Second Series *June, 1928* *Vol. 31, No. 6*

THE

PHYSICAL REVIEW

NEW PRECISION IN COSMIC RAY MEASUREMENTS; YIELDING EXTENSION OF SPECTRUM AND INDICATIONS OF BANDS

By R. A. Millikan and G. H. Cameron

Abstract

Method of measurement of capacities of the order of one cm.—A method is described which leads to greatly increased precision in the measurement of capacities of the order of one electrostatic unit.

More penetrating cosmic rays than we have previously found are indicated by a new absorption curve obtained in Gem Lake (9080 ft.) and Lake Arrowhead (5125 ft.) with much greater precision than hitherto possible.

Cosmic-ray spectrum.—The new curve affords definite evidence for the existence of bands in the spectrum of cosmic rays. The measurements indicate that the cosmic rays consist chiefly of two bands about three octaves apart of mean absorption coefficients 0.35 and 0.04 to 0.05 per meter of water.

The total energy of cosmic rays at the top of the atmosphere is found to be very nearly one-tenth that due to starlight and heat as computed from Seares' data.

I. Introductory

Until about a year ago we had been so occupied with the exploratory phase of the cosmic-ray work, that is, with testing on a given instrument for variations with altitude, with direction, with geographical position, with penetration through absorbing materials, etc., that we had had little time to devote to improving the precision of measurement, and our own former electroscope readings have not yet shown a consistency and a duplicability which was as good as we thought obtainable. Also with two different electroscopes which we have designated as No. 1 and No. 3, we have obtained under the same conditions 1.4 and 1.6 ions per cc per sec. respectively, as the value of the sea-level ionization due to the cosmic rays. Such differences, whatever their cause, are tolerable only in the initial phases of work on any given physical quantity.

Further, when the findings of different observers on any cosmic-ray constants are compared the differences are more striking than the agreements. At high altitudes Millikan and Bowen got a total discharge of an electroscope about one-fourth that computed from the curves of Kolhörster and of Hess; on Pike's Peak we find from our curves 5.2 ions per cc per sec. per atmosphere due to cosmic rays, while Swann publishes 0.75 ions per cc per sec. per atmosphere in the same spot; at sea-level Kolhörster and most observers following him still take 2 ions per cc per sec. as the cosmic-ray ionization in the atmosphere, while we publish the value 1.4. Such differences are quite like those found, for example, in the early determinations of e/m which showed fluctations of 100%, but they cannot long be permitted.

