TIME-LAG IN MAGNETIZATION

By Richard M. Bozorth

Abstract

An investigation has been made of the time-lag in magnetization in a permalloy wire to determine whether lag can be satisfactorily accounted for as due to eddy-currents alone or whether permalloy shows a marked magnetic viscosity such as has been observed by Ewing in iron wires. *Eddy-current lag* has been calculated approximately in a manner which takes into account the changing slope of the magnetization curve. A comparison of the calculated and observed magnetization-*vs*-time curves indicates that the effect is well accounted for as eddy-current lag alone. The eddy-current lag has also been calculated for an iron ring, for which the time-lag has been reported recently in a number of papers by Lapp. The time-lag which he observed is satisfactorily accounted for as eddy-current lag instead of as magnetic viscosity as he had supposed.

1. Introduction

When a magnetic force is applied to a ferromagnetic body an appreciable time is required for the magnetization to reach its final state. One contributing cause for this time-lag in magnetization is, of course, theopposing magnetic force produced by eddy-currents within the body. Time-lag has been observed, however, which cannot well be accounted for as due to eddy-currents. For example, the measurements made by Ewing[1] with an iron cylinder 4 mm in diameter show that the magnetization changes appreciably even after the applied magnetic force has been constant for over ten minutes, as shown in Fig. 1. Now Wwedensky[2] has calculated the time rate of change of magnetization delayed by eddy-currents, for the case in which dB/dH, the rate of change of magnetic induction with magnetic force, is constant, and his results indicate that to explain Ewing's results as eddy-current lag we must assume for his sample a value of dB/dH of over 10^7, whereas at the steepest part of the B-*vs*.-H curve for iron a reasonable value for dB/dH is 25,000. This phenomenon, the nature of which is still obscure, is best called "magnetic viscosity," after Ewing.

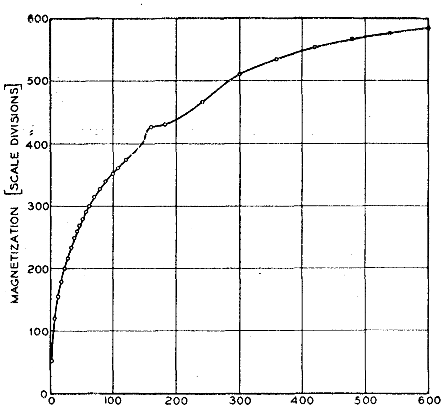

Fig. 1. Magnetic viscosity observed by Ewing, not explicable as due to eddy-currents. When this experiment had been under way for 150 seconds the laboratory door slammed, producing temporarily a rapid increase of magnetization as shown by the hump on the curve.

[1] J. A. Ewing, Proc. Roy. Soc. **A46**, 269–286 (1889).

[2] B. Wwedensky, Ann. d. Physik [4] **64**, 609–620 (1921).

Many experimenters have observed and described magnetic lag.[3] In some cases it seems evident that the lag is due to eddy-currents, and Smith[4] and Wwedensky[5] have interpreted their own results in this way. To determine whether certain strikingly large time-lags, which have been observed in these laboratories in measuring permalloy wires, could all be accounted for as due to eddy-currents or whether some of the effects must be ascribed to magnetic viscosity, the writer undertook the experiments described in Section 3 below. Having found that eddy-currents suffice to explain the lag observed in permalloy, the writer was then led to question the conclusions of Lapp,[6] who has recently made extensive measurements of magnetic lag in iron rings, and who regards the phenomenon which he has studied as magnetic viscosity and not as eddy-current lag. It is shown in Section 4 below that his measurements can be satisfactorily explained as due to eddy-currents alone.

