zero. Thus, large circulating currents of the type originally suggested by Stewart seem to be very impropable except perhaps in regions of short free path where the ionic densities are small. Preliminary examination seems to indicate that such an anisotropic conductivity may assist in explaining the diurnal variation of the earth's electric field.

The spiralling of the electrons and ions about the earth's magnetic field due to their thermal agitation or mass motion of the associated neutral molecules offers an explanation of the diurnal magnetic variations which appears to fit the observed facts very well.

It will be assumed in this paper that the regions in the upper atmosphere are highly ionized by ultra-violet light or by some other agent. In particular, we shall confine our attention to those ionized regions where the free path of the ion is greater than the diameter of the circle described by the ion moving freely in the earth's magnetic field. We shall assume this layer to be ionized but nearly neutral and shall investigate the field produced on the surface of the earth by the presence of such ions which are maintained in motion by thermal agitation or by momentum communicated to them by mass motion of the associated neutral molecules. To explain certain minor phenomena we shall assume the presence of certain small radial electric fields which may be modified directly or indirectly by some outside influence, presumably by corpuscles shot off from the sun.

Schroedinger[5] has examined the theory of diamagnetism due to free electrons in metals which tend to spiral around the impressed magnetic

[5] Schroedinger, Wien Ber. **66,** 1305 (1912). Also Bulletin No. 18 of National Research Council.

field, and it can be readily shown that his result for long free paths agrees (after considerable reduction) with the simple derivation below except for a numerical factor due to the different method of averaging. Bohr (thesis) and Van Leeuwen[6] have examined Schroedinger's work and concluded that if reflections of the electrons at the boundary of the metal were taken into account then the net diamagnetic effect outside the conductor would be zero. These objections do not hold at the boundary of the diamagnetic region situated at high altitudes, since the free paths are very long and the transition layer is thousands of times deeper than the circular path described by even the heaviest ion. A calculation of this diamagnetic effect due to ions having a long free path may be made as follows: The magnetic moment M due to one ion describing a circular orbit of area S in time τ is:

$$M = eS/\tau \tag{11}$$

where e is in e.m.u. Moreover, since the centrifugal force and the magnetic force on the ion are equal we may write:

$$mV^2/r = HeV \tag{12}$$

where m is the mass of the ion, V is the component of velocity perpendicular to the magnetic field, e its charge, H the impressed magnetic field and r the radius of the circular path. Solving

$$r = mV/He.$$

Now

$$S/\tau = rV/2 = mV^2/2He \tag{14}$$

So

$$M = mV^2/2H \tag{15}$$

Now, if there are N ions per cm^3, then the intensity of magnetization becomes:

$$I = -NmV^2H_1/2H \tag{16}$$

where H_1 is the unit vector in the direction of the impressed vector field. If one temporarily discards the possibility of mass motion of the neutral molecules associated with the ion bank, and takes the absolute temperature of the bank to be T, then the above relation may be transformed to $I = -(2N\epsilon T/3H)H_1$, where ϵ is the constant of molecular energy. It follows from this expression for I that the diamagnetic effect in the upper layers will vary greatly from midnight to noon local time, since the number of ions per cm^3 will increase greatly under the influence of the ultra-violet light of the sun. Moreover, the longer radiations will raise the absolute temperature of the region by a factor of nearly two. Thus, for a given value of H the intensity of magnetization of the diamagnetic layer will increase greatly in the daytime.

The cause of the solar component of the magnetic variation may then be attributed to a diamagnetic layer or cap covering the terrestrial hemisphere

[6] Van Leeuwen, Jour. d. Physique **2,** 361–377 (1921).

that faces the sun. The earth in its rotation brings given points on its surface successively under the influence of this diamagnetic cap, and then well away from it. The maximum effect of the diamagnetic cap would be expected to be at the point on the earth's surface which is nearly immediately under the region of maximum diamagnetic intensity. Experiment seems to indicate that this maximum intensity is overhead about two hours before noon local time.