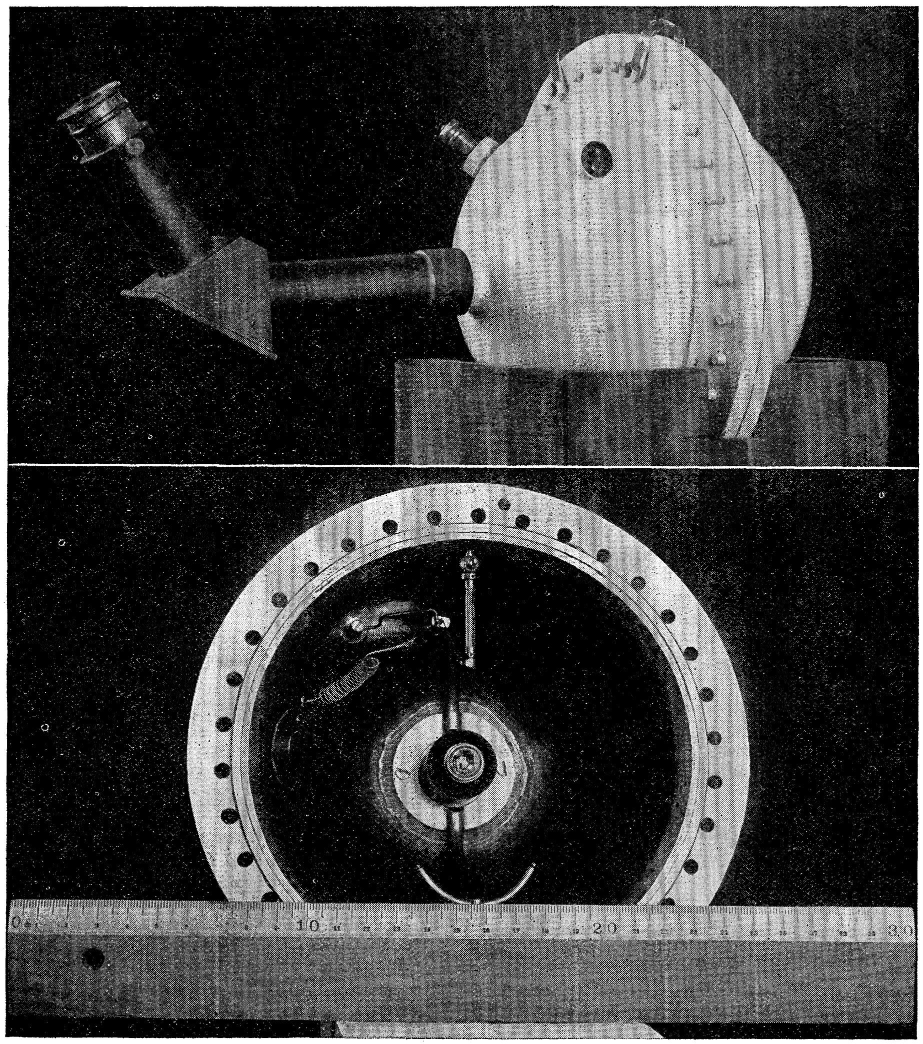

Fig. 1. Photographs of electroscope.

Upon our return from Bolivia in September, 1926, we at once set at work to endeavor to improve our instrumental technique with a view to initiating in our own work at least a period of something like precision measurements on cosmic-ray effects.

II. Precision in Cosmic-Ray Electroscopes

The uncertainties in our own preceding absolute determination of cosmic-ray ionization have been due (1) to lack of precision in the measurement of capacities of the order of one electrostatic unit, (2) to lack of electroscope sensibility and (3) to irregularities due to difficulties in eliminating "soaking-in" effects.

In the new electroscopes, designed primarily for under-water work at depths of many meters, we have used the spherical form as shown in Fig. 1. The volume is about one and a half liters, accurately 1.58 l; the wall-material is steel of thickness 0.6 mm. The heavy flanges, 7mm thick, on each hemisphere are bolted together by 34 bolts with the aid of which a tongue 2 mm high, 3 mm wide, and running entirely around the face of one flange is squeezed into a groove in the face of the other flange. At the bottom of this groove is a rubber washer. This renders the sphere completely air-tight, so that the pressure within it may be pumped up to many atmospheres, thus increasing largely the sensibility. The fibers are of quartz, platinized by cathode sputtering. They are attached as in our preceding electroscopes, save that the upper quartz supporting rod is quite small (2.1 cm long, 2 mm diam.) so as to reduce "soaking-in" effects. The charging is done by a little electromagnetic plunger operated by a battery current entering the coil through the two electrodes just visible inside the opening seen in the face of the sphere in Fig. 1. The device is sketched in Fig. 2.

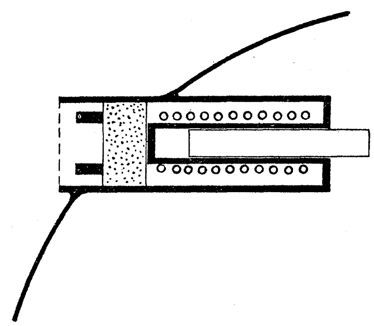

Fig. 2. Device for charging the electroscope.