2. Calculation of Eddy-Current Lag

When the relation between magnetic induction, B, and magnetic force, H, is linear, the change in B produced by a rapid change in H at the time $t=0$ can be calculated as a function of time for a cylindrical sample by taking into account the magnetic fields due to the eddy-currents produced by the changing induction. According to an approximate calculation by Rayleigh,[7] letting b represent the fraction of the total change in B which occurs after the time t,

$$\ln b = -\frac{1.44t}{\pi r^2 \sigma dB/dH},$$

where r is the radius of the cylinder and σ is the electrical conductivity in absolute electromagnetic units. More recently Wwedensky[2] has calculated exactly the relation between b and t when dB/dH is constant and has expressed his results in the form of a table. Rayleigh's relation agrees closely with Wwendesky's table for values of b between 0 and 0.9, and approximately for values of b between 0.9 and 1.0.

If dB/dH changes during the change in induction, no accurate method is available for calculating the time lag. A rough estimation of the magnitude of the effect can be made, however, by assuming that Rayleigh's equation holds for each element of the B-*vs.*-H curve, that is,

$$d \ln b = -\frac{1.44dt}{\pi r^2 \sigma f(B)},$$

[3] An extensive bibliography is given by C. Lapp, Ann. de physique [10] **8,** 278–395 (1927). An important addition to this list is H. Tobusch, Ann. d. Physik [4] **26,** 439–482 (1908).

[4] A. W. Smith, Phys. Rev. [2] **9,** 419–424 (1917).

[5] B. Wwedensky, Ann. d. Physik [4] **66,** 110–129 (1921).

[6] C. Lapp, Jour. phys. rad. [6] **4,** 349–369 (1923); **4,** 169S (1923); **6,** 166–176 (1925); **7,** 230–239 (1926); **8,** 29S–30S (1927); Rev. gen. elec. **15,** 88 (1924); Ann. de physique [10] **8,** 278–395 (1927); Rev. de met. **24,** 496–508 (1927); Rev. scient. **65,** 631 (1927).

[7] Lord Rayleigh, Rep. Brit. Assn. **52,** 446–447 (1882).

where $f(B)$ gives the value of dB/dH for each value of B (or b), and is determined from the B-*vs.*-H curve. The time, t, for any value of b, is then given by the expression

$$t=2.2\sigma r^2 \int_b^1 \frac{dB}{dH} \frac{db}{b},$$

in which the integration is performed graphically. This expression is based on the assumption that the flux is uniform over the cross-section of the wire. The effect of the non-uniformity of distribution, which really exists, can be predicted qualitatively, and will be discussed in the next sections where experimental and calculated values of t are compared.

3. Measurements on a Permalloy Wire

The sample of permalloy with which time measurements were made contained by analysis 78.1 percent Ni and 21.3 percent Fe, and a little Mn. The diameter of the sample was 0.102 cm and its length 67 cm. It was enclosed in a permalloy yoke[8] and the magnetic measurements were made by the ballistic method using a galvanometer with a half-period of seven seconds.

For measuring the residual change in induction, bB, a change in magnetic force was produced in the magnetizing coil surrounding the sample, and a measured time, t, afterwards the search coil was connected to the galvanometer. These magnetizing and galvanometer connections were each made by relays which were operated by two metal brush contacts on a rapidly rotating shaft having alternate conducting and insulating segments. Other relays prevented the operation of the relay controlling the galvanometer circuit until the relay controlling the magnetizing circuit had already closed, and still others held these relays closed once they had been tripped. The relays were put into operation by closing a switch connecting them to a battery, and after the galvanometer deflection was observed they were all released by raising this switch.

The time which elapsed between the closing of the two circuits was calculated from the rate of rotation of the shaft, which was measured by a tachometer and read directly on a voltmeter in revolutions per second. One of the brushes making contact with this shaft could be moved around the shaft with respect to the other. The zero position of this brush—that is, its position when the magnetizing and the galvanometer circuits closed simultaneously ($t=0$, $b=1$) —was carefully checked in a separate circuit in which a slight departure from the zero position caused a deflection of a galvanometer, and correct adjustment caused none. Special care was necessary in insulating the contacts in the relay controlling the galvanometer circuit.

