STARK EFFECT AND SERIES LIMITS[1]

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ABSTRACT

Energy of conditionally periodic and aperiodic orbits in hydrogen-like atoms under the influence of an external electric field.—It is shown that the energy of a conditionally periodic orbit in a hydrogen-like atom under the influence of a homogeneous external field F must be less than $-(3/2)[eFp_\phi/m^{1/2}]^{2/3}$. Aperiodic orbits of less, as well as of greater, energy than this value can exist, and in one class of these the electron may approach the nucleus within distances comparable with the dimensions of periodic orbits. The lower limit of the energy of these orbits is approximately $-2e(eF)^{1/2}$.

Applications to spectra.—Since the energy of an aperiodic orbit can assume any one of a continuum of values, it is concluded that the line spectrum of such an atom must end at a point on the long wave-length side of the normal series limit, and that the continuous spectrum, arising from transitions involving an aperiodic orbit, may extend even further within the normal series limit. Application to the mean field in actual gases gives the position of the maximum of the continuous spectrum in good agreement with the observed values. Further experiments for checking the theory are described.

THE behavior of a hydrogen-like atom under the influence of an external homogeneous electric field has been discussed, from the standpoint of the older quantum theory, by Epstein,[3] Schwarzschild,[4] Bohr,[5] Lenz[6] and others. The aspects of the problem which they have considered relate, however, to states of the atom so far from ionization that the theory of perturbations can be validly applied. It is the purpose of the present paper to investigate, also on the Bohr theory, the nature of the mechanically possible orbits of small negative energy, and to discuss the bearing of the results obtained on series limits and continuous atomic spectra.

ORBITS IN ATOM IN HOMOGENEOUS FIELD

The motion of an electron in the field of a core of residual charge $+e$ and a homogeneous electric field of strength F is determined by its kinetic energy $\frac{1}{2}mv^2$ and potential energy $-e^2/r+eFz$ where z is the component of its distance r from the nucleus in the direction of the field. In parabolic coordinates (ξ, η, ϕ)[7] defined by

$$x=(\xi\eta)^{1/2}\cos\phi,\ \ y=(\xi\eta)^{1/2}\sin\phi,\ \ z=\tfrac{1}{2}(\xi-\eta)\ ;\ (\xi,\eta\geqq 0) \tag{1}$$

the Hamiltonian function is

[1] Preliminary report by Dewey and Robertson in Nature 1928, p. 709.
[2] National Research Fellows.
[3] P. S. Epstein, Phys. Zeits. **17**, p. 148 (1916); Ann. d. Physik **50**, p. 489 (1916).
[4] K. Schwarzschild, Berl. Ber. p. 548 (1916).
[5] N. Bohr, "Quantentheorie der Linienspektren", p. 101 (Braunschweig, 1923).
[6] W. Lenz, Zeits. f. Physik **24**, p. 197 (1924).
[7] The notation is essentially that of W. Pauli, Hndbch. d. Physik **23**, p. 130 (Berlin, 1926).

$$H(\xi,\eta,\phi\,;p_\xi,p_\eta,p_\phi)=\frac{1}{2m(\xi+\eta)}\left\{4\xi p_\xi{}^2+4\eta p_\eta{}^2+\left(\frac{1}{\xi}+\frac{1}{\eta}\right)p_\phi{}^2+meF(\xi^2-\eta^2)-4me^2\right\}$$

and the Hamilton-Jacobi equation

$$H(\xi,\eta,\phi\,;\partial S/\partial\xi,\partial S/\partial\eta,\partial S/\partial\phi)=W \tag{3}$$

is soluble by the method of separation of variables.[8] The momenta are then given by

$$p_\xi=\frac{\partial S}{\partial\xi}=\frac{1}{2\xi}[f(\xi)]^{1/2},\qquad p_\eta=\frac{\partial S}{\partial\eta}=\frac{1}{2\eta}[g(\eta)]^{1/2},\qquad p_\phi=\frac{\partial S}{\partial\phi}=\text{const}. \tag{4}$$

where

$$f(\xi)=-meF\xi^3+2mW\xi^2+2\alpha_1\xi-p_\phi{}^2,\quad g(\eta)=meF\eta^3+2mW\eta^2+2\alpha_2\eta-p_\phi{}^2 \tag{5}$$

and α_1, α_2 are constants subject to the relation

$$\alpha_1+\alpha_2=2me^2. \tag{6}$$

The orbits which can be quantized, whose energy W is consequently limited to a discrete set of values, must be *conditionally periodic.* The theory of such orbits requires that each of the functions $f(\xi)$ and $g(\eta)$ then have two non-negative roots between which they are themselves positive (or, in some cases, a multiple root; a coordinate for which this occurs must retain this

