COLLISIONS BETWEEN ELECTRONS AND GAS MOLECULES

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ABSTRACT

Using a simple method in which positive ion sheaths serve as perfect grids we have obtained data on the *probability of various types of collisions* occurring between Hg, H_2, N_2, He, Ne and A molecules and electrons of moderate velocity, i.e., 30–250 volts. A theory has been developed and the *mean angular deflection for elastic and inelastic collisions between electrons and gas molecules has been calculated.* Evidence on the mechanism of ionization is presented. The current densities of the various Maxwellian groups of electrons and the average temperatures of these groups are calculated from the data and are shown to be in good agreement with data obtained by us using Langmuir and Mott Smith's probe wire method. The *current density of positive ions* has been measured by both plane and probe wire collectors and the *space potential* both measured and calculated. The evidence in support of negative anode drops has been pointed out and the ions flowing to the anode measured separately by means of a perforated collector. The *maximum number of positive ions produced by an electron* with a given velocity before losing its ionizing power has been measured and shown to be independent of current density and pressure being dependent only on the velocity of the electron and the nature of the gas.

The *mean free path of high velocity electrons* (i.e., above the ionizing potential) for various types of collision can be evaluated from the probability data. The mean free path for inelastic impact has been calculated separately and independently of the reflection of electrons from the collectors. The *small angular scattering of electrons* is discussed in detail. Phenomena, similar to Langmuir's high speed electrons in mercury vapor, have been observed and the curves showing the energy transfer between primary electrons in H_2 and A are given. The *resonance and ionization potentials* observed in the work are also tabulated.

MOST quantitative experiments on the ionization of gases involve the use of a stream of electrons moving with homogeneous and definitely known velocity. For this purpose the electrons from a hot cathode are usually accelerated by a positively charged grid or perforated plate into a region which is made as nearly field-free as practicable. To avoid space charge effects and the resultant uncertainties in potential distribution it is necessary to work with very low currents, which usually involve electrometer measurements. Furthermore, the grids must be of very fine mesh or the perforations in the plate of small diameter to avoid transverse velocity components in the electron stream and to make the effective potential of the grid the same as its actual potential. The customary methods are thus not adapted to the study of certain phenomena which appear only when rather large current densities are employed. With the high intensity of ionization accompanying larger currents it becomes possible, however, to use quite different methods for studying ionization phenomena.

In apparatus of ordinary size and with potentials above 40 volts, electron currents of a few milliamperes cause such intense ionization and high electrical conductivity that the interior of a gas at low pressure necessarily

becomes nearly field free. The potential drop between cathode and anode is then largely confined to a thin layer or sheath covering the surface of the cathode in which there is a positive ion space charge. This sheath has a well defined outer edge beyond which the field of the cathode does not extend. The sheath thus acts as an ideal grid for accelerating the primary electrons from a cathode into a field-free region.

The presence of such a homogeneous beam of electrons may be detected by a small disk-shaped collector placed in the electron stream so that the electrons move normal to its surfaces. When the collector is at a negative potential with respect to the ionized gas it is covered by a positive ion sheath. It collects all the positive ions that reach the edge of the sheath and all the electrons (except those reflected at the collector surface) which have velocity components normal to the surface sufficient to carry them against the retarding field in the sheath.

Thus if the collector is at a potential below that of the cathode it will receive only positive ions, but as soon as its potential is higher than that of the cathode, the primary electrons, which have not lost momentum by collision or otherwise, will be able to pass through the sheath, and reach the collector. The current-voltage characteristic of the collector thus shows a break at the cathode potential. If, however, the collector is turned so that its plane is no longer perpendicular to the direction of propagation of the electrons, the collector potential will have to be raised above that of the cathode by an amount ΔE before primary electrons can be collected. This voltage displacement of the break ΔE is thus a measure of the transverse energy of the electrons, viz., the energy component parallel to the surface of the collector.

Apparatus and Method

Unless the dimensions of the collector are large compared to the thickness of the sheath, the sheath area will vary with the voltage, so that the interpretation of the current-voltage characteristic becomes more complicated. If the cathode is a straight filament the collector should be part of a cylindrical surface having the filament as axis in order that the primary electrons may impinge normally at all points of the collector surface. This suggests that the collector may advantageously be made a complete cylinder surrounding the filament and having a length equal to that of the filament. Such a tube is illustrated in Fig. 1 where F is the cathode filament, C is the coaxial cylinder which may be used to collect the primary electrons that reach it, A and B are end-plates which fit closely within the cylinder C and are provided at their centers with small holes through which the cathode filament passes.

In most cases one of the plates (A) is used as anode, its potential being sufficiently high to give strong ionization of the gas within the cylinder C. The electrons from the cathode are then accelerated in strictly radial directions within the cathode sheath, and at low gas pressures a fraction of these electrons reach the cylinder C with normal incidence after having traveled a definite distance through the gas.

The potential of the ionized gas is approximately that of the anode, and is, in fact, usually a couple of volts above it (negative anode drop). We shall see that at low pressures the mobility of the electrons is so high that the ionized gas constitutes a very satisfactory field-free region.

Any electrode which is at a potential below that of the ionized gas will be covered by a positive ion sheath. If C is a few volts negative (we shall take the negative end of the cathode as of zero potential) the primary electrons which move radially across the tube are retarded within the sheath on C, are brought to rest, and are driven out of the sheath again without having lost energy, and still moving in a radial direction. All the positive ions which diffuse to the edge of the sheath on C will be collected, and the positive ion

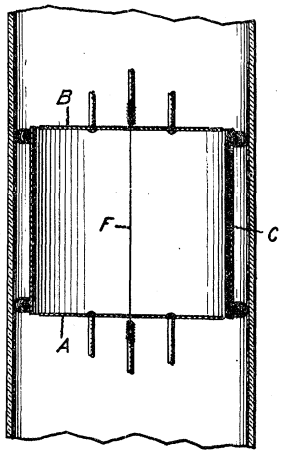

Fig. 1.

current is therefore independent[1,2] of the negative voltage on C. Illustrations of these constant currents may be seen in Figs. 2, 3, 4, etc., for negative voltages. In a similar way the plate B can be made negative and the positive ion current to this electrode measured. Thus by making corrections for the few ions collected by the filament (and by the anode), the total number of ions formed from a given number of primary electrons can be determined. One of the main objects of the investigations to be described in this paper has been to obtain quantitative data on the degree and nature of the ionization produced in a gas by a specified stream of primary electrons of known velocity. A tube of the type of Fig. 1 lends itself admirably to this purpose, not only because of its simple geometry, but also because of the fact that the ionized gas is completely enclosed[3] within metallic electrodes, any one of which may be used as a collector.

[1] Langmuir, Science **58,** 290 (1923).

[2] Langmuir, Gen. Electric Rev. **26,** 731 (1923).

[3] A short preliminary account of the methods of using a tube like that of Fig. 1 to study free paths and ionization phenomena in gases has been published by the present authors in Science **59,** 380 (1924).

If C is made slightly positive the retarding field in its sheath will no longer be able to stop the primary electrons, so that some of these will be collected, and a break in the current voltage curve occurs at the cathode potential (zero) (see Fig. 2). This current discontinuity affords a simple and accurate means of determining electron free paths. The current-voltage curve of the collector C at higher voltages gives data on the velocity distribution (and the distribution of directions of motion) of the various other groups of electrons. Comparisons between the curves obtained with C (Fig. 2) and with B (Fig. 3) as collectors are particularly useful in giving information regarding the directions of motion of the electrons. We can thus estimate the fraction of the primary electrons which make elastic collisions and we also find that practically none of these are scattered through angles as large as 90 degrees.

In a similar way, by analysis of such curves as those of Fig. 2, we find that a certain fraction of the electrons lose an energy corresponding to a resonance potential of the gas (6.7 volts in Fig. 2), but even these electrons are not deflected through large angles. The ionized gas also contains a large number of electrons which move in random directions with velocities distributed according to Maxwell's law.[4,5]

Particularly interesting results are obtainable with tubes like that of Fig. 1 by the use of longitudinal magnetic fields. With weak fields the electron paths are curved so that the electrons reach the cylinder with oblique incidence. The current-voltage curves obtained in this way give good data for measuring the number of electrons that make elastic collisions with gas molecules.

With strong magnetic fields the distribution of electrons and positive ions between two collectors C and B is entirely changed. This furnishes a valuable quantitative method of studying mobilities of electrons and ions. The intensity of ionization near the filament F, even at low pressures of gas, is then many times greater than at C so that relatively large positive ion currents flow to the cathode. This effect, however, is readily studied quantitatively by noting the increase in filament resistance caused by the heating effect of the positive ion bombardment.

Description of Typical Current-Voltage Characteristic Curves

The curves shown in Figs. 2, 3, and 4 were obtained with a tube like that of Fig. 1, in which the cylinder C was 3.1 cm in diameter and had an effective length of 3.8 cm (area 37.0 cm^2). The plates A and B were 3.0 cm in diameter (area 7.0 cm^2 each), and had holes 0.35 cm in diameter through which the filament (of 0.178 mm diameter) passed. To light the filament to a temperature at which 5 milliamperes of electrons were emitted required about 2.5 amperes and 3.3 volts. The electron emission was fairly uniform along the filament up to within about 5 mm of the ends of the filament and then

[4] Langmuir, Jour. Franklin Inst. **196,** 751 (1923).