The most important element in the design is found in the simple method used for measuring the capacity of the electroscope, a method capable of determining a capacity of the order of one electrostatic unit with an accuracy of about one part in a thousand. It is easily intelligible from Fig. 3. After the fibers have been mounted in the face of the hemisphere that carries their supports, there is screwed on in place of the other permanent hemisphere a third, dummy hemisphere just like the permanent one, save that it carries the cylindrical projection shown in cross-section in Fig. 3.

With this dummy hemisphere the steps in the process of finding the capacity of the fibers are then as follows: Two thin and perfectly straight steel wires are taken 1.06 mm in diameter, but one exactly 6 cm longer than the other. Each of these is ground slightly conical at one end and that end can be tightly pressed into a little conical hole in the minute brass piece to which the fibers are attached at the top, thus adding to the capacity of the fibers alone the capacity of this wire (or "cylindrical condenser") in one or the other of the positions shown in Fig. 3. After the careful centering of the wire, both cylindrical condensers may be closed by an accurately fitted cap of an inner diameter precisely the same as that of the cylinders. A constant source of gamma rays is placed a couple of meters away and, when neither of the wires is in place, the time t_1 of discharge through, say, twenty scale divisions is noted. The short steel wire is then inserted and centered as shown in the right-hand diagram of Fig. 3, the cap put on, and the time t_2 of discharge

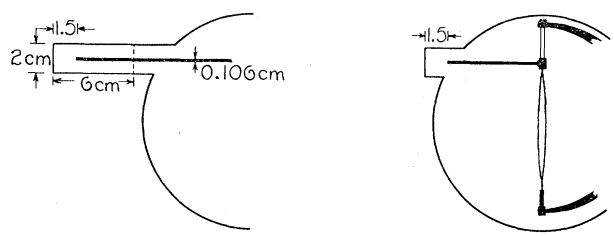

Fig. 3. To illustrate method of measuring capacities.

through the same range is again taken. Then the long steel wire and the long cylinder each having 6 cm of additional length, replace the shorter ones. After the centering of the wire, the cap of the long cylinder is put on, thus assuring that the "end effect" is precisely the same with the long wire and cylinder as with the short ones. Then the time t_3 of discharge through the same twenty divisions is again taken. The capacity x of the fibers alone is then easily seen to be given by

$$x = ct_1/(t_3 - t_2)$$

in which c, the capacity of the added 6 cm of cylindrical condenser is given by

$$c = \frac{1}{2}\frac{l}{\log_e(a/b)} = \frac{1}{2}\frac{6}{\log_e(2/0.106)} = 1.019 \text{ e.s.u.}$$

It is obvious not only that this method eliminates completely the end effect, but that, if dimensions are so chosen that t_1 is about equal to $(t_3 - t_2)$ the accuracy may be made very high. The volume of air in the added cylindrical condenser has been neglected in the foregoing analysis since it is small, but it can of course be easily taken into account if desired, the time t_3 being multiplied by the ratio of the volume without the long cylinder to the volume with it, for the sake of reducing all three t's to conditions of equal volumes.

III. New Depth-Ionization Curve

With the foregoing electroscope pumped up to a pressure of about 8 atmospheres and therefore having a sensibility, for these very hard rays, about eight times as great as we had used in preceding under-water work, the depth-ionization curve shown in Fig. 4 was plotted from readings taken in Arrowhead Lake (140 ft. deep) and in Gem Lake (225 ft. deep) and tabulated in Table I.

Although the elevations of Arrowhead and Gem Lakes are 5,100 ft. and 9,080 ft., respectively, and the latter 250 miles north of the former, as in the case of all our former under-water work, when the rates of discharge are