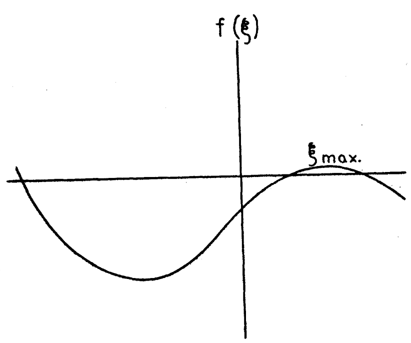

Fig. 1.

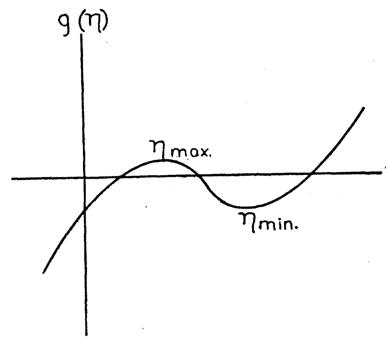

Fig. 2.

constant value throughout the motion). Orbits which do not satisfy these conditions, which we shall for the sake of brevity refer to as *aperiodic,*[9] cannot be quantized, and their energy may assume any of a continuum of values. The present investigation concerns itself primarily with interpreting these conditions in terms of the *mechanically* permissible values of $W<0$,

[8] Strictly speaking, m is the reduced mass $m_{el}m_{nu}/(m_{el}+m_{nu})$ of the electron.

[9] We disregard motions of *limitation.*

and not with the actual determination of the quantized values. With this knowledge of the range of energy allowed in these two types of motion we can predict the nature and position of spectra produced by such an atom, transitions between two quantized states giving rise to a line spectrum and transitions in which one of the states involved is aperiodic to a continuous spectrum.

Conditionally periodic orbits. The conditions mentioned above for orbits of this type require, with regard for the fact that $f(0)=g(0)=-p_\phi^2<0$, that all roots of the cubics $f(\xi)$ and $g(\eta)$ be real and, with the exception of the least root of $f(\xi)$, positive.[10] This can be expressed analytically (cf. Figs. 1 and 2):

$$\xi_{\max} \text{ real and } >0\ ;\quad f(\xi_{\max})\geqq 0 \tag{7}$$

$$\eta_{\max} \text{ real and } >0\ ;\quad g(\eta_{\max})\geqq 0,\ \ g(\eta_{\min})<0 \tag{8}$$

where the subscripts max and min indicate that value of the variable for which the corresponding cubic has a maximum and minimum respectively.

We first note that since the sum of the three (positive!) roots of $g(\eta)$ must equal $-2W/eF$, W must be negative:

$$W<0. \tag{9}$$

For simplicity we introduce the notation

$$\begin{aligned} &X=2mW^2/3eF,\ \ i.e.\ \ W=-(3eFX/2m)^{1/2}\\ &P=(3/8)(6meF)^{1/2}p_\phi^2,\ \ B=2me^2 \end{aligned} \tag{10}$$

The extrema of the cubics are then

$$\xi=2/(6meF)^{1/2}[\mp(X+\alpha_1)^{1/2}-X^{1/2}],\ \eta=2/(6meF)^{1/2}[X^{1/2}\mp(X-\alpha_2)^{1/2}]. \tag{11}$$

From these, the first conditions of (7) and (8), and the relation (6) the restrictions on α_1 and α_2 are found to be

$$\left.\begin{matrix}0<\\ B-X\leqq\end{matrix}\right\}\alpha_1<B,\qquad 0<\alpha_2\left\{\begin{matrix}<B\\ \leqq X\end{matrix}\right.,\quad \alpha_1+\alpha_2=B. \tag{12}$$