[5] Langmuir and Mott-Smith, Jr. Gen. Electric Rev. **27,** 449, 538, 616, 762, 810 (1924). See also; Phys. Rev. **28,** 727, (1926) and Zeit. f. Physik, **46,** 271 (1927).

dropped off rapidly because of the cooling effect of the leads, which extended very close to the holes in the plates.

Curve A (Fig. 2) was obtained with mercury at a pressure of 25 baryes (51.5°C), while B corresponds to 102 baryes (75°C) of mercury vapor. The

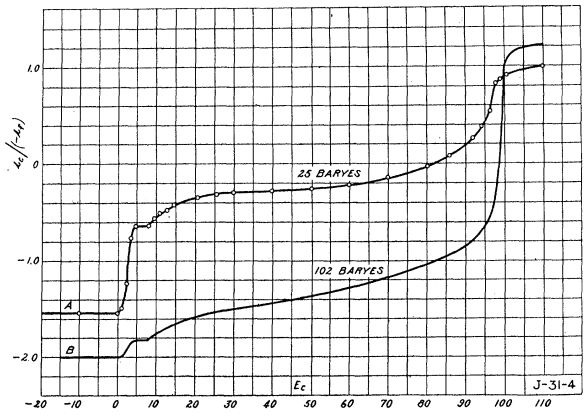

Fig. 2.

two plates A and B were connected together and were used as anode at 100 volts, while the potential of the cylinder C was varied and the current i_C was measured.[6] The filament F was heated by a storage battery and the current i_F of electrons emitted (and positive ions received) was measured

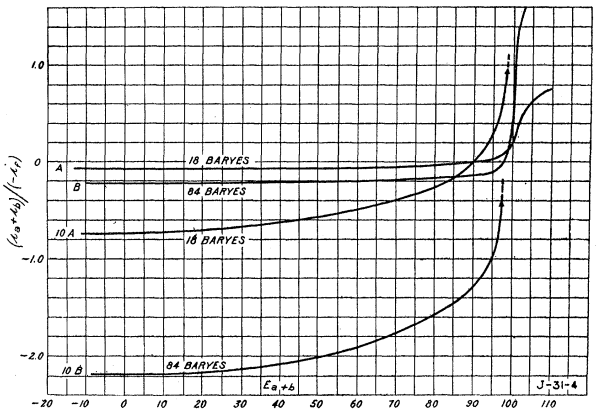

Fig. 3.

directly. This current was maintained at approximately -5 milliamperes. The ordinates in Fig. 2 represent the ratio $i_C/(-i_F)$ and the abscissas are

[6] We adopt the convention that the sign of the current to any electrode is that of the charged particles that it would have to emit in order to carry that current. Thus if F emits electrons or receives positive ions i_F is negative, while i_C is positive if C receives electrons and negative if it receives positive ions.

the collector potentials E_C. It was found in nearly all cases that for moderate variations in i_F the ratio $i_C/(-i_F)$ remained nearly constant.

The curves of Fig. 3 were obtained under similar conditions to those of Fig. 2, except that the functions of the two electrodes $(A+B)$ and C were interchanged. Thus the cylinder C was used as anode and was maintained at $+100$ volts, while the two plates A and B (connected together) were used as collector. The lower curves marked $10A$ and $10B$ are the same as the curves A and B except that the ordinates are multiplied by 10.

The data in Fig. 4 were obtained with argon at a pressure of 30 baryes. The plate A was used as anode at a potential of $+50$ volts, while B and C

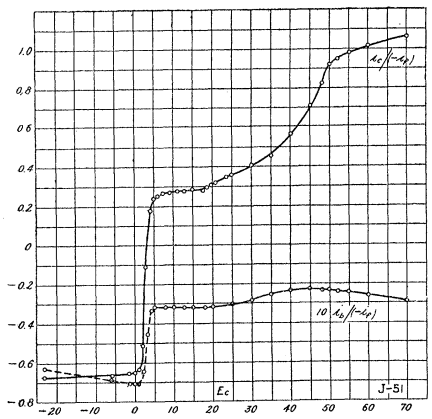

Fig. 4.

were used as collectors. The potential E_C of the cylinder was varied, but the plate B was maintained at a constant potential of -23 volts. The ordinates of the dotted curve representing the current i_B have been multiplied by 10.

Positive Ion Current Density I_P

The practically constant currents that are observed in Figs. 2, 3 and 4 when the collector is at negative potentials measure the rate at which ions diffuse to the edge of the sheath on the collector. These currents divided by the effective areas of the collectors give us a measure of the positive ion current densities I_P in the ionized gas. If the velocities of the positive ions are distributed according to Maxwell's laws corresponding to a temperature T_P, the number of ions per cm^3 can be calculated from the equation (reference 5, p. 455, Eq. (44))

$$n_p = 4.03 \times 10^{13} I_p (m_p/mT_p)^{1/2} \tag{1}$$

where m_P/m is the ratio of the mass of the ion and the electron, and I_P is to be measured in amps. per cm^2. From data which we shall discuss later we may assume T_P is roughly constant and equal to 3000°K, so we are able to estimate the intensity of ionization, i.e., the number of ions per cc, from measurements of the positive ion current densities.

In case the thickness of the positive ion sheath on a collector is appreciable, the effective area of the collector may be different from its actual area. Let us, therefore, determine the thickness of the sheaths in the examples we have cited. In general, the sheath thickness may be calculated from the space charge equations, and these values agree excellently with those obtained by direct visual observation of the dark space surrounding the collector (reference 5, pp. 545, 622). For electrodes having the small curvature of those we are now considering, the space charge equation for parallel planes is applicable:

$$I_p = 2.34 \times 10^{-6} (m/m_p)^{1/2} V^{3/2}/x^2 \qquad (2)$$

Here V represents the voltage drop through a sheath of thickness x (cm).

Table I contains a summary of the data on positive ion currents and sheath thickness from the curves of Figs. 2, 3, and 4. The first column refers to the number of the figure and the designation of the curve, and the second column gives the gas and its pressure in baryes.

TABLE I

Positive ion currents and sheath thicknesses. The currents i_F, I_B and I_C are in milliamperes. Data of Figs 2, 3 and 4.

1	2	3	4	5	6	7	8	9	10
Fig.	Gas	β_B	β_C	i_F	V	Sheath thickness x_B	x_C cm.	Positive current density I_B	I_C
2A	Hg 25	—	1.54	−5.4	105	—	0.19	—	0.256
2B	Hg 102	—	1.99	−6.3	105	—	0.16	—	0.375
3A	Hg 18	0.073*	—	−4.5	105	0.63	—	0.023	—
3B	Hg 84	0.218*	—	−5.5	105	0.33	—	0.086	—
4	A 30	0.071	0.65	−4.9	73	0.31	.022	0.054	0.112

* These values of β_B include the currents flowing to the electrode A which in this case was connected to B.

The sheath thicknesses in cm as calculated from Eq. (2) are given in columns 7 and 8. In the case of the data from Figs. 2 and 3 these thicknesses were calculated for an assumed collector potential of −5 volts, but for the data of Fig. 4, E_C was taken to be the same as E_B, which was kept at −23 volts. In columns 3 and 4 β_B and β_C are taken to represent the nearly constant values of $i_B/(-i_F)$ and $i_C/(-i_F)$ corresponding to the negative collector potentials.

The values of the positive ion current densities I_B and I_C in columns 9 and 10 were obtained by multiplying β_B or β_C by i_F (Column 5) and dividing by the areas of the sheaths on the collectors calculated from the values of x_B and x_C.

The intensity of ionization n_P as given by Eq. (1) may now be calculated from these values of I_P. For example, the value $I_C = 0.256$ ma. per cm^2 from the data of Fig. 2 corresponds to 1.1×10^{11} ions per cm^3 if we assume $T_P = 3000°$.

We see from the data of Fig. 4 in Table I that I_C is about twice I_B. This effect is caused by radial electric fields which accelerate the ions outwards. Since the rate of production of ions per unit volume near the filament is greater than near the cylinder, the concentration of ions must decrease as the distance from the filament increases. The electron concentrations must everywhere in the body of the gas be nearly equal to those of the positive ions, and as the electrons have high mobilities they will set up potential differences which are related to their concentrations according to the Boltzmann equation

$$n_1/n_2 = \epsilon^{Ve/kT} \tag{3}$$

where n_1 and n_2 are the electron concentrations in two regions, T is the temperature of the electrons, and V is the potential of the first region with respect to the second. The ratio e/k is equal to 11,600 degrees per volt. Thus if the positive ion space charge is neutralized principally by electrons having a temperature of 10,000° there will be a potential difference of 0.60 volt between two regions in which the positive ion concentrations differ in the ratio of 2 to 1. Since concentration differences of this order of magnitude are to be expected even at the lowest pressure[7] there must be radial potential differences of the order of one volt. The velocity of the ions in a radial direction will therefore be greater than in the direction of the cylinder axis, and this explains the fact that I_C is greater than I_B in Fig. 4.