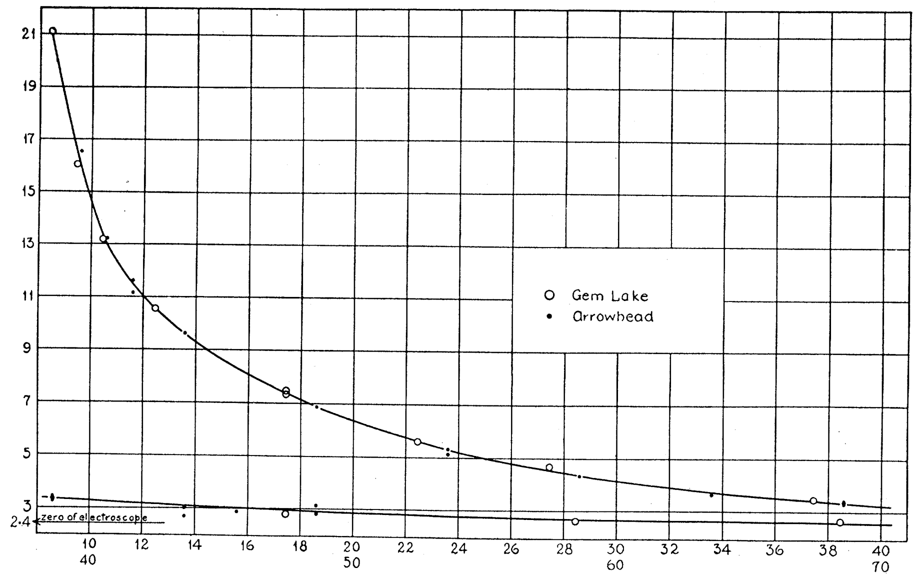

Fig. 4. Depth-ionization curve. Ordinates: Cosmic-ray ionization in ions per cc per second. Abscissas: Depth beneath top of atmosphere in equivalent meters of water. Lower curve: Continuation of upper between 40 and 70 meters.

plotted against depth in equivalent meters of water beneath the top surface of the atmosphere, *the readings all fall upon a smooth ionization-depth curve* (see Fig. 4). Also in spite of the fact that the reading at any particular depth

TABLE I. *Ionization in ions per cc per sec. at various depths below surface of the atmosphere. Readings taken in Arrowhead and Gem Lakes.*

Depth below surface of atmosphere	Arrowhead	Gem	Depth below surface of atmosphere	Arrowhead	Gem
8.45		21.1	27.45		4.68
9.45		16.07	28.6	4.37	
9.6	16.52		33.6	3.63	
10.45		13.19	37.45		3.47
10.6	13.22		38.6	3.36, 3.38, 3.38	
11.6	11.17, 11.65		43.6	2.68, 2.98	
12.45		10.56	45.6	2.87	
13.6	9.64		47.45		2.79
17.45		7.35, 7.48	48.6	2.83, 3.12	
22.45		5.59	57.45		2.62
23.6	5.33, 5.14		67.45		2.63

is now about eight times greater than in our preceding work (since for these very hard rays ionization is at least roughly proportional to pressure) yet the readings are now seen to show much less spread in ions per cc per sec.

than heretofore, so that the new curve shows that the precision has been multiplied fully tenfold, probably more. Not a single reading was discarded during the taking of this particular series of thirty readings. It will be seen that there are but two points in the thirty that now depart from the curves by as much as four-tenths of an ion, while all save three or four are closer than one-tenth of an ion. The resolving power of this curve is therefore very much greater (say tenfold) than that of any we have heretofore published, and we can now draw more definite conclusions about the spectral distribution of cosmic rays than have heretofore been possible.

IV. Spectral Distribution of Cosmic Rays

The first point to be noted about the present curve is that it goes quite definitely down to a depth of at least 58 meters (190 ft.) before the readings cease to decrease measurably with further immersion. There is, indeed, one erratic reading at 43.6 m but there are eight other consistent ones between 38 m and 48 m, so that the position of the curve in this region is fixed with certainty, while the two consistent readings between 58 m and 68 m show that the curve is measurably decreasing down to at least 58 m. *This means at once that our increased sensibility has brought to light very much harder rays than we had observed before when the readings failed to decrease measurably below 25 m.* How much harder may be determined as follows:

The curve is now sufficiently certain so that it is possible with the aid of the Gold table[1] to analyze it meter by meter. This table gives the value of I/I_0 in which I is the observed ionization at depth h below the top of the atmosphere and I_0 the value at the top, for each value of μh. Inserting two values of h (depth beneath top of atmosphere) a meter apart and the observed

Table II. *Portion of the Gold table used in this work.*

μh	I/I_0	μh	I/I_0	μh	I/I_0
0.01	0.94967	0.45	0.35623	1.5	0.07310
0.02	0.91311	0.50	0.32665	1.6	0.06380
0.03	0.88168	0.55	0.30010	1.7	0.05578
0.04	0.85354	0.60	0.27618	1.8	0.04882
0.05	0.82784	0.65	0.25456	1.9	0.04279
0.06	0.80404	0.70	0.23495	2.0	0.03754
0.07	0.78184	0.75	0.21711	2.2	0.02898
0.08	0.76096	0.80	0.20085	2.4	0.02246
0.09	0.74125	0.85	0.18599	2.5	0.01982
0.10	0.72255	0.90	0.17241	2.6	0.01746
0.15	0.64104	0.95	0.15994	2.8	0.01360
0.20	0.57420	1.0	0.14850	3.0	0.01064
0.25	0.51773	1.1	0.12828	3.5	0.00580
0.30	0.46912	1.2	0.11110	4.0	0.00320
0.35	0.42671	1.3	0.09644	4.5	0.00179
0.40	0.38937	1.4	0.08389	5.0	0.00099
				6.0	0.00032

values of I_1 and I_2 we obtain at once the corresponding μ. For convenience we reproduce in Table II the part of the Gold table which we use in this

[1] Gold, Proc. Roy. Soc. **A82,** 43 (1909).

work, (i.e., for $n = 2^1$). Table III gives the series of values of μ thus obtained for successive values of h from 9.5 m down to 60 m, the lower steps, however, being 5 m or 10 m apart instead of 1 or 2.

TABLE III. *Absorption coefficients at various depths, in meters of water, below top of atmosphere.*

Depth in meters of water beneath top of atmosphere	Absorption coefficient μ	Depth in meters of water beneath top of atmosphere	Absorption coefficient μ
8.45– 9.5	0.22	15–20	0.065
9.5 –10.5	0.20	20–30	0.057
10.5 –11.5	0.11	30–40	
11.5 –12.5	0.09	40–50	0.05
12.5 –15	0.07	50–60	

Before attempting to interpret this table it is important to recall that both at Muir Lake (11,800 ft) and at Lake Miguilla (15,000 ft) we obtained the value[2] $\mu = 0.25$ and also that it was necessary, in order to fit Millikan and Bowen's sounding-balloon measurements,[3] which reached more than nine-tenths of the way to the top of the atmosphere, to assume that up to that height the mean absorption coefficient remained about 0.25, in other words that the cosmic rays consisted of a band of frequencies an octave or more wide which, however, did not extend in appreciable intensity into the region between that for which $\mu = 0.25$ and that of gamma rays.

The present measurements enable us to make still further inferences about the character of this band. The fact that an absorption coefficient of between 0.2 and 0.3 appears to hold from the top of the atmosphere down to a depth of 10.5 m and that in going down 1 meter more the mean μ drops suddenly to 0.11, while in going but 3 meters more it drops to 0.07, obviously means that the cosmic rays are not at all continuously distributed between $\mu = 0.2$ and $\mu = 0.07$. Indeed we find that between 30 m and 60 m one single coefficient viz. $\mu = 0.05$ will reproduce our curve within the limits of observational error, so that at the lower end the cosmic rays are more or less monochromatic. *The sudden change in μ at a depth of about 11 m means that at that point a band of long wave-length, or of relatively large absorption coefficient, is dropping out and one of much shorter wave-length is left to cause the bulk of the ionization.* The suddenness of this change is very illuminating. In the preceding paper we showed that if we assumed a mean absorption coefficient 0.25 to be valid from about 9 m up to the top of the atmosphere we accounted perfectly not only for our own data but also for the Millikan-Bowen results at very high altitudes. In precisely the same way we can calculate these ionizations using now as a starting point the intensity observed at 9 m in this new set of observations. If we now subtract from this curve the theoretical values for $\mu = 0.05$, having the intensity necessary to fit the lower end of the curve (from 30 m down), the differences in the range 9 m to 2 m are fitted to a very rough first approximation by $\mu = 0.35$. But the sum of the