Finally, the remaining conditions of (7) and (8) become

$$\begin{aligned} -\tfrac{1}{2}\alpha_1X^{1/2}+(X+\alpha_1)\,[(X+\alpha_1)^{1/2}-X^{1/2}]&\geqq P\\ \tfrac{1}{2}\alpha_2X^{1/2}-(X-\alpha_2)\,[X^{1/2}-(X-\alpha_2)^{1/2}]&\geqq P\\ \tfrac{1}{2}\alpha_2X^{1/2}-(X-\alpha_2)\,[X^{1/2}+(X-\alpha_2)^{1/2}]&<P \end{aligned} \tag{13}$$

These inequalities define the range of W within which the energy of a conditionally periodic orbit must lie.

In unperturbed hydrogen only orbits with negative total energy can be quantized, and this occurs in such a way that there exist (highly excited) orbits whose energies are arbitrarily near zero (ionization). We now ask if

[10] $p_\phi=0$ must be excluded, as in this case the conditionally periodic orbits would approach arbitrarily near to the nucleus. This exclusion is effected automatically in the new quantum mechanics (cf. E. Schroedinger, Ann. d. Physik **80**, p. 463 (1926)).

there exists an analogous upper limit for the energy of quantized orbits in the case under consideration; this will be characterised by the smallest value of X compatible with the above inequalities, so we first investigate the possibility of orbits for which $X<B$ (the least restrictive case). (12) then restricts α_1 to values not greater than X, and from the second inequality of (13)

$$\tfrac{1}{2}\alpha_1 X^{1/2} \geqq P, \text{ i.e. } X^{1/2} \geqq 2P/\alpha_2 \geqq 2P/X$$

whence

$$X \geqq (2P)^{2/3} \tag{14}$$

Furthermore, although orbits with energy corresponding to the equality sign in (14) can be shown to be either unstable or constitute a motion of limitation, the inequalities (13) can be satisfied by conditionally periodic orbits whose energy is less but arbitrarily near this limiting value provided

$$P < 4B^{3/2}/27 \tag{15}$$ [11]

We may consequently state: *There exist no quantizable orbits whose total energy is greater than*

$$-\frac{3}{2}\left(\frac{eFp_\phi}{m^{1/2}}\right)^{2/3} \tag{14'}$$ [12]

and only in case

$$|p_\phi| < \frac{8}{9}\left(\frac{me^3}{(3eF)^{1/2}}\right)^{1/2} \tag{15'}$$

can there exist quantizable orbits with energy in the neighborhood of (14′). The complete discussion of cases in which F or $|p_\phi|$ is so large that the inequality (15) is not satisfied is more complicated; it will be found expedient in any given case to apply the method of successive approximation to the inequalities (13) in order to obtain the limitation imposed on the energy. In any case the energy must be so small that (14) and

$$X > (2P/B)^2, \text{ i.e. } W < -9Fp_\phi^2/8me \tag{16}$$

(which follows from the fact that $\alpha_2 < B$ and the inequality preceeding (14)) are satisfied; the larger of these two lower bounds for X may be taken as a starting point for the method. However (15) is satisfied in the applications which we wish to consider, and consequently the statement above is sufficient for our purposes.

As a consequence of the above and the fact that the lower quantum states suffer only the ordinary Stark effect splitting, the short wave-length limit of the series produced by transitions from or to one of these lower states

[11] This can be proven by noting that if the last two inequalities of (13) are satisfied by values X', α_2' of X, α_2, then they are also satisfied by $X'+s$, $\alpha_2'+\frac{1}{2}s$, where s is any positive quantity.

[12] A simple derivation of the somewhat weaker condition $W < -\frac{1}{2}18^{1/3}(eFp_\phi/m^{1/2})^{2/3}$ can be obtained from the remark above (9) that the sum of the three roots of $g(\eta)$ is $-2W/eF$, together with the fact that each root is certainly greater than p_ϕ^2/α_2, the intercept on the η-axis of the tangent to $g(\eta)$ at $\eta=0$ (cf. Fig. 1). For from (12) $\alpha_2 < B = 2mW^2/3eF$, whence $-2W/eF > 3p_\phi^2/\alpha_2 > 9p_\phi^2 eF/2mW^2$ and the desired inequality follows immediately.

will be displaced toward smaller frequencies by an amount corresponding approximately to (14′). In accordance with the views of Bohr and others concerning continuous atomic spectra, we should expect that a continuous spectrum would extend from this displaced limit toward shorter wavelengths.