In order to make a rough quantitative study of the effect we shall simplify the computation by assuming that the diamagnetic layer is a shell or cap located between 150 and 180 km above the earth's surface; that the magnetic poles coincide with the geographic poles; that the magnetic field is such as would be produced by a small magnet placed at the center of the earth and producing the observed field at the equator and assuming further that the temperature is uniformly 300°K and the layer is 30 km thick. Under these conditions, which will not greatly modify the general result, the field variation at the equator at noon during equinox may be computed in terms of the total number of ions of all kinds per cc. It is reasonable to assume that the number of ions per cc is proportional to the intensity of the incident sunlight or

$$N=N_0 \cos\phi+\delta \tag{17}$$

where N_0 is the number of ions per cc at the equator at noon, ϕ is the angular latitude and δ is the average number of residual ions left during periods of darkness. We are interested here only in the variation of the field so this term will be neglected. The intensity of the field has been taken as that due to a small magnet placed at the center of the earth, therefore the magnetic field H at any latitude may be represented by:

$$H=H_0(1+3\sin^2\phi)^{1/2} \tag{18}$$

where H_0 is the magnetic field at the equator. Thus, an approximate expression for the intensity of magnetization at noon during equinox for any latitude is determined.

$$I_\phi=-\frac{2N_0\epsilon T\cos\phi\ H_1}{3H_0(1+3\sin^2\phi)^{1/2}} \tag{19}$$

where H_1 is the unit vector in the direction of the earth's field at the point where I_ϕ is determined.

The magnetic potential at points on the surface of the earth along a noon meridian may then be determined by the aid of the relation

$$\Omega=-\iiint I_\phi\cdot\nabla(1/r)dV \tag{20}$$

where Ω is the magnetic potential. This expression has been formulated by a graphical integration at several points along a "noon meridian" at equinox.

The result of this integration taking account of the direction of the earth's field is shown in Fig. 1 which is expressed in terms of N_0. The horizontal component of the field due to the diamagnetic distribution was then determined by taking the space derivative of Ω along the earth's surface and its value in terms of N_0 is plotted in Fig. 2. Comparison with the observed values plotted in Fig. 3 shows that the form of the variations are

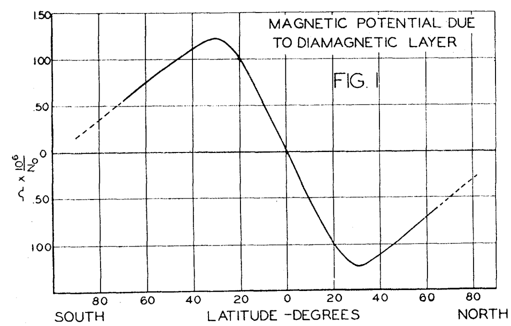

Fig. 1.

nearly identical. It will be observed that the horizontal component of this solar diurnal variation changes sign at about 30° latitude which is in good agreement with experimental observations. Agreement between theory and experiment is equally good in regard to the declination and vertical components of the field.

Measurements at the equator show that the earth's field at noon is on the average nearly 40 gammas (4×10^{-4} gauss) larger than at night. Assuming

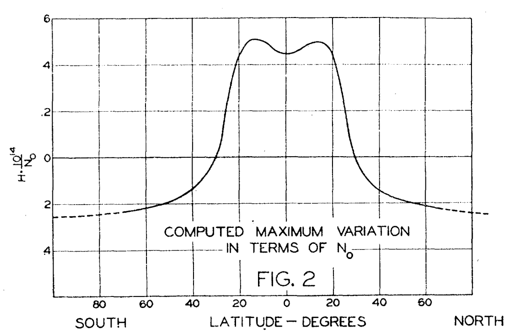

Fig. 2.

this value for the maximum value of the variation field in the foregoing theory the total number of ions of all kinds per cubic centimeter at the equator at noon is found to be $6\times10^{10}=N_0$. That is to say, the total number of ions per square centimeter at the equator at noon extending from the lowest layer where the free path becomes large on out to infinity must equal 18×10^{16}. It must be admitted that this is a large number since it requires that an appreciable fraction of all the molecules present be ionized. Fortunately, no requirement is made in regard to the sign of the charge on the ions and the layer as a whole may be assumed to be nearly neutral. There appear to be no data contradicting the existence of such an intense ionization. Hulburt's[4] calculations of the upper atmosphere indicate that the recombination of ions in the region is slow and that there is sufficient ultra-violet energy

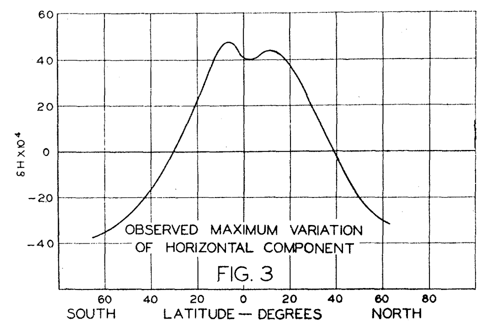

Fig. 3.

from the sun to replace those ions lost by recombination. Further, his calculations show that the number of ions necessary to explain the absorption of the longer radio waves is well within an order of magnitude of the number required on the present theory.