[2] Millikan and Cameron, Phys. Rev. **31**, 163 (1928).
[3] Millikan and Bowen, Phys. Rev. **27**, 353 (1926).

two curves for $\mu = 0.05$ and $\mu = 0.35$ rises above the observed curve in the range between 9 m and 15 m. In other words, this combination of $\mu = 0.05$ and $\mu = 0.35$, while fitting the observed curve well at the lower end (30 m to 60 m) and roughly at the upper end, does not yield so sharp a change of μ in the neighborhood of 10.5 m as is shown in Table I, and yet this is the region in which our observational data are especially reliable. The foregoing sudden change, then, combined with the approximate fitting of the two ends by $\mu = 0.05$ and $\mu = 0.05 + \mu = 0.35$, respectively, requires that the cosmic rays consist chiefly of two bands of wave-lengths even farther apart than correspond to $\mu = 0.05$ and $\mu = 0.35$. No single coefficient much lower than $\mu = 0.05$ will, however, fit the lower part of the curve. We are forced, then, to build up this part of the curve out of components whose joint effect is about that of $\mu = 0.05$, but which contain at least one strong radiation of smaller value of μ than 0.05, for this will tend to sharpen up the "knee" at $h = 10.5$. The net result of this method of inspection and trial is that if we divide the radiation reaching 30 m depth so that two-thirds of it corresponds to $\mu = 0.04$ and one-third to $\mu = 0.08$ we find that we can again fit the lower part of the curve (20 m to 60 m) perfectly. For depths less than 20 m the sum of these two ionization curves is of course much less than the observed ionization. But if we now give the residual ionization at 9 m to $\mu = 0.35$ and from this starting point complete this curve and then add the three curves, we can build up the observed curve everywhere to within about the limits of its experimental uncertainty. Table IV shows how perfectly this particular distribution of frequencies and intensities fits the observed readings. It will be seen that it is only from 9 m up to 2 m that the "sum" departs appreciably from the figures in the last column, and this is the region in which we have no observational data except the reading obtained by Millikan and Bowen in their highest balloon flight which reached an altitude corresponding to about 1 m and had a mean height of about 2 m. If this reading were 30% too high because of (a) observational uncertainty (it might reach nearly this value), or (b) radioactive deposit on the electroscope walls, or (c) soft radiations reaching down a meter or so from the top of the atmosphere, there would be no discrepancies at all between the last two columns. In other words, the departures are not now serious, and they are in the right direction, for while the balloon data may be too high they can hardly be too low.

There is obviously no altogether unique solution to such a curve as that shown in Fig. 4. On the other hand, *every solution must have the approximate characteristics here given.* The sharp "knee" and the characteristics of the curves at the two ends cannot be obtained save by two frequency bands which have their main strength at about $\mu = 0.04$ to 0.05 and $\mu = 0.30$ to 0.40. This conclusion is of course of much importance for the determination of the character of the nuclear changes which give rise to the cosmic rays. It needs checking by new determinations of the ionization-depth curve—determinations which we hope soon to carry out with electroscopes of still greater sensibility—but *the present measurements at least furnish indications that the cosmic rays consist chiefly of three bands for which the mean absorp-*

TABLE IV. *Comparison of the experimental curve with a theoretical curve compounded of three wave-lengths.*