Comparing the values of I_C from Fig. 2 (Table I) with those of I_B from Fig. 3, we find that the ratios in these cases range from 4 to 11. It must be noted, however, that the values of I_C were measured when $A+B$ was anode, while I_B was measured when C was anode. With C as anode a certain fraction (about 50 percent at 18 baryes) of the primary electrons were removed by this electrode, so that the total number of ions produced was less than if B or $A+B$ had been anode. However, allowing for this effect and for the differences in the pressures for the data of Figures 2 and 3, we find that the ion current densities at the cylinder are still 5 times (at 18 baryes) or 3 times at (at 84 baryes) greater than those at the plates. Experiments with mercury vapor with connections like those of Fig. 4 have given ratios I_C/I_B not exceeding 2, so we must conclude that the high values for this ratio in Figures 2 and 3 are an indication that the radial electric field is greater when C is anode than when an end plate is anode. This conclusion is in accord with our theory of the cause of the radial field, for when C is negative the primary electrons that are reflected back into the gas from the sheath on C produce ions near the surface of C. The removal of these primary electrons by making C anode lowers the concentration of ions near the cylinder relatively more than near the filament and thus increases the radial field.

[7] Direct measurements of the ion concentrations in a tube carrying an arc in mercury vapor at low pressures gave concentrations at the axis about 2 to 4 times greater than those at the wall. See loc. cit. (5) p. 769.

The currents flowing to negatively charged collectors are practically independent of the collector voltage according to the curves of Figures 2, 3, and 4. The dotted curve in Fig. 4, where the lower end of the curve shows a marked negative slope, corresponds to a case in which the current to the plate B is altered by changing the negative potential on the cylinder C. It was found, however, that if B and C were connected together so as to be at the same potential, the current to each electrode remained constant as the negative potential changed. The negative slope of $i_B/-i_F$ is very largely explained by considering the thicknesses of the sheaths on the electrodes. When the potential of C is changed from -23 to 0 volt (in Fig. 4), the sheath on C changes from 0.22 to 0.17 cm. This sheath on C partly covers the plate B and prevents positive ions from being collected near the edge of the plate. The effective diameter of the plate (actual diameter 3.1 cm) thus increases from 2.66 to 2.76 cm, as E_C changes from -23 to 0 volts.

The increase in area is 8 percent, while the positive ion current $(-i_B)$ increases about 14 percent. Thus the change in collecting area accounts for only part of the change.

At higher pressures, where the free paths of the electrons become small compared to the radius of the tube, the radial concentration gradient of ions is determined by diffusion.[8] With a given concentration of ions near the axis the rate of flow to the sheath on C should increase as the sheath increases in thickness because of the decreased distance through which diffusion occurs. With a given rate of production of ions near the axis, the concentration should therefore decrease as the sheath becomes thicker. This effect, together with the decreased collector area, probably explains the negative slope of the left hand part of the curve for i_B in Figure 4. It should be noted that the actual decrease in $i_B/(-i_F)$ from -23 to 0 volts is only 0.009, while the increase in $i_C/(-i_F)$ is 0.02.

Number of Ions Produced by Each Electron (β)

Since the ionized gas (Fig. 1) is completely surrounded by metallic electrodes at each of which the positive ion current density can be measured, we can determine the total number β of ions produced by each electron emitted from the cathode, assuming no recombination.

Let us express by β_A, β_B, etc., the ratio of the positive ion current at any electrode to the current i_F that flows from the filament. Then β_F gives the fraction of the current from F which is carried by the positive ions that strike the filament and $1-\beta_F$ is the fraction carried by electrons. If no recombination of ions and electrons occurs within the ionized gas the total ionization β per electron emitted is evidently

$$\beta=(\beta_A+\beta_B+\beta_C+\beta_F)/(1-\beta_F) \tag{4}$$

[8] Schottky and Issendorff, Zeits. f. Physik **31**, 163 (1925) have developed the mathematical theory of the diffusion of electrons and ions subject to the condition that the concentrations of the ions and the electrons shall everywhere be equal.

In experiments like that which gave Fig. 4 we measure β_B and β_C. The value of β_A corresponds to the positive ion current flowing to the anode. If, as is usually taken for granted, the anode were positively charged with respect to the space near it, we should place $\beta_A = 0$ since the ions would be repelled from the anode. But in general, where the anode is of large area and especially where the gas is strongly ionized by high velocity primary electrons from a hot cathode, the anode will be at a negative potential with respect to the ionized gas. Direct measurements of the space potential by small collectors near the anode have shown these negative anode drops (reference 5, page 767). Compton and Eckart,[9] following a suggestion of one of us, have demonstrated their existence in low voltage arcs at higher pressures.

A. F. Dittmer has suggested the use of plane collectors provided with one or more small round holes (of diameter small compared to the sheath on the collector) back of which is a Faraday cage or a second plate. By this arrangement the positive ions and electrons that pass through the holes can be separated. Measurements recently made by us with such a perforated collector have shown that it receives just as many positive ions when it is anode as when it is at a large negative potential.

We may therefore conclude that we should place $\beta_A = \beta_B$. The only remaining unknown quantity in Eq. 4 is β_F. This may be calculated from space charge equations with the degree of accuracy with which I_P in the neighborhood of the cathode may be estimated. Since β_F is usually very small, this method is often permissible. In further experiments now in progress β_F is being measured by the changes in resistance and in the electron emission of the filament due to the temperature rise that results from the positive ion bombardment.

If the positive ion current to F is limited by the rate at which positive ions reach the sheath surrounding the filament we may calculate β_F by the space charge equation

$$i_F\beta_F = 14.68\times10^{-6}L(m/m_p)^{1/2}V^{3/2}/r\beta_0^2 \tag{5}$$

where L is the length and r the radius of the filament, V is the voltage drop through the sheath of radius a, and β_0 is a function of a/r for which tables are available.[10] As we do not know a we cannot use this equation directly to obtain β_F, but since the outside surface of the sheath is the effective collecting area we may use the relation

$$i_F\beta_F = 2\pi aLI_p \tag{6}$$

and can then change Eq. (5) to the form

$$(a/r)\beta_0^2 = 2.34\times10^{-6}(m/m_p)^{1/2}V^{3/2}/(r^2I_p). \tag{7}$$

[9] K. T. Compton and Carl Eckart, Nature **114,** 51 (1924); Phys. Rev. **25,** 139 (1925). See also; Compton, Turner and McCurdy, Phys. Rev. **24,** 577 (1924).

[10] Langmuir and Blodgett, Phys. Rev. **22,** 347 (1923).

All the quantities of the right hand member are known. Thus by preparing a curve giving $(a/r)\beta_0^2$ as a function of a/r from Table III of paper in ref. 10 we may determine a and then by (6) can find β_F.

For the case of the experiment that gave Fig. 4 let us place (see Table I) $I_P = 1.12 \times 10^{-4}$ amps. per cm.[2] Putting $V = 73$, $(m/m_P)^{1/2} = 1/272$ and $r = 0.0089$ cm we find by (7) that $(a/r)\beta_0^2$ is 605, which corresponds to $a/r = 12$. Thus a, the radius of the sheath on the cathode, is 0.107 cm. If we take I_P near the axis of the tube to be twice as great, viz., 2.24×10^{-4}, we find $a/r = 9.4$ and $a = 0.084$ cm. By (6) we now calculated β_F, taking ($i_F = 4.9$ ma.) $L = 3.8$ and find for $I_P = 1.12 \times 10^{-4}$, $\beta_F = 0.058$, while for

$$I_p = 2.24 \times 10^{-4},\ \beta_F = 0.092.$$

When the radius of the sheath around a cylindrical collector is as great as 10 times that of the collector itself, the tangential components of the velocities of the ions when they enter the sheath may cause a considerable fraction of the ions to describe orbits about the collector and thus pass out of the opposite side of the sheath.

Under these conditions the current flowing to a collector of area A is

$$i = AI_p f \tag{8}$$

where f is a function of a/r, V and T_P, whose value approaches a/r when Ve/kT_P is less than $2(a/r) - 2$ and approaches the value $2[(1 + Ve/kT_P)/\pi]^{1/2}$ when a/r and T_P are sufficiently large.[11] For the case in hand we find the following values for f

I_p	$f(T_p = 5000)$	$f(T_p = 10000)$
1.12×10^{-4}	10.52	8.72
2.24×10^{-4}	8.95	7.88

We see that the radius of the sheath, which is proportional to f is comparatively little affected by doubling either I_P or T_P. Taking the higher value of I_P as the more probable, and choosing $f = 8.4$, we find from Eq. (8) by placing $i_F = 4.9$ m. amp., $A = 0.212$ cm^2 and $i_F\beta_F = i$, the most probable value of β_F to be

$$\beta_F = 0.081. \tag{9}$$

Inserting $\beta_C = 0.67$, $\beta_A = \beta_B = 0.062$ and $\beta_F = 0.08$ in Eq. (4) gives us for the total ionization the value $\beta = 0.95$. That is, in an experiment with argon at 30 baryes, each 50 volt electron produces 0.95 ion before losing its ionizing power.

If the conditions of the experiment are such that no recombination occurs, and all the primary electrons give up their energies to gas molecules before reaching the anode, we should expect β to be independent of pressure, the primary electron current and such geometrical factors as the diameter or length of the cylinder C.

[11] The complete expression for f is given in the G. E. Review article reference,[5] p. 454. A discussion and derivation of these equations has been given by Mott-Smith and Langmuir.[5]

Thus we can understand why it is, in experiments such as that of Fig. 2, that the positive ion current to the negatively charged cylinder is so nearly independent of the potential of the cylinder even when the sheath thickness varies considerably. If the number of ions per cc in the gas remained constant, the current to C should be proportional to the diameter of the sheath, so that the voltage-current curve would have a negative slope for negative cylinder voltages. But if β remains constant and if the current i_A is small compared to i_C, i_C must also be constant, and therefore the intensity of ionization of the gas must increase as the sheath on the cylinder becomes thicker. This effect is due to the reflection of the electrons from the sheath edge, which increases the primary electron current density and makes the rate of ionization per unit volume inversely proportional to the volume.