[8] O. E. Buckley and L. W. McKeehan, Phys. Rev. [2] **26,** 269 (1925). This yoke should have little or no effect on the decay of eddy-currents in the sample, for the magnetic flux which passes through it is divided among 10 cylindrical rods. In each of these rods r^2B, which determines the strength of the eddy-currents, is one-tenth of what it is in the sample itself, so that the currents in the yoke should die out much faster than in the sample. No direct experimental test of this point was made.

The time necessary for the magnetic force to attain a constant value was computed from the known resistance and inductance of the circuit to be negligibly small; less than 5 percent of the total change in magnetic force occurred after 10^{-4} sec.

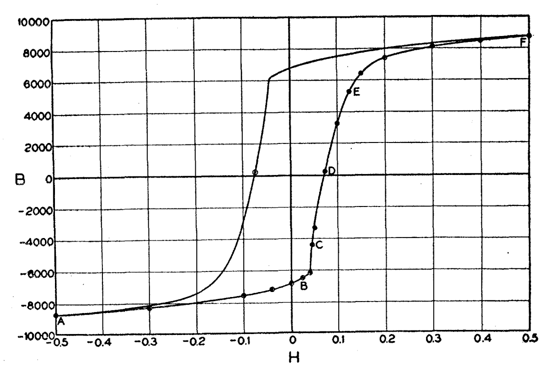

Fig. 2. Hysteresis loop for permalloy containing 78 percent nickel.

The hysteresis loop for a maximum magnetic force of 0.5 gauss is shown in Fig. 2. All lag measurements were made with the magnetic forces chang-

TABLE I. *After the magnetic field has been constant for t seconds, the magnetic induction changes by the fraction b of the total change in induction from A to E in Fig. 2.*

t	b	t	b	t	b
0.000	1.000	0.020	0.609	0.15	0.0726
.001	.920	.030	.502	.20	.0263
.002	.872	.050	.372	.25	.0090
.005	.808	.075	.262	.30	.0044
.010	.740	.100	.194	.40	.0028

ing from −0.5 gauss to various positive values marked on the hysteresis loop with letters. Table I shows the results obtained when the final value

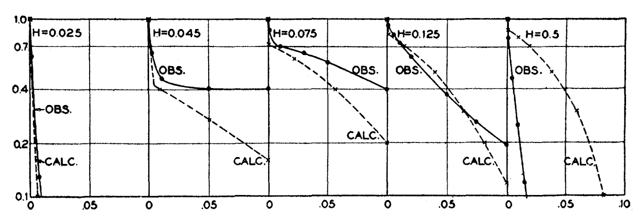

Fig. 3. Time lag in magnetization in a permalloy wire 1 mm in diameter, containing 78 percent nickel. The letter b represents the fraction of the total change in flux which takes place after a certain measured time.

of the magnetic force was +0.125 gauss. The log b-*vs.*-t curves are shown as solid lines in Fig. 3 for various changes in B, and the dashed lines are the corresponding curves calculated as described in Sec. 2 above.

Comparing the calculated and observed curves, it will be noticed that when the magnetic state changes from that represented by the point A to that represented by the point B ($H=0.025$) the induction changes through various values for which dB/dH continually increases. Although in calculating the time lags it is assumed that the whole cross-section of the sample is in the same magnetic state, in reality the outside approaches its final state more quickly than the inside and so dB/dH is greater in the outer layers than is implicitly assumed in the calculation. For a given value of b, then, the measured times will be greater than those calculated. When the opposite situation occurs, and the magnetic state of the sample changes from A to F ($H=0.5$) so that during most of the time dB/dH decreases as the induction changes, the calculated times will be too large. For changes such as A to E ($H=0.125$) the calculated times may be expected, because of the simplifying assumption of uniform flux distribution, to be sometimes too small and sometimes too large. All these expected differences are found between the calculated and experimental curves shown in Fig. 3. It is difficult at first to see how the slope of the flat portion of the second observed curve in the figure can be so different from that of the calculated curve. It seems probable that the large values of dB/dH determined from the hysteresis loop near the point C are in error, and that their true values are even greater. This discrepancy may be due to several causes besides the obvious error in determining a slope so large. For example, if the coercive forces for different parts of the wire are but slightly different, the superpositon of their B-*vs.*-H curves shows that the measured value of dB/dH may be too small by a large factor if dB/dH is already large. Another contributing factor may be the discontinuities in magnetization which as Barkhausen[9] has shown, occur in that part of the magnetic cycle for which dB/dH is large.