The situation in regard to the semidiurnal or lunar component of the magnetic variation is not quite so simple since it involves two independent phenomena; namely, the sun's ionizing effect and a tidal or other semidiurnal motion of the earth's atmosphere. Data available on the atmospheric tides indicate that tidal variations of semidiurnal character are extraordinarily regular. Such does not appear to be the case with the 24 hour or solar variation which is much more erratic. It is logical, therefore, to assume that the lunar variation is related to a true tidal effect or oscillation of the atmosphere while the solar variation is not. To bring the observed lunar diurnal magnetic variation into rough quantitative agreement with observation it is only necessary to assume that the mass motion of the neutral molecules of the diamagnetic layer due to their tidal motion communicate sufficient additional velocities to the ion bank to bring about variations having the same period as the tides. Assuming that on the average the distribution and intensity of ionization is the same as that determined for the solar component and that the ratio of the observed maximum variations is approximately 25, then by aid of Eq. (16) the maximum velocity of the tidal wind is found to be 10^4 cm/sec. This value may not be at all unreasonable at the high altitudes which are under consideration. Chapman[2] pointed out that there appeared to be no simple phase relation between the atmospheric tides and the semidiurnal magnetic variation. This is not at all surprising for it is reasonable to believe that there is a slight radial potential gradient existing in the upper regions which will give rise to a preferential direction of motion of the ions toward the east or west, depending upon whether the field is the same or opposite to that at the earth's surface. Such a transport phenomenon is capable of explaining any reasonable phase shift of the magnetic effect and if we assume that the small residual charge in the upper atmosphere is such as to produce a field in a direction opposite to that at the surface of the earth the phenomena may be invoked to account for the occurrence of the maximum solar variation before high noon.

The type of variation of the earth's field occurring on so-called disturbed days may be qualitatively explained on the present theory in several possible ways. Observational data show that the maximum superimposed diamagnetic effect on disturbed days occurs at about sunset or a minimum effect at about sunrise, the variation being nearly sinusoidal in character with a 24 hour period. The simplest explanation fits with present notions, since it is only necessary to assume the presence of a strong ionizing agent, presumably a vertical ion stream which falls on the P.M. hemisphere of the earth. The presence of such a stream would increase the specific ionization as well as the temperature of the diamagnetic region and cause a large increase in the diamagnetic effect on the P.M. hemisphere. Conversely a sufficiently intense neutral molecular stream incident on the A.M. hemisphere might be expected to increase ion recombination greatly and produce a marked reduction of the diamagnetic field. This possibility is not viewed with favor.

It is difficult to understand just why an ion stream supposed to be nearly neutral as a whole should choose to fall on either the A.M. or P.M. hemisphere. It seems to be more logical, in view of the relative velocities of the ion stream and orbital velocity of the earth to assume the stream is incident for the most part on the hemisphere facing the sun. Such an assumption is capable of explaining the phenomena if we assume further that in addition to increasing the ionization and temperature of the region this ion stream produces either directly or by a redistribution of charge a radial electric field which is in the same direction as the electric field at the surface of the earth. Such a field will superimpose a translational motion in the direction of the earth's rotation on some of the ions formed and carry them well around to the P.M. hemisphere producing an intense ion bank in this region and explaining the observed effect. That such an electric field exists is pure assumption but is perhaps not improbable when we stop to reflect that the conductivity of the layer in the direction of the assumed field is very small.

The magnitude of this field at the equator may be determined from relation (8) by assuming the average transport velocity shall be slightly larger than the peripheral velocity of the earth. Taking, therefore $V_y = 5.0\times10^4$cm/sec. and $H=0.3$ gauss, $E_x=1.50\times10^{-4}$ volts/cm or a field roughly one ten-thousandth that at the surface of the earth which does not seem impossible.

A complete discussion of this theory would necessarily require a consideration of the effect of induced currents in the earth, and a more precise formulation of the ionic density distribution. To completely determine the latter it will be found necessary to know more about the temperatures, electric fields and velocity of ions at high altitudes and more information must be available on the manner by which ionization in these regions increases with the length of exposure to ionizing radiations. Moreover, the direction of diffusion of the ions and the effect of the earth's magnetic gradient on the diamagnetic elements of the upper layers must be considered in a final and decisive analysis.

In view of our comparative ignorance of the physical facts it does not appear worth while to attempt a more detailed analysis of the problem at the present time.

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