Aperiodic orbits. According to the above an orbit whose energy is greater than (14′) will be aperiodic, but it does not follow that orbits of less energy are necessarily conditionally periodic. Returning to equations (4) it is seen that since $g(\eta)$ is a cubic in which the coefficient of η^3 is positive there exist aperiodic orbits, for which η is greater than the largest root of $g(\eta)$, corresponding to any choice of W and α_1 which satisfy conditions (7) (for if these were not satisfied $f(\xi)$ would be negative for all positive ξ and p_ξ would consequently be imaginary). In general, however, these orbits are at a comparatively large distance from the nucleus and may be considered as representing the motion of an essentially free electron in the electric field—as, for example, the orbits for which η is greater than the largest root of $g(\eta)$ in Fig. 2.

Restricting ourselves to aperiodic orbits whose total energy is negative, there does exist a class of such orbits in which the electron may approach the nucleus within a distance comparable with those of the lower quantized orbits. They are characterised by the fact that for them $g(\eta)$ has but one real root and is positive at the point of inflection. These orbits are of two kinds,

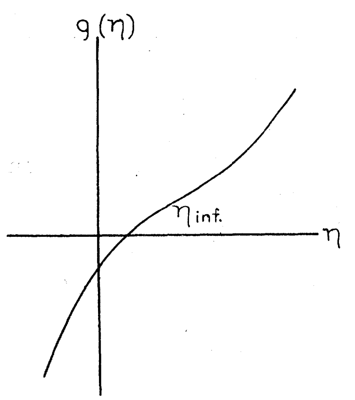

Fig. 3.

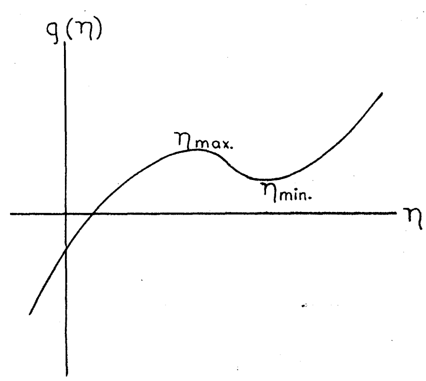

Fig. 4.

(a) those for which $g(\eta)$ has no extrema (Fig. 3), and (b) those in which there exist real extrema at which $g(\eta)$ is positive (Fig.4); in either case the conditions (7) on $f(\xi)$ must be satisfied. The conditions for (a) may be written

$$\begin{gathered} 0<\alpha_1\leqq B-X, \quad X\leqq\alpha_2<B, \\ -\tfrac{1}{2}\alpha_1X^{1/2}+(X+\alpha_1)\left[(X+\alpha_1)^{1/2}-X^{1/2}\right]\geqq P, \end{gathered} \tag{17}$$

and for (b)

$$\begin{gathered} \left.\begin{matrix}0< \\ B-X\leqq\end{matrix}\right\}\alpha_1<B, \quad 0<\alpha_2\left\{\begin{matrix}<B \\ \leqq X\end{matrix}\right., \\ -\tfrac{1}{2}\alpha_1X^{1/2}+(X+\alpha_1)\left[(X+\alpha_1)^{1/2}-X^{1/2}\right]\geqq P, \\ \tfrac{1}{2}\alpha_2X^{1/2}-(X-\alpha_2)\left[(X-\alpha_2)^{1/2}+X^{1/2}\right]>P. \end{gathered} \tag{18}$$

We are now interested in determining the largest X, *i.e.* the smallest energy, compatible with the inequalities (17) or (18); from this the maximum extension of the continuous spectrum *inside* the normal series limit, corresponding to transitions between a lower quantum state and these nearer orbits (*i.e.* ionization or capture of an electron by the ion), can be determined. Again, in the applications considered later an inequality similar to (15) holds, which we write

$$\epsilon^2 = P/B^{3/2} << 1 \tag{19}$$

Examination of the third inequality of (17) shows that for orbits of type (a) X must be less than B by a term of order $B\epsilon$; the largest value of X compatible with this inequality is found by expansion to be, within terms of order $B\epsilon^2$,

$$X = B(1 - 2(6)^{1/2}\epsilon/3)\,. \tag{20}$$

On the other hand, orbits of this type can be found for which X is arbitrarily small.