When A and C are connected together the fact that the curves for i_A and i_C are flat (for negative voltages) is explained by the constancy of β. In the data of Fig. 4, where E_B was kept constant while E_C was varied, we have seen that the negative slope of i_B was about equal to the positive slope of i_C. This is a result of the constancy of β.

Primary Electrons Which Reach the Cylinder C.

If the cylinder C is brought to a positive potential the retarding field is no longer able to prevent primary electrons from reaching it. At pressures low enough for an appreciable fraction γ of the primary electrons to reach the collector C without having collided with atoms, there is thus an abrupt change Δ_C in the value of $i_C/-i_F$ when the potential E_C is raised from negative to positive values. In Fig. 2 we see for mercury vapor at 25 baryes that Δ_C is 0.90, while with a pressure of 102 baryes Δ_C is only 0.18, indicating that as the pressure increases a larger fraction of the electrons from the cathode lose energy by collision.

The change Δ_C is due to two factors: (1) the electrons taken up by C and (2) the decrease in the positive ion current to C resulting from the removal of the primary electrons by C.

The primary electron current from the cathode is $i_F(1-\beta_F)$. Of these electrons the fraction γ reach the cylinder when it is positive, but only the fraction $1-\rho$ of these are collected, where ρ is the reflection coefficient for electrons. Thus the current due to primary electrons which reach the cylinder directly is

$$i_F\gamma(1-\beta_F)(1-\rho) \tag{10}$$

and the current of electrons reflected is $i_F\gamma\rho(1-\beta_F)$. When the potential of the cylinder is only a few volts positive, the primary electrons lose nearly all their energy in traversing the sheath on the cylinder, and for these low velocities the reflection coefficient ρ is small (about 0.2). Although the reflected electrons probably leave the surface of the collector with a distribution of directions approximating to Lambert's cosine law, they are so strongly accelerated by the field in the sheath that they emerge from the sheath moving in a radial direction, so that we may consider the reflection to be specular.

Thus at low pressure some of the reflected electrons pass back across the tube, a fraction γ^2 of them reaching the opposite side of the cylinder, so that the fraction $\gamma^2\rho$ of them are reflected a second time. The total electron current is thus obtained by multiplying that given by (10) by $1+\rho\gamma^2+\rho^2\gamma^4+\cdots=1/(1-\rho\gamma^2)$.

Each primary electron emitted from the cathode produces an ionization which contributes $\beta_C/(1-\beta_F)$ to the positive ion current to C. Similarly, each primary electron removed by the collector C will cause a decrease $\beta_C'/(1-\beta_F)$ in the positive ion current to C. Here β_C' differs from β_C only because the ions produced near the cylinder are more likely to reach the cylinder than if they were produced near the filament, and because the radial potential gradient which draws ions away from the filament is decreased by ions produced near the cylinder. At low pressures it is probable that β_C' and β_C are practically equal.

Since the total change in current to the cylinder, when this becomes positive, is $i_F\Delta_C$ we find

$$\Delta_C=\gamma(1-\rho)(1+\beta_C'-\beta_F)/(1-\rho\gamma^2). \tag{11}$$

Fig. 3 shows that there is no corresponding break in the current to the end-plate if the potential of the plate is varied while C is kept negative. This means that there are no electrons in the ionized gas which move in an axial direction with the velocity of the primaries. However, the current i_B does suffer a break (as in Fig. 4) if E_C, the potential of the cylinder is changed from negative to positive values. Let us represent the change in $i_B/-i_F$ by Δ_B. Each electron from the cathode contributes a fraction of the positive ion current to B equal to $\beta_B/(1-\beta_F)$ and we may assume that each electron taken up by the cylinder C causes a decrease of $\beta_B'/(1-\beta_F)$ in the current to B. Here β_B' and β_B are probably even more closely equal than are the corresponding quantities β_C' and β_C.

By reasoning similar to that used in deriving (11) we find

$$\Delta_B=\gamma(1-\rho)\beta_B'/(1-\rho\gamma^2). \tag{12}$$

By division we obtain from (11) and (12)

$$\Delta_B/\Delta_C=\beta_B'/(1+\beta_C'-\beta_F). \tag{13}$$

The break Δ_B is observed with value unchanged when E_c is raised from a negative to a positive value, whether B is kept at constant negative voltage or is connected to C. While Δ_C is due to changes in both the electron and the ion currents to C, Δ_B is caused solely by the change in ion current.

The experiments show that the ratio of the observed values of Δ_B and Δ_C agrees well with that calculated by Eq. (13). For example, the data from Fig. 4 give $\beta_B=0.071$, $\Delta_B=0.039$, $\beta_C=0.65$ and $\Delta_C=0.92$ and we have already estimated the value of β_F to be 0.08. The value of Δ_B/Δ_C calculated from Eq. (13) by ignoring the distinction between β and β' is thus 0.0451, while the ratio of the observed values of Δ_B and Δ_C is 0.0424. This difference, amounting to 6 percent, is in the direction to be expected, since β_C' should actually be slightly greater than β_C.

Theory of Current-Voltage Characteristic for Primary Electrons Which Make Elastic Collisions.

A detailed analysis of the shape of the current-voltage characteristic of $i_C/(-i_F)$, such as the curve shown in Fig. 4, will enable us to calculate the numbers of electrons which make elastic and inelastic collisions and also to determine the distribution of the angles through which the electrons are deflected or scattered by their collisions with gas molecules.

We have seen that each primary electron moves along a radius until it either reaches the sheath on the cylinder or collides with a gas molecule. Although the kinetic energy of an electron is not appreciably changed by an elastic collision, the velocity no longer remains radial so that when the electron reaches the sheath on the cylinder (used as collector) it is not able to move against so large a retarding potential as if it had not collided. In general, the greater the angle through which the electron has been deflected by the collision, the higher will be the collector potential required to collect the electron. Let us now attempt to calculate what fraction of the electrons which make collisions can reach the collector when this is at any given potential.

In Fig. 5, which shows the tube in cross-section, O is the cathode filament and the circle of radius R is the collecting cylinder. Consider an electron which is projected from the cathode sheath in the radial direction OU and

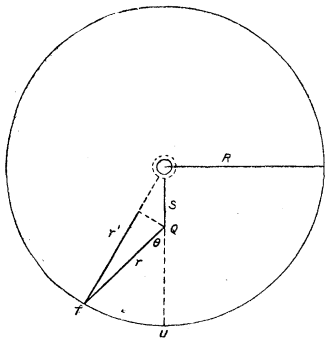

Fig. 5.

let this electron make an elastic collision with a molecule at Q after having moved a distance S from the cathode. After the collision the electron in general will have a velocity component along the axis of the cylinder, but this component will have little effect in determining whether or not the electron will be collected by the cylinder, provided (as we are assuming) that the cylinder is long enough. We therefore need consider only the velocity component perpendicular to the axis. We shall call r the magnitude of this component and θ the angle which its direction makes with the original direction of motion OU. When the electron arrives at the cylinder its radial velocity component is the projection of r on OT, and is thus equal to $r[1-(S^2/R^2)\sin^2\theta]^{1/2}$.

The electron will be collected only if this radial velocity equals or exceeds a certain value u_0 given by

$$u_0 = [2(e/m)(V_0 - E)]^{1/2} \tag{14}$$

where V_0 is the potential of the ionized gas (approximately anode potential) and E is that of the collector, both measured from the cathode. Thus the condition that the electron shall be collected is

$$r^2 - (r^2 s^2/R^2)\sin^2\theta \geqq u_0^2. \tag{15}$$

Scattering through cone of semi-angle θ_M. Let us now consider a large number N_S of electrons which make their collisions with gas molecules in a small element of volume surrounding Q after having traveled a distance S. We will assume provisionally that the velocity vectors after collision are distributed at random through a cone of semi-angle θ_M whose axis is the original direction of motion of all the electrons (OU). In other words the end points of the vectors are distributed with uniform density on a zone of a spherical surface of radius v_0 where v_0 is the magnitude of the original velocity and is given by

$$v_0 = [2(e/m)V_0]^{1/2}. \tag{16}$$

The *projected* velocities r may therefore be represented in a diagram with polar coordinates, r and θ as in Fig. 6, by points lying in a circular segment

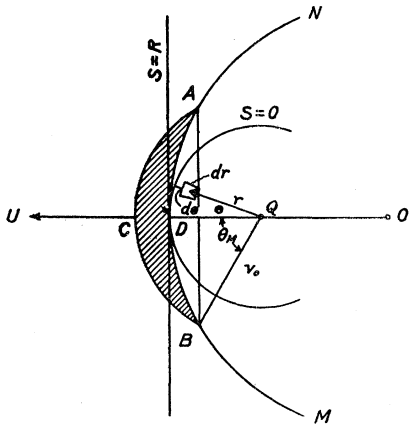

Fig. 6.

