IONIZATION IN POSITIVE ION SHEATHS

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ABSTRACT

It was found that the positive ion current to a plane auxiliary collector placed in a neon discharge had about twice the value expected from the equations of Langmuir and Mott-Smith. This increase must be due to an ionization within the sheath surrounding the electrode or to an emission of electrons from the surface of the electrode.

Four different possible causes of the increase are analyzed and relations between the voltage drop V, total current to the collector i, and sheath thickness x are obtained. Comparison with data shows that the increase in i is probably caused by the ionization of the metastable atoms within the sheath by radiation from the discharge. The relations; $V=A(Bx^{4/3}+Cx^{8/3})$, and $i=i_0+I_0x^2/2$ hold, and check fairly well with the three experimental curves. Considerations of atomic energy states of the metastable atoms show that this ionization would be most marked in the noble gases, and almost nonexistant in mercury vapor, which was the gas investigated by Langmuir and Mott-Smith.

INTRODUCTION

Observations, which will shortly be published in greater detail, have been made by one of us (W.U.) at the Philips Laboratory at Eindhoeven, Holland, on the positive ion current to a plane auxiliary collector placed in a neon discharge. These show values for the current of about twice the values expected from the equations of Langmuir and Mott-Smith.[3]

A possible explanation of this increase is that there is a slight amount of ionization within the sheath; an ionization not small enough, however, to be entirely neglected, as Langmuir and Mott-Smith have assumed.

Since the field applied is sufficient to exclude practically all electrons from the sheath, electrons cannot produce this ionization, but it can be brought about in several other ways.

In Fig. I, x is the sheath thickness, V the potential drop across it, i the total positive ion current reaching the collector, and i_0 the constant current entering the sheath from the discharge. I_z/e is the number of ions formed per second per cubic centimeter at a point z centimeters from the sheath boundary, and V_z, E_z and i_z are values of potential drop, electric intensity and current density respectively at the same point.

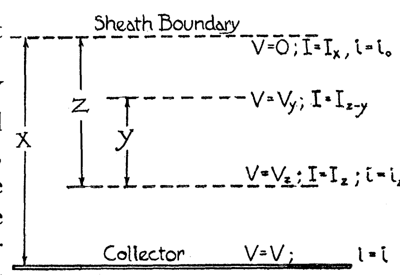

Fig. 1.

Case 1. The radiation from the arc might ionize the neutral atoms within the sheath. In this case, since the radiation density is constant through-

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[3] Langmuir and Mott-Smith, G. E. Rev. **27**, 449 (1924).

out the sheath, the number of ions formed per second would be the same throughout the sheath, and, $I_z = I_0$.

Case II. On the other hand, there are neutral atoms in metastable states streaming in from the discharge in a constant flow. Since there are no metastable atoms formed in the sheath, the law of diffusion requires that the concentration of these atoms at any point be proportional to the distance of the point from the collector plate.

These metastable atoms would have a much greater likelihood of being ionized than the neutral atoms, and in this case the ionization per second per cubic centimeter at a point will be proportional to the point's distance from the collector, and; $I_z = I_0(x-z)$

Case III. A less likely case would be that the metastable atoms, in colliding with each other, would release enough energy to ionize one of them. In this case the ionization would be proportional to the square of the distance, and; $I_z = I_0(x-z)^2$.

Case IV. The metastable atoms will also strike the conductor, and give up their energy to an electron, which will ionize the gas in its path across the sheath. Or a photoelectron might be given off by the collector plate. In either case the number of ions formed will probably be small, since the sheath thickness is of the order of a mean free path; and the probability of photoelectric emission from most metals is less than of emission from the metastable atoms in the sheath. In both cases, however, the ionization at any point will be some function of the electric intensity at that point, $I_z = F(E_z)$,

Very likely all four of these effects are present, but Case II will probably be preponderant. At any rate, by solving each of these cases separately and determining which fits the data best, it can be determined which cause of ionization is the important one.