Depth (meters of water)	$\mu=0.04$	0.08	0.35	Theoretical Curve (Sum)	Experimental Curve
70	0.15	0.02		0.17	0.16
60	0.24	0.03		0.27	0.24
50	0.40	0.09		0.49	0.44
40	0.69	0.23		0.92	0.85
30	1.20	0.60		1.80	1.79
20	2.16	1.70	0.10	3.96	3.95
15	2.98	2.96	0.37	6.31	6.24
12	3.64	4.21	1.21	9.06	8.60
10	4.20	5.35	2.65	12.20	12.20
9	4.52	6.07	4.20	14.79	16.05
7	5.27	7.86	9.64	22.77	30.7
5	6.19	10.4	23.9	40.5	61.0
3	7.49	14.1	63.3	84.9	127.5
2	8.21	16.7	107.3	132.2	192.0

tion coefficients are approximately 0.35, 0.08 and 0.04. Computing as we have heretofore done by the Compton formula our shortest wave-length is now 0.00008A, and the equivalent generating potential approximately 150,000,000 volts.

V. Comparison of the Total Energy in Cosmic Rays and in Starlight

The absolute value of the cosmic-ray ionization at sea-level (10.33 m) inside the present electroscope can be read off with great precision from the foregoing curve. It will be seen to be 11.25 ions per cc per sec. To reduce this to atmospheric pressure we shall provisionally divide through by the pressure, namely 8. This gives 1.4 ions, the value previously found with the use of two other electroscopes. This procedure involves a small error since we have obtained evidence that even for these very hard rays there is not strict proportionality between pressure and ionization. We shall make a later report upon the amount of this uncertainty as experimentally determined. But for the present purpose we shall make no significant error if we take 1.4 as the sea-level ionization at atmospheric pressure, and with this as a starting point compute with the aid of the absorption coefficients given in Table III the ionization as far as this curve goes, namely up to 9.5 m. From there up we have already found by observations up to 15,000 feet in Bolivia that the apparent absorption coefficient does not rise above 0.25, a value which also fits the Millikan and Bowen sounding balloon data up to practically the top of the atmosphere. The ionization at all heights in the atmosphere above 8.5 m can then be obtained with the aid of Table II and $\mu=0.25$, the values thus found being practically identical with those given in a preceding paper.[2] By then plotting the successive values of these ionizations against depth in equivalent meters of water beneath the surface from 0 to ∞, it is easy to make a graphical integration and determine the total ionization due to cosmic rays in a column of water of 1 sq. cm area, and extending from the top of the atmosphere down to say 50 m or 60 m, where the ionization has practically ceased. For such an integration it is important to remember that the number of ions per cc of water at sea-level is 1.4×850, the last factor being the ratio of the densities of water and air. This factor must of course be inserted at all depths to obtain the total number of ions per sq. cm column. The result is 12,850,000 ions.

Taking the average ionizing potential of the external shell of electrons in nitrogen as 15 volts (if K electrons are ejected K radiation is produced which spends itself in ionizing the outer shell) we find that the average energy required to produce an ion is $(15/300)\times 4.774\times 10^{-10} = 2.4\times 10^{-11}$ergs. The total energy content, then, of the cosmic rays coming into the earth per sq. cm is $12.8\times 10^{6}\times 2.4\times 10^{-11} = 3.07\times 10^{-4}$ergs.

To compare this with the heat and light coming into the earth from the stars we have the solar constant at the top of the atmosphere $=1.94$ cal/sq. cm/min. $=1.35\times 10^{6}$ ergs/sq. cm/sec. The sun has a magnitude of -26.72 and hence has the effect of $10^{27\cdot 72\times 0.4}$ or 1.22×10^{11} first magnitude stars. According to Seares,[4] starlight is equivalent to 1092 first mag. stars. Hence the intensity of starlight is $1092/4\pi$ first mag. stars per sq. radian. Call this I. Then the total energy E flowing per sq. cm into the top of the atmosphere in starlight is given by

$$E = I\int_0^{2\pi} d\phi \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = \pi I \therefore E = \frac{1092}{4}\times\frac{1.35\times 10^{6}}{1.22\times 10^{11}} = 3.02\times 10^{-3} \text{ ergs}$$

i.e. the *total energy coming into the earth in the form of cosmic rays is very close to one-tenth the total energy of starlight.*

The whole of this cosmic-ray work has been done with the aid of funds provided by the Carnegie Corporation of New York and administered by the Carnegie Institution of Washington.

