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THE BEHAVIOR OF GLASS AS A DIELECTRIC IN ALTERNATING CURRENT CIRCUITS:
I. RELATION OF POWER FACTOR AND DIELECTRIC CONSTANT TO CONDUCTIVITY

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ABSTRACT

Power factor, dielectric constant at 1000 cycles and apparent resistivity for six glasses of varying composition, both lead glasses and borosilicates with and without lead, all at 20°C. The values of the apparent resistivity at 20°C were found by linear extrapolation, by means of the equation $\rho = e^{k/T}$, from values of the apparent resistivity calculated from measurements of the resistance of small rods at temperatures ranging from 100°C to 500°C. In two cases studied the power factor of strained samples was about 50 percent greater than after annealing. No explanation is offered. For glasses of relatively similar composition both the power factor and the dielectric constant are found to increase with increasing conductivity. The results indicate a close relation between the phenomena of absorption and of ionic conductivity and are shown to be qualitatively in agreement with the theory of Joffé that the phenomena of absorption are due to a counter-electromotive force of electrolytic polarization. Possible alternative molecular explanations are suggested.

OF THE six glasses chosen for investigation numbers 1 and 3 were lead glasses; numbers 2, 4 and 5 were borosilicates without any heavy metals and number 6 was a lead borosilicate. These glasses were chosen because they are known to vary widely in power factor and conductivity. Measurements were made: (1) of the power factor of each glass at 1000 cycles per second, at approximately 20°C; (2) of the dielectric constant at 1000 cycles per second and approximately 20°C; (3) of the apparent resistivity at temperatures ranging from 100°C to 500°C.

Methods of Measurement

The values of the power factor were calculated from measurements of the equivalent series resistance and capacitance made with a capacitance bridge elsewhere described.[1,2] A potential difference in the neighborhood of 100 volts was applied to the bridge terminals. Power was furnished by an electron tube generator of the Hartley circuit type with a 50-watt tube, placed in another room to avoid stray coupling. The test condensers were beakers with mercury electrodes. The beakers were thoroughly washed, rinsed with distilled water and dried in an oven. The mercury was also cleaned and dried. The test beaker was floated in a second beaker containing the mercury which formed the outer electrode and held in place by a wire frame dipping into the mercury and grounded. To maintain constant temperature the entire condenser was placed in an electric furnace with heating coil grounded.

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[1] H. J. MacLeod, Phys. Rev. **21,** 53 (1923).

[2] L. S. McDowell, Phys. Rev. **23,** 507 (1924).

The frequency was measured by a frequency meter consisting of a large four-step inductor and a variable oil condenser, calibrated at intervals during the experiments by comparison with tuning forks. This meter was placed about ten feet from the oscillator and the resonance frequency found by telephones connected unilaterally. Usually measurements were made at several frequencies and the value of the power factor at 1000 cycles obtained by interpolation.

For the measurements of dielectric constant, the glass was in the form of plates, 16 cm square, ground plane parallel to a thickness of about 2 mm. The average thickness was determined by means of a Randolph and Stickney dial gauge reading to a thousandth of a millimeter. Readings were taken at thirteen points fixed by a pattern which divided the area of the upper electrode into equal parts. The dried plates were floated on mercury which formed the lower electrode; the upper electrode was an amalgamated copper ring about 13 cm in diameter, 7 millimeters in height, filled with mercury. The capacitance was measured at 1000 cycles, by means of a series inductance bridge with a third arm, since the capacitance bridge used in the power factor measurements was not at the time available. No correction was made for the effect of lack of uniformity of the field at the edge.

The measurements of resistivity at temperatures above 100°C were made upon samples in the form of rods from 2 to 6 cm in length, and about 6 mm in diameter. Contact was made by Dumet wires pushed into the ends of the heated rod. The specimen to be measured was suspended vertically in a cylindrical furnace by the Dumet wires which extended through Pyrex tubes embedded in asbestos stoppers, at opposite ends of the furnace, and were soldered outside to small copper rods. The temperature was measured by a chromel-alumel thermocouple in conjunction with a Wolff potentiometer. The resistance was calculated from the applied electromotive force, usually 50 volts, and the current. For the current measurements a Leeds and Northrup galvanometer of high current sensitivity was used with suitable shunts so calibrated as to permit the calculation of the resistance directly from the applied electromotive force and the galvanometer deflection. To test for errors due to leakage through the furnace a trial run was made with the specimen removed. The error from this cause was found to be less than one percent and at the higher temperatures amounted to only one or two parts in a thousand. To minimize errors in the resistivity, calculated in ohms per cubic centimeter, due to the shape of the specimen and the consequent impossibility of measuring the dimensions with accuracy, at least two samples of different length were measured for each glass, and the effective length found by comparison.