General Considerations

In any of these cases there will be a relationship between n_z, the concentration of ions at point z, V_z and z. The current density i_z will be $n_z eu$, where u is the average drift velocity of the ions at point z. Since the sheath thickness is of the order of one mean free path, we can consider the ion as falling through free space, and;

$$u = [2e(V_z - V_y)/M]^{1/2}$$

where $(V_z - V_y)$ is the potential difference between the point under consideration and the point where the ion was formed. For the constant current density coming from outside the sheath, we have $(V_z - V_y) = V_z$, and;

$$n_z'e = i_0/u_0 = (M/2e)^{1/2} i_0/V_z^{1/2} \tag{1}$$

The number of ions n'' due to ionization within the sheath will be such that the current at z due to them will be proportional to the number formed per second between the sheath boundary and z:

$$i_z'' = n''eu'' = \int_0^z I_{z-y}\, dy \tag{2}$$

The average drift velocity of these ions, u'', will be the sum of all their velocities divided by their total number;

$$u''=\frac{(2e/M)^{1/2}\int_0^z (V_z-V_y)^{1/2}I_{z-y}\,dy}{\int_0^z I_{z-y}\,dy} \tag{3}$$

Then the total ion concentration at any point will be;

$$n_z e=\frac{i_0}{u_0}+\frac{i_z''}{u''}$$

and from Poisson's Equation;

$$\frac{d^2V}{dz^2}=4\pi\left(\frac{M}{2e}\right)^{1/2}\left[\frac{i_0}{V_z^{1/2}}+\frac{\left(\int_0^z I_{z-y}\,dy\right)^2}{\int_0^z (V_z-V_y)^{1/2}I_{z-y}\,dy}\right] \tag{4}$$

However, we are interested in conditions over the whole sheath. When we let $z=x$, we get two equations; one for the total current flowing in the conductor;

$$i=i_0+\int_0^x I_{x-y}\,dy \tag{5}$$

giving;

$$di/dx=I_x \tag{6}$$

and one relating V and x. Letting $4\pi(M/2e)^{1/2}=A^{1/2}$, then;

$$\frac{d^2V}{dx^2}=A^{1/2}\left[\frac{i_0}{V^{1/2}}+\frac{\left(\int_0^x I_{x-y}\,dy\right)^2}{\int_0^x (V-V_y)^{1/2}I_{x-y}dy}\right] \tag{7}$$

This last equation cannot, in general, be solved exactly. However, since we shall find that the second term will be much smaller than the first, it being of the nature of a correction to the first term, an approximate solution will be obtained by setting d^2V/dx^2 equal to each term separately, and then letting the final solution be the sum of the two partial solutions.

The solution of the first term will be the same for all four cases, and will be the Langmuir and Mott-Smith equation;

$$V'=A^{1/3}\left(\frac{9\,i_0}{4}\right)^{2/3}x^{4/3} \tag{8}$$

The solution of the second portion will depend on our choice of I_z.

Case I. Here I_z is constant, and the numerator of the fraction;

$$\left(\int_0^x I_{x-y}dy\right)^2=I_0^2x^2.$$

The denominator is undetermined until V is known. However V is probably a function of x to some power between one and two. If we let $V_y=V(x-y)/x$, *i.e.*, a linear function of y, then;

$$I_0\int_0^x (V-V_y)^{1/2}dy=2I_0xV^{1/2}/3$$

and if we let $V_y = V(x-y)^2/x^2$, then;

$$I_0 \int_0^x (V-V_y)^{1/2} dy = \pi I_0 x V^{1/2}/4$$

In other words, if V is proportional to some power of x between one and two, the denominator becomes $I_0 x (V^{1/2}/a)$, where a is a factor between $3/2$ and $4/\pi$. Then the equation becomes;

$$\frac{d^2V''}{dx^2} = A^{1/2} \frac{aI_0}{2} \frac{x}{V''^{1/2}}$$

This has a solution; $V'' = A^{1/3}(aI_0/2)^{2/3} x^2$

and the complete approximate solution will be in this case;

$$V = V' + V'' = A^{1/3}[(9i_0/4)^{2/3} x^{4/3} + (aI_0/2)^{2/3} x^2] \tag{9}$$

and;

$$i = i_0 + I_0 x \tag{10}$$

These two relations will completely determine the relationship between i, V and x.