The inequalities (18) require that for orbits of type (b) X be greater than the two lower bounds $(2P)^{2/3}$ and $(2P/B)^2$ found in the investigation of periodic orbits. The left hand side of the last inequality of (18) is a monotonically decreasing function of X with a root at $X = 4\alpha_2/3$; X must consequently be less than $4B/3$. Examination of the behavior of this inequality when α_2 and X are in the neighborhood of B and $4\alpha_2/3$ respectively gives as the maximum of X

$$X = (4B/3)(1 - 4(3)^{1/4}\epsilon/3) \tag{21}$$

We thus arrive at the conclusion that there can exist in addition to aperiodic orbits with positive energy certain others, having small perihelion distances, whose energies are negative and range down to approximately

$$W = -2e(eF)^{1/2}[1 - 2(3)^{1/4}\epsilon/3] \tag{21'}$$

Transitions involving a lower quantum state and these orbits will give rise to a continuous spectrum which extends *within* the normal series limit a distance corresponding to (21′). Superposed on a portion of this continuous spectrum there may occur lines arising from transitions involving periodic orbits of comparable energy. A discussion of the relative probability of these various transitions would necessitate applying the new quantum mechanics, and would carry us beyond the scope of the present paper.

Spectral Applications

That only an isolated atom in the absence of any field can have spectral terms of energy arbitrarily close to zero has been known since the earliest days of the quantum theory and is in accord with observations on spectra. Bohr[13] first showed this from spacial considerations and Franck[14] pointed

[13] Bohr, Phil. Mag. **26,** 9 (1913).
[14] Franck, Zeits. f. Physik **1,** 2 (1920).

out that the electric field of nearby atoms and ions must be the controlling factor. Oldenberg[15] has suggested that a Stark effect displacement of the series limit in the direction of lesser energy would explain the observation of continuous spectra to the red of the normal limit. We see the effect in the small number of lines observed in turbulent discharges and the large number observed in absorption in diffuse vapors.[16] Also it has been observed that stars with sharp spectral lines show maxima of continuous absorption nearer the normal limit than do those with broad spectral lines.[17]

The quantitative application of conditions (14′), (15′) and (21′) to an actual gas is at best a rough approximation, for the fields in a gas arise from the presence of ions and from the electric moments of the atoms themselves and are unhomogeneous. In the case of the strongest mean fields which we shall consider, however, (8 e.s.u. per cm.) the mean distance of the charged particles from each other is of the order of 10^{-5} cm and the radius of the largest periodic orbit that can exist is of the order of 10^{-6} cm; the variation in the field is therefore in general only a fraction of its magnitude. The variation due to the motion of the charges is small compared to the variation in space for stationary charges as long as the velocity of the electron which forms part of the atom is large compared to that of the charges which set up the external field. Holtsmark[18] calculated the width of spectral lines from the Stark effect displacements for homogeneous fields and obtained good agreement with observed widths. The approximation is probably better in the cases he considers than near the series limit because of the smaller size of the orbits.

To determine of the displacement of the limit to be expected the formula are given in numerical form. From (14′) $W<-9.9\times10^{-16}(MF)^{2/3}$ ergs, where M is the "equatorial" (magnetic) quantum number and the field F is expressed in electrostatic units per centimeter, or

$$\nu>5.2(MF)^{2/3}\text{cm}^{-1} \tag{14''}$$

Where ν is the term frequency of the last mechanically possible periodic orbit. For the aperiodic orbits of smallest energy we obtain

$$\nu<(109F^{1/2}-0.8F^{3/4}|\,M\,|)\text{cm}^{-1} \tag{21''}$$

We cannot expect to observe all the lines allowed by condition (14″) for they may be too weak to be photographed, and for this reason it is not useful for quantitative application. All we can say is that no lines have been observed which it would exclude. We can apply condition (21″) to the observed continuous atomic spectrum of hydrogen.