ABC of radius v_0 and semi-angle θ_M and it is readily found that the number of points in a specified element of area $rdrd\theta$ is

$$\frac{N_s}{2\pi v_0^2(\sin\frac{1}{2}\theta_M)^2}\cdot\frac{rdrd\theta}{[1-(r^2/v_0^2)]^{1/2}}. \tag{17}$$

Not all of these representative points correspond to electrons which can be collected, but only those which satisfy the condition (15). Thus for any given

value of S the points corresponding to the electrons that can be collected lie, in Fig. 6, to the right of a certain curve, (such as MDN) whose equation is found by taking the equality sign in (15). This curve is in general an ellipse, whose center is at Q and which as indicated in Fig. 6, degenerates into a circle for $S=O$ and into a straight line for $S=R$. For any given distance S the number of electrons collected (N_C) is therefore found by integrating the expression (17) over the shaded region indicated in Fig. 6. This integration is best carried out by expanding the expression (17) into a series in terms of E/V_0. Taking only the first order terms and restricting ourselves to values of θ_M less than about 30° we thus find (approximately)

$$N_C=\frac{N_S R}{S}\cdot\frac{E/V_0}{\sin^2\theta_M} \tag{18}$$

provided that S is greater than a critical value S_1 defined by

$$S_1=R(E/V_0)^{1/2}/\sin\theta_M \tag{19}$$

For this value of S the ellipse MDN of Fig. 6 passes through the points A and B. When S is greater than S_1 the expression for N_C/N_S is the same (except for the proportionality factor $1/\sin^2\theta_M$) as it would be if the electrons were scattered uniformly through *all* angles instead of through a cone of semi-angle θ_M. Thus Eq. (18) is relatively simple.

When, however, S is less than S_1 the limits of integration depend upon θ_M and the expression for N_C/N_S becomes very complicated, but it is found that the function can fairly well be represented by

$$N_C=N_S(E/V_0)^{1/2}k(S)/\sin\theta_M \tag{20}$$

where $k(S)$ is a quantity which is nearly independent of E and of θ_M but varies from $4/\pi$ to 1 as S increases from 0 to S_1.

For collisions taking place near the filament, that is, for $S=0$, Eq. (20) gives for small values of θ_M and E/V_0

$$N_C=(4/\pi)N_S(E/V_0)^{1/2}/\sin\theta_M. \tag{21}$$

In the scattering occurs uniformly through all angles so that $\theta_M=180°$ as in the case of electrons colliding with heavy elastic spheres, a rigorous expression for N_C can readily be worked out by integrating over the whole annular ring between the circles indicated in Fig. 6. We find in this case

$$N_C=N_S(E/V_0)^{1/2}. \tag{22}$$

For collisions occurring near the cylinder, that is, for $S=R$, a rigorous expression for N_C is obtainable:

$$N_C=\frac{N_S[1-(1-E/V_0)^{1/2}]}{2\sin^2(\theta_M/2)} \tag{23}$$

provided that E/V_0 is not greater than $\sin^2\theta_M$. If E/V_0 is greater than $\sin^2\theta_M$ then we have

$$N_C = N_S \quad \text{for} \quad \theta_M < 90°$$

or

$$N_C = N_S\left[1-\frac{(1-E/V_0)^{1/2}}{\sin^2(\theta_M/2)}\right] \text{ for } \theta_M > 90°. \tag{24}$$

Distribution of collisions along the radius. We must now take into account the fact that the collisions of the electrons with molecules can occur at all points along the radius. If λ is the mean distance that an electron travels through the gas before making a collision then $1/\lambda$, which we shall denote by P is the probability of collision per unit length of path. For each particular type of collision that may occur in a gas we shall have a different value of P. We shall use the following notation for the probabilities per unit length of path; P for collisions of all kinds, P_e for elastic collisions, P_k for inelastic collisions, P_i for collisions that cause ionization, P_r for resonance collisions, i.e., those that cause the impinging electrons to lose a definite energy corresponding to a resonance potential of the gas.
The first three of these quantities are related:

$$P = P_e + P_k. \tag{25}$$

We have used γ, according to Eq. (11), to denote the fraction of the primary electrons which reach the sheath on the cylinder C after having undergone some specified change as a result of collisions. In accordance with this use we may let: γ_0 be the fraction of the primaries which reach the sheath[12] on C without colliding with gas molecules; γ_n be the fraction of the primaries which reach the sheath on C after having made *n elastic* collisions but no inelastic collisions (we have γ_1, γ_2, etc., corresponding to $n=1$, $n=2$, etc.); γ_e be the fraction of the primaries which reach the sheath on C after having made one or more elastic collisions, but no inelastic collisions; and γ_r be the fraction of the primaries which reach the sheath on C after making one resonance collision. This includes the electrons which may also have made one or more elastic collisions (either before or after the resonance collision). From these definitions we see that

$$\gamma_e = \gamma_1 + \gamma_2 + \gamma_3 + \cdots \tag{26}$$

$$\gamma_0 = \epsilon^{-RP} \tag{27}$$

We calculate γ_1 as follows. If N_F primary electrons leave the cathode sheath then $N_F P_e \epsilon^{-SP} ds$, of these make elastic collisions with molecules in the distance ds after they have traveled a distance S without having pre-

[12] In these definitions it is specified that the electrons shall reach merely the edge of the sheath, i.e., the boundary between the sheath and the field-free ionized gas, for at present we do not wish to distinguish between the electron that can reach the *cylinder* against the retarding field in the sheath and those that are not able to do so.

viously made inelastic collisions. In accordance with experiments which will be described, we assume that the electrons are deflected by the elastic collisions only through rather small angles so that after the collision the distance the electron must travel before reaching the cylinder is $R-S$. Thus the fraction $d\gamma_1$ of the primary electrons which reach the cylinder after a single elastic collision (but no inelastic collision) is

$$d\gamma_1 = P_e\epsilon^{-SP}\epsilon^{-(R-S)P}ds = P_e\epsilon^{-RP}ds \tag{28}$$

and therefore by integration from $S=0$ to $S=R$

$$\gamma_1 = RP_e\epsilon^{-RP}. \tag{29}$$

In general we find in a similar manner

$$\gamma_n = (1/n!)(RP_e)^n\epsilon^{-RP}. \tag{30}$$

Thus by Eqs. (26) and (27) we obtain

$$\gamma_e = \epsilon^{-R(P-P_e)} - \gamma_0 \tag{31}$$

and by (25)

$$\gamma_0+\gamma_e = \epsilon^{-RP_k}. \tag{32}$$

By elimination of P between Eqs. (29) and (31) we obtain

$$\gamma_0/(\gamma_0+\gamma_e) = \epsilon^{-RP_e}. \tag{33}$$

For the resonance collisions we have, by the method used for Eq. (29),

$$\gamma_r = RP_r\epsilon^{-RP_k} \tag{34}$$

and thus by Eq. (32)

$$\gamma_r = RP_r(\gamma_0+\gamma_e). \tag{35}$$

Let N_e be the average number of elastic collisions that are made by the electrons which reach C after having made *at least one elastic but no inelastic collisions.*

Let N_r be the average number of *elastic* collisions that are made by the electrons which reach C after having made *one resonance* collision.
We see that

$$N_e = \frac{\gamma_1+2\gamma_2+3\gamma_3+\cdots}{\gamma_1+\gamma_2+\gamma_3+\cdots} = \frac{RP_e}{1-\epsilon^{-RP_e}} \tag{36}$$

and

$$N_r = \frac{\gamma_1+2\gamma_2+3\gamma_3+\cdots}{\gamma_0+\gamma_1+\gamma_2+\gamma_3+\cdots} = RP_e. \tag{37}$$

By combining (36) and (33) we obtain the more convenient equation

$$N_e = RP_e(\gamma_0+\gamma_e)/\gamma_e \tag{38}$$

Analysis shows that the electrons which reach the cylinder after having made n elastic collisions have traveled an average distance $R/(n+1)$ since the last collision. In fact it is readily seen if we consider only electrons that reach C after having made one or more elastic collisions, that these collisions are distributed with equal probability along the radius.

Scattering by collisions uniformly distributed along a radius. We wish to consider the current-voltage curve due to electrons, collected by C, which have made single elastic collisions distributed at random along a radius. We have defined γ_1, as the fraction of the primary electrons that reach the sheath on C after having made single elastic collisions. In general, because of the retarding field in the sheath, only the fraction γ of the primary electrons will reach the surface of C. Evidently γ will become equal to γ_1 if the retarding field should disappear, that is if E, the potential of the collector is made equal to V_0, the potential of the gas. The value of γ is found by integrating the expression for N_C/N_S from Eqs. (18) and (20) along the radius. Thus

$$\gamma=(\gamma_1/N_S R)\int_0^R N_C ds. \tag{39}$$

This integration must be carried out in two steps, from $S=0$ to $S=S_1$, by Eq. (20) and from S_1 to R by Eq. (18). Placing for convenience

$$E/V_0=\nu \tag{40}$$

we thus find[13]

$$\gamma=\gamma_1(\nu/\sin^2\theta_M)\left[1+\log(\sin\theta_M/\nu^{1/2})\right]. \tag{41}$$

This equation, like (18) and (20) is based on the assumption that ν and θ_M are small quantities. However, from an examination of the second and third order terms in the expansions it appears that Eq. (41) should give a good approximation (as accurate as the experiments) up to values of ν as high as 0.6 and values of θ_M as high as 30°.