As a check upon the measurements at high temperatures, the resistivity in the case of glasses numbers 1 and 2, at room temperature, was calculated from values of the resistance of the thoroughly annealed glass plates used in the measurement of the dielectric constant. The same mercury electrodes were used with the addition of a guard ring connected to the grounded side of the galvanometer, to prevent the measurement of the current leaking over the surface. The applied electromotive force was 270 volts. The ratio of the current through the glass to the current through a resistance of one megohm was found by direct substitution, with the aid of an Ayrton shunt to alter the sensitivity of the galvanometer. For the glasses of higher resistivity the galvanometer was not sufficiently sensitive to permit the use of this method.

The power factor was calculated from the relation

$$\text{Power factor} = \cos\theta = \sin\phi$$

where ϕ is the phase difference. For small values of ϕ, $\sin\phi \approx \tan\phi$ and $\tan\phi = R\omega C$, where R and C are respectively the equivalent series resistance and capacitance of the condenser.

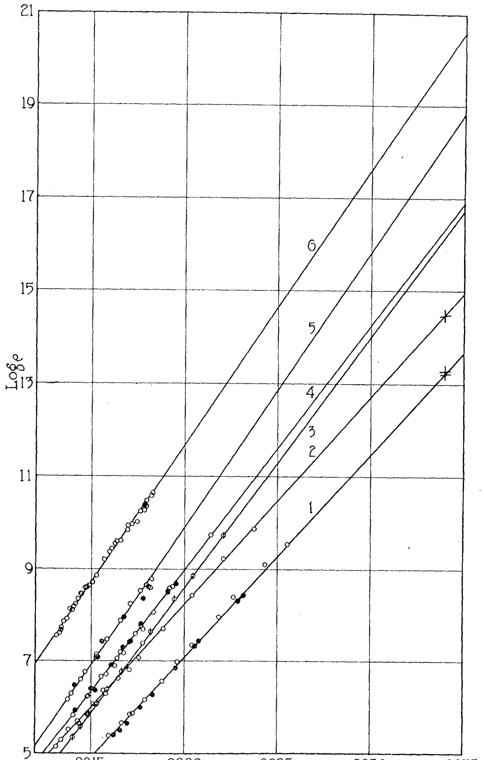

Fig. 1. Logarithm of the resistivity as a function of the reciprocal of the absolute temperature.

Results

Fig. 1 shows the values of the logarithm of the resistivity plotted as a function of the reciprocal of the absolute temperature. Within the errors of observation the points lie on a straight line in accordance with the relation

$$\rho = e^{k/T}$$

first found by Rasch and Hinrichsen[3] to hold true for various insulators in which the conductivity is now considered to be electrolytic. The values measured at room temperature for glasses 1 and 2 are indicated by the crosses. The agreement between these measured values and the extrapolated values is considered to be sufficiently good to warrant the use of the extrapolated values for the other glasses.

Table I gives the values of apparent resistivity at 20°C, of the tangent of the phase difference at 1000 cycles, and of the dielectric constant at 1000 cycles, for the six glasses divided into two groups according to composition.

TABLE I

Values of the resistivity, of the tangent of the phase angle and of the dielectric constant for six glasses

No.	$\rho(20°C)$	Tanϕ(1000 cycles)	K(1000 cycles)
a. Lead glasses			
1	2.3×10^{13}	0.0132	7.5
3	2.0×10^{15}	0.0031	6.8
6	8.1×10^{19}	0.0022	5.3
b. Borosilicates without lead or any heavy metal			
2	3.1×10^{14}	0.0082	5.0
4	2.5×10^{15}	0.0039	4.7
5	3.5×10^{16}	0.00077	4.1

In each group the order of decreasing conductivity is also the order of decreasing power factor and decreasing dielectric constant.