Case II. In this case $I_{x-y} = I_0 y$. By a process similar to that in case I, we obtain;

$$\frac{d^2V''}{dx^2} = A^{1/2} \frac{b}{4} I_0 \frac{x^2}{V''^{1/2}}$$

where b is between 15/4 and 3. This equation has a solution which, added to Eq.(8), gives as complete solution;

$$V = A^{1/3}\left[\left(\frac{9i_0}{4}\right)^{2/3} x^{4/3} + \left(\frac{9bI_0}{160}\right)^{2/3} x^{8/3}\right] \tag{11}$$

and the other equation will be;

$$i = i_0 + I_0 x^2/2 \tag{12}$$

Case III. This has as solutions;

$$V = A^{1/3}\left[\left(\frac{9i_0}{4}\right)^{2/3} x^{4/3} + \left(\frac{cI_0}{210}\right)^{2/3} x^{10/3}\right] \tag{13}$$

where c is between 105/16 and $16/\pi$. The other equation is;

$$i = i_0 + I_0 x^3/3$$

Case IV. If we take $F(E) = I_0 E = I_0\, dV/dy$, then;

$$\frac{d^2V''}{dx^2} = 3A^{1/2} I_0 V''^{1/2}$$

giving as complete solution;

$$V = A^{1/3}\left[\left(\frac{9i_0}{4}\right)^{2/3} x^{4/3} + A^{2/3}\left(\frac{I_0}{8}\right)^2 x^4\right] \tag{14}$$

If $F(E)$ were taken as proportional to any higher power of E, V'' would be proportional to an even higher power of x. So, in general, for $I = F(E)$; $V = Bx^{4/3} + Cx^n$ where n is probably higher than 4. Since; $E = B'x^{1/3} + Cx^{n-1}$ then I will vary as the cube or higher power of x.

Experimental Confirmation

Thus four different assumptions as to the cause of ionization in the sheath give four different relations between i, V and x, if one of these relations fits the experimental data with tolerable exactness, then it may be considered as the preponderant cause of the ionization.

The experimental data give the relationship between i and x as shown in Fig. (2). The slope di/dx for different values of x is a straight line going through the origin. From Eq. (6) it is seen that $di/dx = I_x$, so I_x in this case must equal Ax. Then, letting $A = I_0$,

$$I_z = I_0(x - z)$$

which corresponds to Case II.

Solving the data by least squares to fit Eq. (12), i_0 is found to be 693,000 e.s.u., and I_0 to be 23,160 e.s.u per mm².

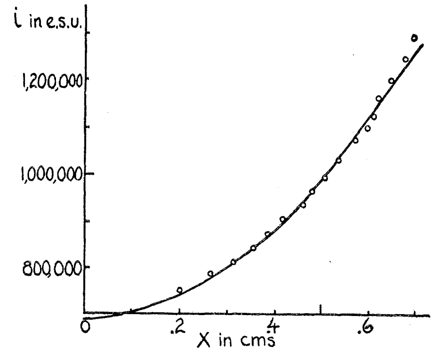

Fig. 2. Variation of positive ion current with sheath thickness. Observations of Uyterhoeven; U5-H, Neon; 450 ma.; pressure 0.02 mm; nickel collector.

Putting these values into Eq. (11) a curve is obtained for V in terms of x, which is of the form; $V = C(623x^{4/3} + 57.8x^{8/3})$.