From Fig. 6 we see that all electrons which are scattered through angles of less than θ_M will be collected by the cylinder, no matter where the collisions occur along the radius, if the voltage of the cylinder is raised sufficiently to make the point D meet the line AB or pass to the left of it. This will occur if

$$u_0 \leqq v_0 \cos\theta_M. \tag{42}$$

By Eqs. (14), (16) and (40) we find that this condition is equivalent to

$$\nu \geqq \sin^2\theta_M. \tag{43}$$

When ν becomes equal to $\sin^2\theta_M$, γ should be equal to γ_1, and in fact we see that Eq. (41) gives this value under these conditions. For values of ν

[13] The logarithmic term in the brackets comes from the integration of Eq. (18) between S_1 and R, while the term unity was obtained from the integral of Eq. (20) after having calculated the value of $k(S)$ with some care.

greater than $\sin^2\theta_M$ of course Eq. (41) does not hold but γ remains constant and equal to γ_1.

Although Eq. (41) is not applicable when θ_M is a large angle, it is possible to derive by similar methods the following approximate equation for the case that $\theta_M = 180°$, that is when electrons are scattered equally in all directions by collisions uniformly distributed along a radius

$$\gamma = \gamma_1 \nu [0.88 + \tfrac{1}{4} \log (1/\nu)]. \tag{44}$$

Scattering according to probability laws. We have thus far taken the directions of the scattered electrons to be uniformly distributed within a cone of semi-angle θ_M. If we have n electrons scattered in this way, the angular density η (expressed as electrons per steradian) can be calculated from the relation

$$n = 2\pi\eta(1 - \cos\theta_M) = 4\pi\eta \sin^2(\theta_M/2). \tag{45}$$

When θ_M is small this becomes

$$n = \pi\theta_M^2\eta. \tag{46}$$

The error in using (46) in place of (45) is only 2.3 percent for $\theta_M = 30°$ and 1 percent for 20°.

Let us now assume that the scattered electrons instead of having uniform angular density are distributed according to a type of probability law such that

$$\eta = \eta_0 \epsilon^{-\theta^2/\theta_0^2} \tag{47}$$

where η_0 is the maximum angular density and θ_0 is the angle at which the angular density of the scattered electrons is $1/\epsilon$ of η_0. It can be readily proved that θ_0 is the root-mean-square deflection of the electrons. The average angle θ_{av} and the most probable angle θ_P of scattering are related to θ_0 as follows

$$\theta_{av} = \tfrac{1}{2}\pi^{1/2}\theta_0 \text{ and } \theta_p = \tfrac{1}{2}2^{1/2}\theta_0. \tag{48}$$

The total number n of electrons that are scattered according to Eq. (47) is approximately (for not too large values of θ_0) given by

$$n = \int_0^\infty 2\pi\theta\eta d\theta = \pi\theta_0^2\eta_0. \tag{49}$$

Comparing this result with Eq. (46) we see that the total number of electrons is the same as in a cone of semi-angle θ_0 having electrons distributed with the uniform angular density η_0.

From the results already obtained we may now by an integration process derive an equation for the current collected by C when this is at a voltage E for the case that the electrons are scattered according to probability laws by collisions uniformly distributed along the radius.

If N_F primary electrons leave the cathode $N_F\gamma_1$ of them reach the sheath on C after making single elastic collisions, and we will assume that these are

scattered according to Eq. (47). We may divide these $N_F\gamma_1$ electrons into an infinite number of groups, each group consisting of $N_F d\gamma_1$ electrons scattered through a cone of semi-angle θ (variable for different groups) with a uniform angular density $d\eta$. The number of electrons in a group is thus

$$N_F d\gamma_1 = \pi\theta^2 d\eta .$$

Let the collector C be maintained at a given voltage E, then from Eqs. (43) and (40) we see that if $\theta < \nu^{1/2}$ all the electrons in the group will be collected, but if $\theta > \nu^{1/2}$ only a part will be collected, the fraction collected being, by Eq. (41) equal to

$$(\nu/\theta^2)\left[1+\log(\theta/\nu^{1/2})\right].$$

Thus we find that the total number $N_F\gamma$ of electrons reaching C is

$$N_F\gamma = \int_{\eta_c}^{\eta_0} \pi\theta^2 d\eta + \int_0^{\eta_c} \pi\nu\left[1+\log\left(\theta/\nu^{1/2}\right)\right]d\eta \tag{50}$$

where η_c is the value of η which, by Eq. (47) corresponds to the critical angle $\theta = \nu^{1/2}$. Thus

$$\eta_c = \eta_0 \epsilon^{-\nu/\theta_0^2}. \tag{51}$$

We may express $d\eta$ in terms of θ by differentiating Eq. (47), and by Eq. (49) we find that

$$\eta_0 = N_F\gamma_1/(\pi\theta_0^2). \tag{52}$$

If we then substitute $y = \theta^2/\theta_0^2$, Eq. (50) becomes

$$\gamma/\gamma_1 = \int_0^x y\epsilon^{-y}dy + x\int_x^\infty \left[1+\tfrac{1}{2}\log\left(y/x\right)\right]\epsilon^{-y}dy$$

where

$$x = \nu/\theta_0^2 = E/(V_0\theta_0^2). \tag{53}$$

Carrying out the integration we get

$$\gamma/\gamma_1 = 1-\epsilon^{-x}+\tfrac{1}{2}x\int_x^\infty \epsilon^{-x}dx/x. \tag{54}$$

The integral in Eq. (54) is equal to $-Ei(-x)$ where Ei stands for the "exponential integral" given for example in Glaisher's Tables (Phil. Trans. *140*, 367 (1870)). By means of the series expansion for $Ei(-x)$ we find for small values of x that

$$\gamma/\gamma_1 = x(0.7114-\tfrac{1}{2}\log x)$$

which gives only 3 percent error when $x=1$. For larger values of x a good approximation is

$$\gamma/\gamma_1 = 1-\tfrac{1}{2}\epsilon^{-x}.$$

Table II gives values of γ/γ_1 as a function of x, calculated from Eq. (54).

TABLE II

The fraction of collectable electrons as a function of $x=E/(V_0\theta_0^2)$ *as given by Eq* (54)

x	γ/γ_1	x	γ/γ_1	x	γ/γ
0.001	0.0042	0.2	0.3035	1.5	0.8519
0.01	0.0301	0.3	0.3950	2.0	0.9136
0.02	0.0534	0.5	0.5334	3.0	0.9698
0.005	0.1105	0.75	0.6553	4.0	0.9892
0.10	0.1863	1.00	0.7418	5.0	0.9961

It is of interest to compare Eq. (54) which is for electrons scattered by probability laws, with Eq. (41) which is for electrons scattered uniformly through a cone. Replacing $\sin\theta_M$ by θ_M in Eq. (41) and by analogy with Eq. (53) placing

$$x'=\nu/\theta^2_M \tag{55}$$

we have

$$\gamma/\gamma_1=x'[1+\tfrac{1}{2}\log(1/x')]. \tag{56}$$

A double logarithmic plot of the function of the right member of Eq. (54) is given in Fig. 8 by the full line ST (also marked $\delta=0$). The corresponding function of Eq. (56) is represented in this figure by the dotted curve. By means of the function given in Table II we shall be able to analyze current-voltage curves such as those of Fig. 2 and thus determine from the

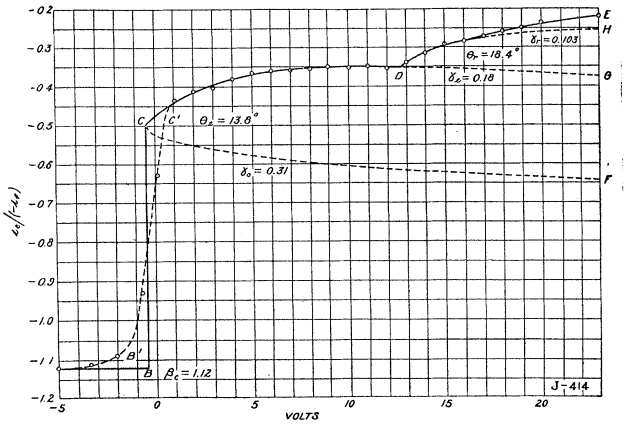

Fig. 7.

experiments the number of electrons that make elastic collisions and the mean angle θ_0 through which they are scattered by the collisions.

Application of the theory for electrons which have suffered collisions. To illustrate the method of using the equations let us consider in some detail a typical example which gave the data recorded in Table III and Fig. 7.

In this experiment nitrogen was used at 50 baryes and 650°K (in the cylinder C). This pressure was corrected for the effect of thermal effusion as will be described later. The cylinder C_r was of molybdenum, 5.0 cm long, 5.2 cm in diameter, and the end-plates were 4.85 cm in diameter and 5 cm apart. One of them, A, was used as anode at 100 volts while the other, B, was maintained at -6.0 volts. The filament F was of tungsten, 5 cm long, 0.0254 cm in diameter, heated to about 2300°K by an intermittent current, which read 5.95 amps. on an A.C. ammeter and about 3.92 on a D.C. meter. A rotating commutator operated by a 60 cycle synchronous motor was used to produce the intermittent filament current and to connect the circuit for measuring the current i_C only in the intervals during which no current flowed through the filament. Thus the effects of voltage drop along the filament were avoided. The circuit for C was closed 43.6 per of the time so that the observed currents as read by a D.C. microammeter were multiplied by 1/0.436 to get the instantaneous current i_C.