NORMAN BRIDGE LABORATORY OF PHYSICS,
CALIFORNIA INSTITUTE,
March 19, 1928.

[4] Seares, Astroph. Jour. **62**, 373 (1925).

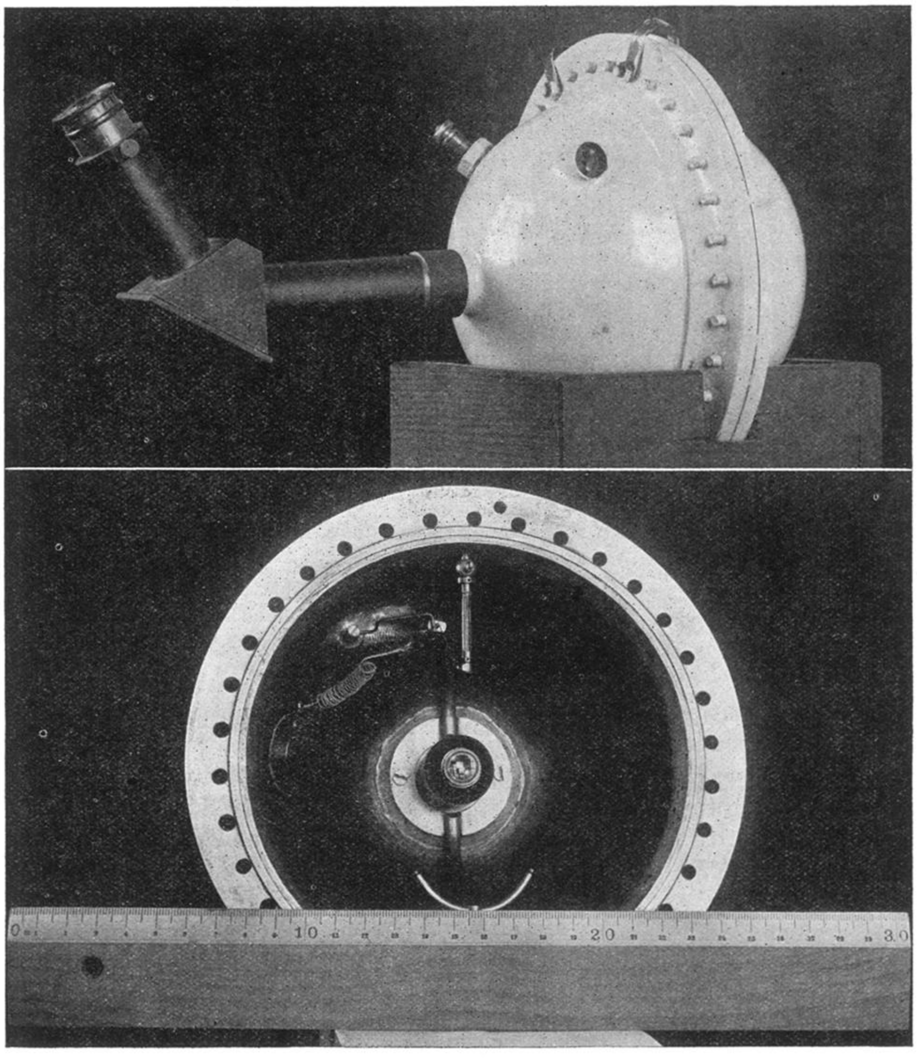

Fig. 1. Photographs of electroscope.