4. Measurements by Lapp on Iron Rings

Recently Lapp[6] has measured time-lag in rings of pure iron. The cross-sections of these rings were rectangular and varied in area between 0.1 and 1.0 cm^2 in the various samples used. Although Lapp states that induced currents are negligible[10] when the cross-sectional area is 1 cm,^{2} nevertheless the calculations described below indicate very definitely that his measurement of change of induction with time, and also of the effects of cross-sectional area and shape and of temperature on the induction-*vs.*-time curves, are satisfactorily explained as due to eddy-currents.

Variation of induction with time. Fig. 4 shows the hysteresis loop plotted from Lapp's tabulated data for a ring having a cross-section of about 2.0×0.4 cm,^{2} and Fig. 5 his observed values of log b and t (solid lines) and my values (dashed lines) calculated as described in Section 2. The differences between the calculated and observed curves are of the same nature as those discussed above in connection with the measurements on the permalloy

[9] H. Barkhausen, Phys. Zeits. **20,** 401–403 (1919).
[10] C. Lapp, Ann. de physique [10] **8,** 280 (1927).

wire, and seem to be caused by the simplifying assumption made in the calculation. It is at once obvious from the form of both the observed and

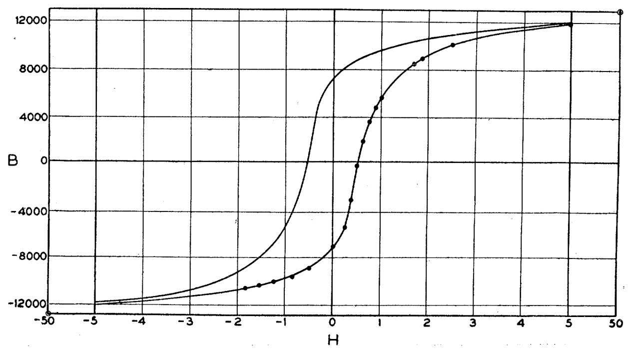

Fig. 4. Hysteresis loop for pure iron according to Lapp.

calculated curves, why it is that when Lapp extrapolated his curves to $t=0$ he found a value of b less than unity; for the smallest time which Lapp

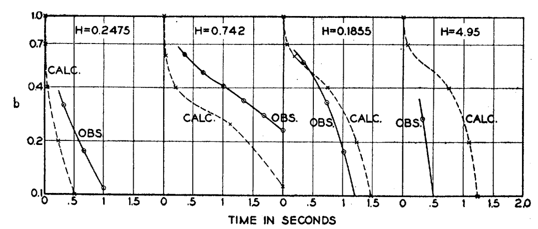

Fig. 5. Time lag in magnetization in a ring of pure iron having a cross-sectional area of 0.82 cm², as measured by Lapp (solid lines) and as calculated by the writer (dashed lines).

measured was 1/3 second, and the calculated curves show that for smaller values of t the curves bend upward ending at the point $t=0$, $b=1$.