Table III, in columns 1 and 2, gives the instantaneous currents to the cylinder C at various cylinder voltages. The currents are expressed as a fraction of the current i_F from the cathode. As these data are given only to illustrate the method used for calculation, alternate readings have been

TABLE III

Typical current voltage data. Nitrogen at 50 baryes and 650°K; $R=2.60$ cm; anode at 100 volts; $\beta_c=1.12$; $\beta_F=0.11$; $\gamma=0.498\Delta/(1-\rho)$; $\Delta=i_c/(-i_F)-1.12$

E_c	$i_c/-i_F$	$1/(1-\rho)$	γ	obs. $\gamma-\gamma_0$	cal. $\gamma-\gamma_0$	Diff.	γ'	E'
2	−0.413	1.100	0.387	0.077	0.076	+ .001	—	—
4	− .384	1.135	.416	.106	.107	− .001	—	—
6	− .362	1.163	.439	.129	.128	+ .001	—	—
8	− .354	1.187	.453	.143	.142	+ .001	—	—
10	− .352	1.208	.462	.152	.152	.000	—	—
12	− .354	1.228	.468	.158	.160	− .002	—	—
14	− .318	1.245	.497	.187	.166	+ .021	.0176	1.2
16	− .286	1.262	.524	.214	.171	+ .043	.0380	3.2
18	− .259	1.278	.548	.238	.173	+ .065	.0583	5.2
20	− .240	1.292	.566	.256	.175	+ .081	.0735	7.2

omitted from the table; all the observed points between -5 and $+23$ volts are indicated by the small circles in Fig. 7. The same data over a wider range of voltages are also given in curve V of Fig. 12.

The points in Fig. 7 lie along a broken curve $AB'C'DE$. The section AB corresponding to $\beta_C=1.12$ gives the positive ion current. The break $B'C'$, which corresponds to γ_0, is due to electrons which reach C without collisions. Along the curved portion $C'D$ the electrons that have made elastic collisions are being collected, while in the range DE the electrons which have made resonance and ionizing collisions are also collected.

We wish now to distinguish between the electron current and the ion current. To do this we let Δ represent the increase in current[14] (at any given

[14] For convenience we shall frequently refer to the current i_C when we really mean $i_C/(-i_F)$

voltage) over the value -1.12 which corresponds to β_C. Then by Eq. (11) we may calculate γ the fraction of the primary electrons that are able to reach the surface of the cylinder at any given voltage. We may neglect the small quantity $\rho\gamma^2$ and thus write Eq. (11) in the form

$$\gamma=\Delta[1/(1-\rho)]/(1+\beta_C-\beta_F). \tag{57}$$

Reflection coefficient for electrons. To use this equation we must know ρ, the reflection coefficient for electrons. Since ρ is a function of the velocity of the electrons, it might seem at first that for groups of electrons which are not all collectable at a definite voltage, the value of ρ should enter Eq. (57) as a complicated integral. For example, if the collector is at $+5$ volts, the electrons collected include those that are just able to move against a retarding field of one volt as well as those that can move against five volts.

However, the electrons in these different groups, since they cannot have made inelastic collisions, must all have the same total energy as the primary electrons, and thus they differ only in their angles of incidence on the collector. There is considerable evidence, experimental and theoretical, that the reflection coefficient, at least on smooth clean surfaces, depends mainly on the total energy of the electrons rather than upon the normal component of the energy. Therefore, for electrons which have made elastic collisions, we are justified in taking ρ the same for all those that are collected at a given voltage. Eq. (57) may thus be used if we take ρ as the reflection coefficient for electrons which are moving with an energy corresponding to the voltage E (the energy of impact on C). Farnsworth[15] has given data for the reflection coefficient ρ of electrons from tungsten surfaces the values 0.20 for 2 volts, 0.32 for 6 volts and 0. 38 volt electrons. Hull's observations with dynatrons indicate that the reflection from molybdenum is considerably less than from tungsten. Petry[16] has published data on the reflection of electrons from molybdenum, but only relative, not absolute values of ρ were given. Mr. Petry has kindly furnished us with the notes of his original observations and from these we have calculated values of 0.11 at 5 volts, 0.17 at 10 volts, 0.19 at 15 and 0.22 at 20 volts. Based partly on these data and partly on other data in this laboratory[17] we have adopted values for ρ which give

$$1/(1-\rho)=1+0.0645E^{1/2}. \tag{58}$$

The potential E in this equation should correspond to the energy of the electrons which reach the collector. Examination of Fig. 7 shows that electrons begin to be collected when E_C is -2 volts and a large fraction are collected before C becomes positive. The initial velocities of the electrons as they leave the filament and any contact potential difference that may exist between the tungsten filament and the molybdenum cylinder tend to produce small displacements of the curves parallel to the voltage axis.

[15] H. E. Farnsworth, Phys. Rev. **25,** 41, (1925).

[16] R. L. Petry, Phys. Rev. **26,** 346, (1925).

[17] We shall see that the data from the present experiments at a series of pressures all indicate that $1(1-\rho)$ at about 15 volts is close to 1.25 in agreement with Eq. (58).

Therefore we take, as our zero point of potential in calculating E, the mid-point on the steep rise $B'C'$ corresponding to the electrons that come through without collisions. Thus $E=E_C+0.4$ volt, not only in Eq. (58) but when we calculate x or ν by Eq. (53).

The data in the 3rd column of Table III are obtained from Eq. (58) in this way. The effect of possible errors in the value of ρ will be discussed later. Taking $\beta_C=1.12$ and assuming $\beta_F=0.1$, $\beta_A=0.11$ we thus find from Eq. (57) the values of γ given in the 4th column. From our theoretical study of the current-voltage curves we concluded that the relation between γ and the voltage E can be expressed by Eqs. (54) or (56) according as the electrons are scattered according to probability laws, or are scattered uniformly through a cone.

Let us now compare our experimental data for γ with those calculated from these equations.

In the range from C' to D in the curve of Fig. 7, the electrons collected, corresponding to γ consist of two parts: those that have made no collisions (γ_0) and those that have made elastic collisions. It is only this second part that corresponds to γ in Eq. (54). From Eq. (53) we see that x is proportional to E. By plotting the logarithms of γ/γ_1 and of x, as in Fig. 8, we thus see

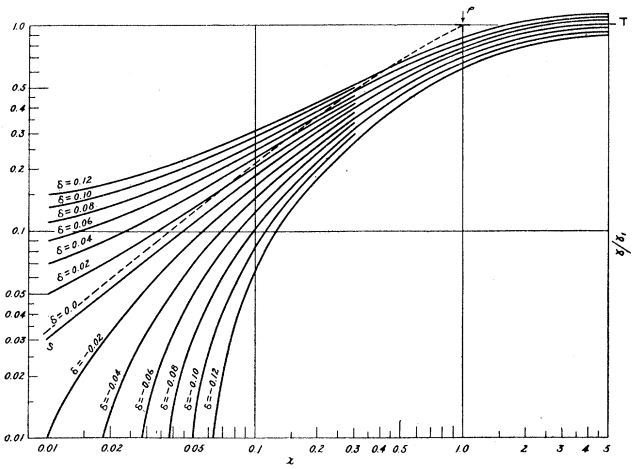

Fig. 8.

that changes in the values of the "unknowns" γ_1 and θ_0 do not alter the shape or size of the curve but result merely in vertical or horizontal displacements of the curve as a whole. On the other hand a change in the value of γ_0 corresponds to the addition or subtraction of a quantity and this does change the shape of the curve.

These latter changes may be studied by plotting a family of curves (as in Fig. 8) from the data of Table II, corresponding to $(\gamma/\gamma_1)\pm\delta$, where δ is given successively the values 0.02, 0.04, 0.06, etc.

We may now use this plot in a very simple way, to determine from the experimental data for γ (Column 4 of Table III) the values of γ_0, γ_e and θ_0. We proceed as follows:

By examination of a plot of γ (from Table III) against E, or directly from a plot such as Fig. 7, we make a rough preliminary estimate of the value of γ_0 which must correspond to the steep initial rise $B'C'$ in current near $E=0$. Suppose we assume $\gamma_0'=0.33$ for this value. We then plot on double-logarithmic paper (using the same scale as in Fig. 8) the values of $\gamma-\gamma_0'$ as ordinates and the voltages E as abscissas, so that we obtain the points which lie near the curve $A'C'$ in Fig. 9. By superposing[18] Fig. 9 on Fig. 8,

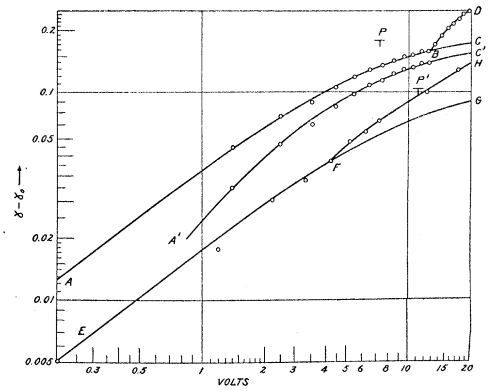

Fig. 9.

keeping the edges of the curve sheets parallel, one figure can be displaced with respect to the other until these points are made to fit as well as possible with one of the family of curves in Fig. 8. We find that the curve $\delta=-0.10$ gives the best agreement. Tracing this curve through on to Fig. 9 we obtain the curve marked $A'C'$.

