THE THEORY OF THE HERSCHEL-QUINCKE TUBE

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Abstract

The well known simple explanation of the Herschel-Quincke interference tube is inadequate. It is herein shown that the ratio of the transmitted to incident acoustic energy is

$$[4(\sin(\alpha_3+\alpha_2)/2)(\cos(\alpha_3-\alpha_2)/2)]\{[1-2\cos(\alpha_3+\alpha_2)+\cos(\alpha_3-\alpha_2)]^2+4\sin^2(\alpha_3+\alpha_2)\}^{-1/2}$$

wherein α_2 and α_3 are the phase changes over the two branches of the tube. This ratio is zero not only when $\alpha_3-\alpha_2=(2n+1)\pi$, as formerly explained, but also when $\alpha_3+\alpha_2=2n\pi$, provided $\alpha_3-\alpha_2\neq 2n_1\pi$, where n and n_1 are independent integers.

Comparison of the new theory with experiment.—The transmission in an interference tube having branches 10.0 and 14.3 cm in length was studied and showed satisfactory agreement with the new theory. Practically zero transmission was found at frequencies of 1100 and 3300 d.v. and this is in accord with the simple theory of difference in path, as well as the revision here presented. But there was also serious interference at 1000, 2000, 3000 and 4000 d.v. and this fact is in accord only with the revised complete theory.

The Herschel-Quincke tube[1] consists of a branched conduit as shown in Fig. 1 constructed with constant total cross-sectional area. Assume a wave travelling to the right. If the difference in path is $(2n+1)\lambda/2$, n an integer and λ the wave-length, then the excess pressure at the distant junction point is zero and the wave therefore is not transmitted to the right but is reflected back. This is the accepted simple explanation. That it is not sufficient was found by the writer in an experimental test shown by the indicated points in Fig. 1. The frequencies A and B are those given by $(2n+1)\lambda/2$. Clearly the actual frequencies at which there is interference are much more numerous. The purpose of this article is to outline the theory, and to make a comparison of theory with experiment.

Theory. Assume a plane wave incident in conduit 1 at junction 123. There are seven possible waves to be considered; two each in conduits 1, 2, and 3, and one in conduit 4. The waves to the right in Fig. 1 will each have the displacement amplitude indicated by A and a subscript designating the conduit. Waves to the left will be similarly designated by B. Excess pressure, p, and displacement, ξ, may be represented as follows:

$$p=\rho a\omega A e^{i\omega t}, \quad \xi=Ae^{i\omega t}, \tag{1}$$

wherein ρ and a are respectively the density of and velocity in the medium, ω is 2π times the frequency and $e^{i\omega t}$ represents the simple harmonic nature of the oscillations. In the waves B, it will be necessary to use the positive sign for pressures and negative signs for displacements, for a displacement

[1] Herschel, Phil. Mag. **3**, 401 (1833); Pogg. Annal. **31**, 245 (1834); Quincke, Pogg. Annal. **128**, 177 (1866).

to the right is positive. At the junction 123, the continuity of pressures and displacement may be written:

$$\left.\begin{array}{ll}\text{Pressures:} & (A_1+B_1)=(A_2+B_2)=(A_3+B_3)\\ \text{Displacements:} & S_1(A_1-B_1)=S_2(A_2+A_3-B_2-B_3)\end{array}\right\} \tag{2}$$

wherein S indicates the area and the subscript the conduit.