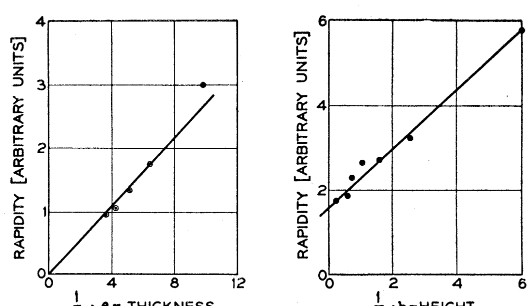

Fig. 6. Variation of rapidity with the thickness and height of the ring sample, according to measurements made and plotted by Lapp.

Effect of cross-sectional area and shape. Lapp's measurements show the effect of the area and shape of the sample on the "rapidity," β, which may be defined in accordance with Lapp's equation as the slope of the ln*b*-*vs*.-*t* curve. Fig. 6 shows Lapp's curves of rapidity-*vs*.-thickness, *e*, and rapidity-*vs*.-height, *h*, indicating that the rapidity varies inversely as the squares of the thickness and of the height. Taking account of the rectangular cross-section of the sample, analysis shows[11] that $1/r^2$ in the equation for circular cross-sections may be replaced by $(h^2+e^2)/h^2e^2$. This factor, it will be noted reduces to $1/e^2$ for thin sheets ($h>>e$). Now assuming the lag to be caused by eddy-currents, the rapidity can be calculated as a function of thickness and height and compared with Lapp's experimental data. Fig. 7 shows the

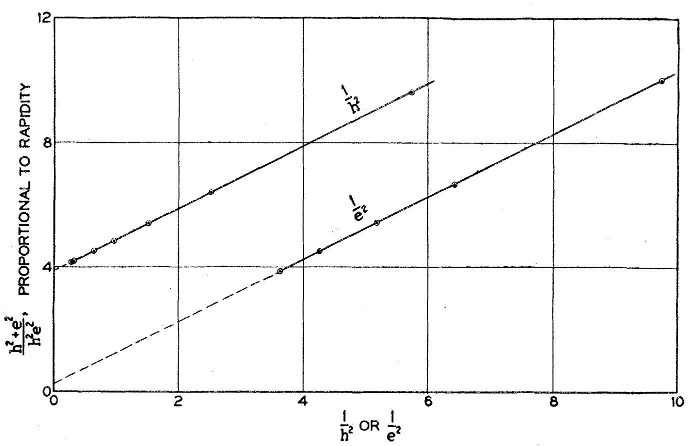

Fig. 7. Calculated variation of rapidity with thickness, *e*, and height, *h*, of a ring sample. Compare the straightness of these lines with that of the experimental lines of Fig. 5.

result of such a calculation using the values of *e* and *h* for Lapp's cores, and comparison with Fig. 6 shows that the relations between cross-sectional dimensions and rapidity which he observed are those to be expected from my analysis.

Inspection of the factor $(h^2+e^2)/h^2e^2$ indicates that for similar cross-sections ($h/e=$constant), the rapidity varies inversely as the area of the cross-section. On the other hand, if h/e is large and h is kept constant while e is varied the rapidity varies with the square of the area of cross-section as Lapp has stated. It is clear that no general statement can be made regarding the dependence of the rapidity upon the area of cross-section.

Effect of temperature. The dependence of rapidity upon absolute temperature observed by Lapp ($\beta \propto T^2$) can also be explained if the resistivity, $1/\sigma$, which we have seen is proportional to β, is nearly enough proportional to T^2. To show how nearly the relation $1/\sigma \propto T^2$ is satisfied experimentally, the data[12] for pure iron are plotted in Fig. 8 and a straight line drawn for which this relation is exactly satisfied.

[11] Assuming that the current paths are rectangular, Dr. J. J. Gilbert of these laboratories has derived this factor depending upon the dimensions of the cross-section.

[12] J. Dewar and J. A. Fleming, Phil. Mag. [5] **36,** 271 (1893); L. Holborn, Ann. d. Physik [4] **59,** 145–169 (1919); W. Meissner, Zeits. f. Physik **38,** 647–658 (1926).