In laboratory sources, hydrogen gives a continuous spectrum which extends over the entire visible and ultra-violet region and may be ascribed to

[15] Oldenberg, Zeits. f. Physik **41**, 1 (1927), see also Saha, Nature, **114**, 155 (1924).
[16] Wood, Astrophys. J. **29**, 97 (1909; **43**, 73 (1916).
[17] See, for instance, Payne, Stellar Atmospheres, Cambridge, Mass., 1925, p. 43.
[18] Holtsmark, Ann. d. Physik **58**, 577 (1919); Phys. Zeits. **25**, (1924).

the molecule. In the stars, however, because of their high temperatures, very few molecules can be present and we can assign the continuous spectrum observed near the limit of the Balmer series to the atom. Wright[19] attempted to explain the appearance of the portion of the continuous spectrum within the normal series limit by assuming that there exists a last quantized orbit, beyond which the electron becomes free and has less energy. This is, however, impossible from energetic considerations; his hypothesis neglected the potential energy of the quantized orbits. Wright's theory was extended by Crew and Hulburt[20] and applied to their measurements on the continuous spectrum of a hydrogen discharge tube. In their form of the theory it is not clear from what source the electron obtains the energy necessary to remove it to any distance from the nucleus when the atom is ionized by light of a frequency less than the limiting frequency. According to the ideas presented here this energy is taken up from the electric field or, in the case of stray fields of ions, from the kinetic energy of the ion or ions in whose field the atom lies. As this energy is small (0.04 volts at most in our applications) it will in general be available; if it is not, the ion and electron will recombine with emission of light of the same or longer wave length than that absorbed. The continuous spectrum of hydrogen has been discussed by various authors and complete bibliographies are given by Anderson[21] and by Herzberg.[22]

The most recent and extensive data on the continuous atomic spectrum of hydrogen are those of Yü[23] who measured the absorption as a function of wave length in the neighborhood of the limit of the Balmer series for a large number of stars. Hartmann[24] also gives photometric measurements on a few stars. According to these authors the maximum of absorption in many class A and class B stars occurs at 3700A and absorption is appreciable 100 to 150A to the red of this wave length. Russell and Stewart[25] and Eddington[26] estimate the mean field in the sun to be 6 e.s.u. per centimeter. Since according to Payne[27] the ion and electron concentrations in these stars are of the same order as in the sun, we shall take this estimate for these stars also. In α cygni, a giant star, as well as in the chromosphere[28] and the nebula N. G. C. 1499,[29] the limit of continuous absorption is close to the normal limit, as we should expect from the rarified condition of these spectral sources. Herzberg[30] has recently succeeded in obtaining the continuous spectrum of atomic hydrogen distinct from the molecular spectrum in an electrodeless ring discharge. On the reproductions of plates which

[19] Wright, Nature, **109**, 810 (1922).
[20] Crew and Hulburt, Phys. Rev. **28**, 936 (1926).
[21] Anderson, Zeits. f. Physik **38**, 530 (1926).
[22] Herzberg, Ann. d. Physik **34**, 555 (1927); Phys. Zeits. **28**, 727 (1927).
[23] Yü, Lick Bull, **12**, 104 (1926).
[24] Hartmann, Phys. Zeits. **18**, 429 (1917).
[25] Russell and Stewart, Astrophys J. **59**, 197 (1924).
[26] Eddington, The Internal Constitution of the Stars, Oxford 1926, p. 356.
[27] Payne, loc. cit. pages 110 and 173.
[28] Evershed, Phil. Trans. **197**, 381 (1901).
[29] Hubble, Pub. A. S. P. **32**, 155 (1920).
[30] Herzberg, loc. cit.

accompany his article the continuous spectrum overlaps the lines to about 300 cm^{-1} within the limit and the last lines appear 200 cm^{-1} within the limit. There is now way of estimating the field in an electrodeless discharge. It is probably appreciable as the continuous spectra will not be emitted with observable intensity unless the ion concentration is large. Balasse[31] reports the appearance of forbidden lines in a similar discharge in caesium.