It should be noted that the values of resistivity given are those of apparent initial resistivity, taken a brief, unmeasured interval of time after the application of the potential difference, which is doubtless considerably less than the final conductivity. Race,[4] for example, obtains for "PYREX" laboratory glass (glass No. 2) a value of the resistivity of 8.6×10^{16} ohms per cubic centimeter, 5000 minutes after the application of the potential difference and of 4.9×10^{15} ohms per cubic centimeter, one minute after the application of the potential difference.

To show to what extent the power loss might be supposed to be due to conduction, the power factor was calculated from the relation

$$1/R_p = P/E^2 = \omega C \tan \phi$$

where R_p, the resistance of the condenser, was calculated from the measured resistivity and the dimensions of the condenser. For glass No. 1 the power factor so calculated was 1.05×10^{-5}, for No. 6, 5.41×10^{-10}. These values are negligibly small in comparison with the measured power factors, and if the final resistivity had been used would have been still smaller.

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[3] Rasch and Hinrichsen, Zeits. f. Elektrotech. **14,** 41 (1908).

[4] H. H. Race, Thesis, Cornell University, 1927 (to be published).

Since the dielectric constant was measured at a frequency of one thousand cycles it might be supposed that the correlation between the dielectric constant and the conductivity is to be accounted for entirely by the increase in the apparent dielectric constant due to the absorbed charge. However, the rate of change of the dielectric constant with frequency does not seem to be sufficiently great for the glasses of large conductivity and power factor to alter the order as given in Table I. For instance, for glass No. 1 at the frequency of one million cycles per second, the value of the dielectric constant was found to be 7.3.

THE EFFECT OF ANNEALING

To test the effect of annealing, the power factor of two glasses was measured before and after annealing. The results are given below:

Glass	Condition	Power Factor
2	Strained	0.0113
	Annealed	0.0074
4	Strained	0.0055
	Annealed	0.0034

No explanation is offered for the large decrease in the power factor produced by annealing. Despite the rather large increase due to strain, the values of the power factor for different specimens of glass of fairly uniform composition, when well annealed, agree reasonably well. This is illustrated by the following values for "PYREX" laboratory glass measured, at intervals extending over more than a year, and upon specimens from different melts:

Specimen	Form	Power Factor
a	Beaker	0.0085
b	"	0.0089
c	Flat dish, 6mm thick	0.0083
d	" " " "	0.0090
e	" " " "	0.0084
f	" " " "	0.0078
g	" " " "	0.0074
h	Plate 2mm "	0.0084
i	Test tube	0.0073

Specimens *f* and *g* were used in the experiments to determine the effect of annealing and were probably annealed with unusual thoroughness.

INTERPRETATION OF RESULTS

The results indicate a close relation between the phenomena of absorption and ionic conductivity. They seem therefore less favorable to the theories of Maxwell and Wagner[5] which explain absorption as due to a variation in the ratio of the dielectric constant to the resistivity for the different layers or constituents which compose the dielectric and more favorable to an explanation on the basis of the behavior of the individual molecules and ions. Qualitatively the results are in agreement with the theory of Joffé, as set forth by Sinjelnikoff and Walther,[6] that the phenomena of absorption are

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[5] K. W. Wagner, Archiv. f. Elektrotech. **2**, 371 (1914).

[6] Sinjelnikoff and Walther, Zeits. f. Physik, **40**, 786 (1927).

due to a counter electromotive force of electrolytic polarization, P, such that the final current, $i=(V-P)/R$, where R, the true resistance, is constant and of magnitude sufficient to account for the power loss in an alternating field, if the effect of space charge be taken into account. As alternative explanations, it seems possible to suppose either that the ions, because of assymmetry, possess an electric moment and are slowly rotated as they drift, or that there are present some dipole molecules the orientation of which is retarded because of the influence of the fields of force of the ions, due to the close packing of molecules and ions in a solid. If either the theory of Joffé or the first of the suggested alternative molecular explanations is accepted, however, it would seem difficult to account for the complete absence of absorption phenomena in liquids.

A second paper, in preparation, will discuss the effect of frequency and temperature upon the power loss.

The measurements of resistance were made in the Physics Laboratory of Cornell University and the measurements of dielectric constant in the Capacitance Laboratory of the Bureau of Standards during the year 1925–26, while the senior author was a guest in the laboratory. It is a pleasure to acknowledge the many courtesies extended in both laboratories and in particular the friendly interest in the investigation shown by both Dr. H. L. Curtis and Dr. Chester Snow of the Bureau of Standards.

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