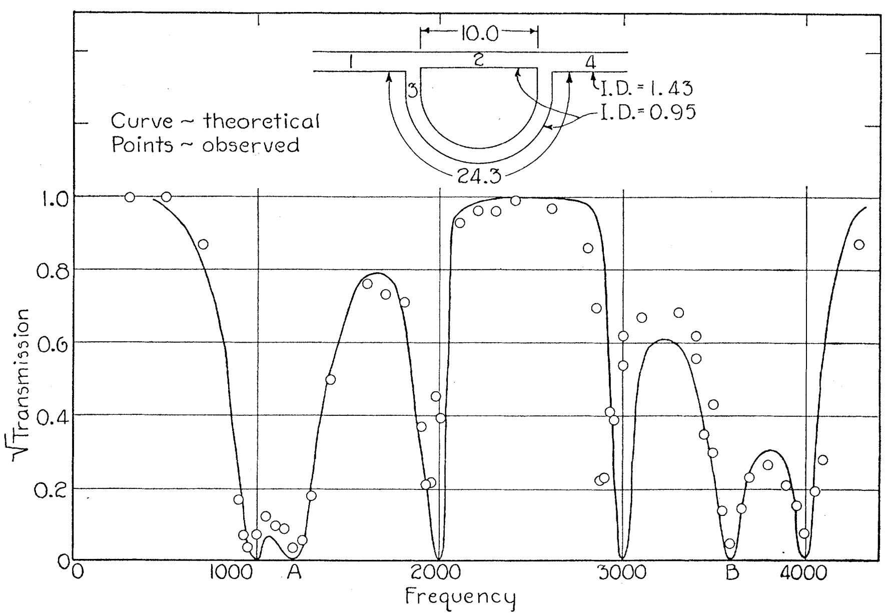

Fig. 1. Diagram of Herschel-Quincke tube and the relation between percentage transmission and the square root of the frequency.

If α_2 is the change of phase in conduit 2 and α_3 in conduit 3, then corresponding equations for junction 234 are:

$$\left.\begin{array}{l}A_2e^{-i\alpha_2}+B_2e^{i\alpha_2}=A_4=A_3e^{-i\alpha_3}+B_3e^{i\alpha_3}\\ S_2(A_2e^{-i\alpha_2}-B_2e^{i\alpha_2}+A_3e^{-i\alpha_3}-B_3e^{i\alpha_3})=S_1A_4\end{array}\right\} \tag{3}$$

There are in Eqs. (2) and (3) seven unknowns and six independent linear equations. By solving, the ratio between A_4 and A_1 can be found as follows:

$$|A_4/A_1| = \{[4\sin(\alpha_3+\alpha_2)/2][\cos(\alpha_3-\alpha_2)/2]\}\{[1-2\cos(\alpha_3+\alpha_2)+\cos(\alpha_3-\alpha_2)]^2+4\sin^2(\alpha_3+\alpha_2)\}^{-1/2} \tag{4}$$

This is the square root of the transmission or of the ratio of the transmitted to the incident intensity.

Inspection of Eq. (4) shows that: transmission $=0$, if $\alpha_2-\alpha_3=(2n+1)\pi$, $n=0, 1, 2, 3$, etc. or if $\alpha_2+\alpha_3=2n\pi$, $n=0, 1, 2, 3$, etc., provided in each case that $\alpha_2-\alpha_3\neq 2n_1\pi$, $n_1=0, 1, 2, 3$, etc. The former condition is well known, but the latter is new.

Measurement of transmission. The apparatus was the same as that described[2] in the measurement of acoustic impedance. The Herschel-Quincke tube was inserted and removed from the conduit and the ratio of the two intensities taken as the transmission of the interference tube.

Comparison of theory and experiment. Fig. 1 is self-explanatory. The frequencies A and B have the well known conditions $\alpha_3-\alpha_2=(2n+1)\pi$. The other minima, at 1000, 2000, 3000 and 4000 d.v., are more numerous and comply with the condition $\alpha_3+\alpha_2=2n\pi$, $\alpha_3-\alpha_2\neq 2n_1\pi$. In fact the curve is drawn by computation from Eq. (4) assuming $a=34300$ cm/sec. The observed values of the minima are not in exact agreement with the theoretical curve and for several reasons. In the first place the junction points are not made with sufficient care to conform to the ideal condition of constancy of total area. The junction should be a Y-tube. Moreover, no consideration is given to viscosity in the theory. In view of these considerations experiments and theory agree very satisfactorily.

The question arises as to why the incorrectness of the theory of the tube, known for almost a century, was not noticed. Apparently the explanation in terms of the difference between α_3 and α_2 was so simple that no one observed that the waves in 2 and 3 travelling to the left do not experience constancy of area on arrival at junction 123 and consequently suffer a reflection. In fact, in general, the waves travel about the circuit in a very complex manner, impossible to follow without resorting to the use of equations.

The conclusion is that the theory of the Herschel-Quincke tube as herein derived is in satisfactory agreement with experiment.

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[2] G. W. Stewart, Phys. Rev. **28**, 1038 (1926).