Since any atom becomes hydrogen-like as one of its electrons is removed to higher quantum states we can apply these considerations to any continuous atomic spectrum. Paschen[32] observed three continuous spectra in helium using a discharge of high electron concentration. He reports that the spectra appearing near the $2s$ and $2p$ limit extend 275 cm^{-1} to the red of the normal limit. The maximum of the $2s$ spectrum appears on the reproduction to lie about 190 cm^{-1} to the red of the limit. Mohler[33] has recently made photometric measurements on the continuous spectra of caesium and observed maxima in the intensity 250 and 300 cm^{-1} to the red of the limits $2P$ and $2D$ respectively, with an appreciable intensity extending at least 1500 cm^{-1} to the red in the later case; the $2P$ spectrum overlaps the $2D$ so that it is not possible to judge its extension to the red of the maximum. In tubes similar to the one in which he observed this emission he has measured an ion concentration of 10^{14} ions per cubic centimeter.[34] From this ion concentration one obtains by the formula of Holtsmark[17] a field of 8 e.s.u. per centimeter. Table I collects these data and gives the limit, calculated from

TABLE I

Element	Limit	Authority	Field (e.s.u.)	Distance from Normal Limit (cm^{-1}) Calc. Limit	Obs. Maximum	Obs. Limit
H	Balmer	Yü	6	270	350	1200
H	Balmer	Herzberg			200	300
He	2s	Paschen	3	175	190	275
He	2p	Paschen	3	175		275
Cs	2P	Mohler	8	310	250	
Cs	2D	Mohler	8	310	300	>1500

formula (21''), of the continuous spectrum which would be emitted were all the atoms exposed to a homogeneous field of the magnitude of the mean stray field in the gas. A continuous spectrum emitted in the absence of field would have a maximum intensity just to the violet of the normal limit. If we assume that the intensity in a homogeneous field would be distributed in somewhat the same way and consider the spectrum emitted in a gas as the sum of spectra emitted in fields distributed around the mean, the maximum intensity will lie just to the violet of this new calculated limit, since most of the atoms are exposed to fields close to the mean. The extension of the

[31] Balasse, C. R. **184,** 1002, (1927).

[32] Paschen Berl. Sitzb. 1926, p. 135.

[33] Mohler, Phys. Rev. **31,** 187 (1928).

[34] Reported at the meeting of the American Physical Society, Feb. 25, 1928 in New York City.

spectrum to the red of this limit is to be expected since at any given time a large number of atoms are exposed to fields larger than the mean.
The excellence of the agreement in all but the first case must be regarded as fortuitous as, even if the theory were strictly applicable, the fields are not estimated with accuracy. In addition the observed continuous spectrum is the sum of the true continuous spectrum and of the line spectrum which is spread over the whole background by the unhomogeneous field. In the reproductions of Paschen's and of Herzberg's plates and on a plate Dr. Mohler was kind enough to send us the continuous spectrum overlaps a number of apparently sharp lines of considerable intensity so that it is unlikely that very much of the continuous spectrum arises from the spreading out of the lines. We must also consider the question of probability: it is not necessarily true that the maximum probability of transition occurs for energies very close to the displaced limit; it may be that this probability is a maximum for some frequency greater than this. Even for frequencies to the violet of the normal limit and neglecting the effect of the field on changing the selection principle we cannot expect the observed variation of intensity with wave-length to conform to that predicted by the theory for an unperturbed atom,[35] for, because the atoms are exposed to varying fields, a given frequency is not associated with any given kinetic energy of the free electron but may be emitted or absorbed in various types of transitions giving the same energy. These difficulties must be introduced for all emission spectra and all absorption spectra not associated with the normal state of the atom, except as we observe them in nebulae or other rarified sources of enormous volume, since in a laboratory source a large ion concentration is necessary to obtain observable intensity.

Experiments are under way to investigate the continuous spectrum of helium in a tube in which measurements of electron concentration can be made. It is also planned to investigate the absorption spectrum of rarified caesium vapor in a homogeneous electric field, so that a direct check on the theory can be obtained.

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PRINCETON, N. J.,
March 6, 1928.

[35] Kramers, Phil. Mag. **46,** 386 (1923); Oppenheimer, Camb. Phil. Soc. **23,** 422 (1926); Zeits. f. Phys. **41,** 268 (1927).