By comparing this with the experimental curve, Fig. (3), the average value of C is found to be .000075. But C is $(8\pi^2 M/e)^{1/3}$, which, for neon, is about .0001, giving a further check.

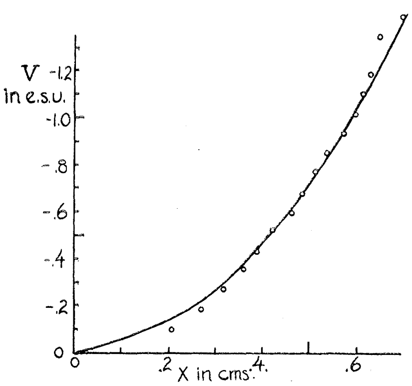

Fig. 3. Variation of potential drop with sheath thickness. Data taken from Uyterhoeven's observations and used in conjunction with Eq. (11).

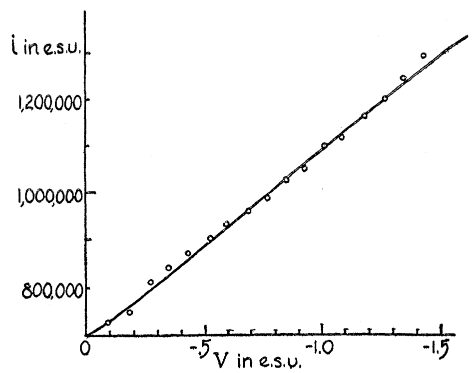

Fig. 4. Variation of positive ion current with potential drop.

Fig. (4) is the experimental curve for i in terms of V. In all these figures the smooth curve represents the curve calculated from Eqs. (11) and (12), and the small circles represent the experimental data.

Conclusion

There are several objections which might be raised against these deductions, but which seem to be satisfactorily answered. The question might be brought forward as to whether there are enough metastable atoms flowing into the sheath to cause this current. However comparison with an unpublished estimate of the number of metastable atoms diffusing through the sheath, made by W. de Groot, indicates that the number of metastable atoms ionized according to Case II above is about one-tenth of the total number passing into the sheath.

Since the thickness of the sheath is of the order of magnitude of a mean free path of an atom, the validity of the diffusion equation might be questioned. Inasmuch as the mean free path of an excited atom is considerably less than that of the unexcited atom, and as the sheath is but a small portion of the total region across which diffusion is occurring, for lengths of time much larger than a mean free time the diffusion equation will hold with reasonable accuracy.

The approximation made in the integration of Eq. (11), that of solving the two parts of the equation separately, is equivalent to assuming that the total current due to ionization in the sheath is less than i_0. But since i_0 is about 700,000 e.s.u. and I_0 only 23,000 e.s.u./mm^2, it is only for values of x greater than about 6 mm or of V greater than 300 volts that an appreciable deviation from the solution might be expected.

Langmuir and Mott-Smith, in their data on mercury[4] found no such variation from their equation, or, rather, they found a very slight variation (see his Fig. 4; the portion AB of the curve only deviates slightly from the parallelism to the V axis required by his equation). This means that there are less metastable Hg atoms ionized by radiation from a mercury discharge than there are Ne metastable atoms ionized by radiation from a neon discharge. The ionization potential of most of the metastable atoms of both Hg and Ne is about 5 volts. But since there is very little radiation from a mercury discharge of that high a frequency, while a considerable portion of the energy of radiation from a neon discharge has this energy or higher, it is to be expected that the phenomenon will be exhibited in neon to a much more marked degree than in mercury.

This markedly greater ionization of metastable atoms is to be expected in helium and argon also, and may serve to explain why the behavior of the electric discharge through the noble gases has seemed anomalous.

Thus the data available at present seem to show that the ionization within the sheath is due to the ionization of metastable atoms by radiation from the discharge, and that therefore the relations between i, V and x follow Eqs. (11) and (12).

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[4] Langmuir and Mott-Smith, G. E. Rev., Vol. 27, 538 (1924).