It appears probable that Lapp's observations on the effects of aging and annealing of iron, and his experiments on cobalt and nickel, can also be explained as due to eddy-currents, the different behavior with various treatments and materials being attributed to the different character of their *B-vs.-H* curves, and perhaps to the magnetic non-homogeneities produced by heat treatments such as quenching.

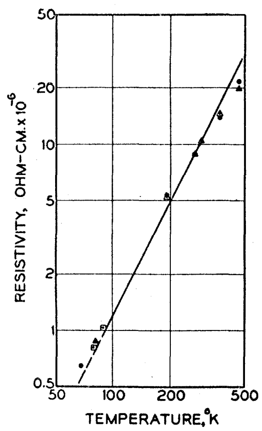

Fig. 8. Resistivity of iron at various absolute temperatures. The data are those of Dewar and Fleming (circles), Holborn (triangles), and Meissner (squares). For the straight line the resistivity is exactly proportional to the square of the absolute temperature, the relation necessary to explain the variation of rapidity with temperature observed by Lapp.

5. Magnetic Viscosity

The nature of the phenomenon of magnetic viscosity, exemplified by the experiments of Ewing, still remains obscure. It is barely possible that slight statistical fluctuations in the supposedly steady current from the battery, or in a superposed magnetic field, could account for these observations. Another possibility is that the sample is continually subjected to slight vibrations which, as Ewings's data show, would produce a change in induction in the right direction.

To test this latter point, to find the course of the induction vs. time curve when the sample was continually vibrated, a 4 cm length of 0.5 mm soft iron wire was placed in a magnetometer[13] and an ordinary electric buzzer attached rigidly to the framework on which the magnetometer was mounted. The field was changed rapidly, with the buzzer not operating, to a new value such that dB/dH was constant over a considerable region; then the buzzer was operated for various lengths of time and the corresponding deflections of the magnetometer noted. The results are shown in Fig. 9, curve A, where the logarithms of the deflections (1 unit corresponds to 0.01 gauss) are plotted against the logarithms of the time. The fact that the curve is a straight line over a large part of its course shows that the nature of the phenomenon is entirely different from that of eddy-current lag,[14] and suggests that it is very similar to the effect observed by Ewing, which is replotted in Fig. 9, curve B, for comparison.

It is very doubtful, however, whether the lag observed by Ewing could have been due to external mechanical shock, because the shock necessary to reproduce approximately Ewing's magetization vs. time curve is so large that it could not well have escaped his notice. The most reasonable supposition, therefore, is that the phenomenon that Ewing observed should be properly termed magnetic viscosity, and the suggestion is made that it is

[13] R. M. Bozorth, J.O.S.A. & R.S.I. **10**, 591–598 (1925).

[14] Assuming any reasonable value for the final induction, the log *b-vs.-t* curve departs widely from a straight line.

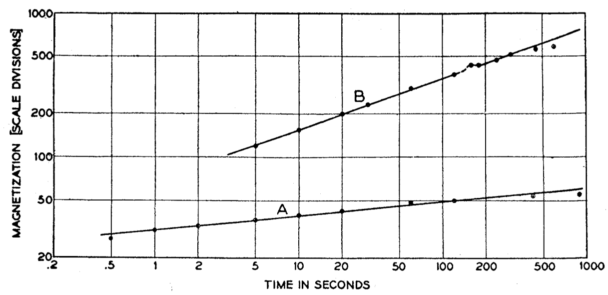

Fig. 9. Curve *A* shows the effect of continuous mechanical vibration on magnetization as a function of time. In curve *B* Ewing's data of Fig. 1 are replotted to show that in this case, as well as when the sample is vibrated, the magnetization vs. time curve is a straight line when plotted with double logarithmic scales.

caused by the disturbances of unstable configurations of molecular magnets by thermal vibrations, in much the same way that such configurations are disturbed by mechanical vibrations produced from the outside.

Bell Telephone Laboratories, Incorporated,
New York, N. Y.
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