The curve ST in Fig. 8, for which $\delta=0$, when transferred through to Fig. 9 gives ordinates in Fig. 9 about 0.02 greater than those of the observed points. Therefore we see that if we had chosen 0.31 instead of 0.33 for the value of γ_0, the observed points for $\gamma-\gamma_0$ would lie on the theoretical curve ($\delta=0$). Using this revised value of γ_0 and plotting $\gamma-0.31$ as given in Col. 5 of Table III, we obtain the points indicated in Fig. 9 by circles along the curve ABD. This curve can now be superposed upon ST in Fig. 8. The curve ABC in Fig. 9 is a theoretical curve obtained in this way by tracing through curve ST from Fig. 8. The point P in Fig. 8 at $\gamma=\gamma_1$, and $x=1$ when transferred to Fig. 9, gives the point P marked by the intersection of the short horizontal and vertical lines. The coordinates of this point give $\gamma_e=0.180$ and $E_E=7.2$ volts where E_E is the value of E corresponding to $x=1$. From Eq. (53) for $x=1$,

[18] This is conveniently done by placing one curve sheet on the other, holding both against a glass window by daylight, so that the curves can be seen by transmitted light.

$$E_E = V_0\theta_0^2 \tag{59}$$

and thus, since $V_0 = 100$ volts, we find $\theta_0 = 0.268$ radians or 15.4° as a measure of the angle through which the electrons are scattered by the elastic collisions. Let us denote this value by θ_E.

The good agreement between the observed points and the theoretical curve in Fig. 9 proves that the electrons are scattered at least approximately according to probability laws. By no choice of γ_0, γ_e or θ_M can the experimental data be made to agree with a curve like that shown by the dotted line in Fig. 8 which corresponds to uniform scattering of the electrons through a cone of semi-angle θ_M, according to Eq. (56).

The values of $\gamma - \gamma_0$, as taken from the theoretical curve ABC in Fig. 9, are given in Col. 6 of Table III. Up to $E_C = 12$ volts the agreement is good, the errors (Col. 7) being only about 0.2 percent for the original values of γ. Beyond 12 volts the differences increase rapidly because of electrons which are collected after having made resonance collisions. Let γ' be the fraction of the primary electrons reaching C at any given voltage after they have made one resonance collision. As E_C increases, γ' will approach the limit γ_r which we have already defined. Below the resonance voltage V_r, γ' is zero. In the present case, from inspection of the break at B in Fig. 9, we will take $V_r = 13.2$ volts. Let us place $E' = E - 13.2 = E_C - 12.8$ as in Col. 9. The energy of the electrons which reach C after single resonance collisions is evidently measured by E' and therefore the reflection coefficient for these electrons will be less than for the electrons that have made no collisions or have made only elastic collisions. An analysis shows that γ' may thus be calculated by multiplying the difference given in Col. 7 by the ratio of the values of $1/(1-\rho)$ at E' and at E. For example, at $E_C = 18$ we have $\gamma' = 0.065 \times (1.147/1.278) = 0.0583$, where 1.147 is the value of $1/(1-\rho)$ at $E' = 5.2$ volts and 1.278 is the corresponding value at $E = 18$ volts.

The values of γ' have been plotted in Fig. 9 as the small circles which lie along the curve EFH. By parallel displacement, the curve ST of Fig. 8 has been brought to fit the points in as reasonable a manner as possible, thus giving in Fig. 9 the theoretical curve EFG. The coordinates of the point P' give $\gamma_r = 0.103$ and $E_R = 11.2$ volts, where E_R is the value of E' corresponding to $x = 1$. The corresponding value of θ_0 which we shall denote by θ_R may be calculated from the equation

$$E_R = (V_0 - V_r)\theta_R^2 \tag{60}$$

which is derived from Eq. (53) by considering that the velocity of the "resonance" electrons corresponds to $V_0 - V_r$ instead of V_0. Thus we find $\theta_R = 0.360$ radians $= 20.6°$ as the root-mean-square angle through which the electrons are scattered by resonance collisions.

The angles θ_E and θ_R which we have thus determined are the mean deflections produced by the combined effect of all the collisions that the electrons may have made in traveling from F to C. We are more interested, however, in knowing the deflections of electrons which have made only one collision.

Assuming that the scattering is of the probability type according to Eq. (47), we may let θ_e be the mean deflection corresponding to a single elastic collision and let θ_r be the mean deflection for a single resonance collision. In successive collisions which an electron may make with gas molecules the transverse components of momentum delivered by the separate collisions, are distributed in random directions in the plane normal to the original path of the electron. Therefore θ^2 for the resultant deflection is found by addition of the values of θ^2 for the separate collisions.

Thus for the N_e elastic collisions which are made on the average by each of the electrons in the group corresponding to γ_e we have

$$\theta_E^2 = N_e\theta_e^2 \tag{61}$$

where N_e is given by Eq. (38).

The resonance electrons, corresponding to γ_r, have each made only one resonance collision and, on the average, N_r elastic collisions. Therefore

$$\theta_R^2 = \theta_r^2 + N_r\theta_e^2 \tag{62}$$

where N_r is given by Eq. (37).

Our analysis of the data of Table III (Figs. 7 and 9) has given:

$\gamma_0 = 0.310$	$V_r = 13.2$ volts
$\gamma_e = 0.180$	$\gamma_r = 0.103$
$E_E = 7.2$ volts	$E_R = 11.2$ volts
$\theta_E = 15.4°$	$\theta_R = 20.6°$.

From these values by means of Eqs. (27), (33), (25), (35), (37), (38), (61) and (62) we thus derive, placing $R = 2.60$ cm.

$P = 0.450$ cm^{-1}	$N_r = 0.458$
$P_e = 0.176$ cm^{-1}	$N_e = 1.25$
$P_k = 0.274$ cm^{-1}	$\theta_e = 13.8°$
$P_r = 0.081$ cm^{-1}	$\theta_r = 18.4°$.

These results have been obtained from an experiment with nitrogen at 650°K and a pressure of 50 baryes. The angles θ_e and θ_r are presumably determined by the nature of single collisions so they should not depend on the pressure of the gas. But the probabilities, P, etc., should be proportional to the number of molecules of gas per unit volume, so that we may recalculate them for the standard temperature and pressure of 20°C and 1000. baryes by multiplying the foregoing values by (1000/50) × (650/293) = 44.4, obtaining

$P = 20.0$ cm^{-1}	$P_k = 12.2$ cm^{-1}
$P_e = 7.8$ cm^{-1}	$P_r = 3.6$ cm^{-1}.

The ionization probability P_1 has been determined for a number of gases by Compton and van Voorhis.[19] For nitrogen with 100 volt electrons

[19] K. T. Compton and C. C. Van Voorhis, Phys. Rev. **27**, 724 (1926).

they find for 20°C and 1000 baryes $P_i = 7.7$ cm^{-1}. The inelastic collisions must include those that give ionization and those that produce resonance at 13.2 volts. Thus the remainder P_x given by

$$P_x = P_k - P_r - P_i. \tag{63}$$

measures the probability of all other types of inelastic collisions such as those that excite the molecules to higher energy levels than 13.2 volts. For the case under consideration we find

$$P_x = 0.9 \text{ cm}^{-1}.$$

The curves ABC and EFG in Fig. 9 are the theoretical curves corresponding to the foregoing values of P, γ_0, γ_e, θ_e etc. By taking points from these curves we can calculate values of Δ and compare them with the original observations. Such reconstructed curves are shown in Fig. 7, the whole curve $ABCDH$ being a theoretical curve calculated in this way by our equations. Thus AB is the theoretical line corresponding to a positive ion current of -1.12. The curve $ABCF$ is the theoretical curve corresponding to the positive ion current together with the current of primary electrons ($\gamma_0 = 0.31$) which have made *no* collisions. The negative slope of the line CF shows the effect of the reflection of the electrons from the collector C.

The curve $ABCDG$ includes the current due to the electrons ($\gamma_e = 0.18$) which have made elastic collisions and have thus been scattered through angles which average 12.2° (i.e., $\frac{1}{2}\pi^{1/2} \times 13.8°$). Finally the curve $ABCDH$ includes the electrons that are collected after resonance collisions.

The theoretical curves can thus be made to fit the experimental data within the probable error of the measurements except in the regions where E is close to zero and where E is above the ionizing potential (in this case 17 volts).

When an electron ionizes a gas molecule it must lose an energy corresponding to the ionizing potential, but then in general it must divide its remaining energy with the new electron produced by the ionization. Thus the probability is small that an electron that ionizes a molecule shall lose merely the energy which corresponds to the ionizing potential. Therefore at collector voltages above the ionizing potential the electrons that are collected can no longer be resolved into groups having homogeneous velocities. We see in fact from Fig. 7 that the observed points (along DE) begin to depart from the theoretical line DH at the ionizing potential but this departure begins gradually and not abruptly as in the case of the resonance potential at D.

The experimental data indicated by circles in Fig. 10, were obtained with mercury vapor at 44.5 baryes and 650°K in a tube having a cylinder C of 1.55 cm radius, the anode potential being 50 volts. By an analysis involving a double logarithmic plot it was found that

$\gamma_0 = 0.158$ $V_r = 6.3$ volts

$\gamma_e = 0.159$ $\gamma_r = 0.109$

$E_E = 2.2$ volts $E_R = 2.8$ volts

$\theta_E = 12.0°$ $\theta_R = 14.6°$

and from these were calculated

$P = 1.190$ cm^{-1} $N_r = 0.364$

$P_e = 0.443$ cm^{-1} $N_e = 1.20$

$P_k = 0.743$ cm^{-1} $\theta_e = 11.0°$

$P_r = 0.222$ cm^{-1} $\theta_r = 13.0°$.

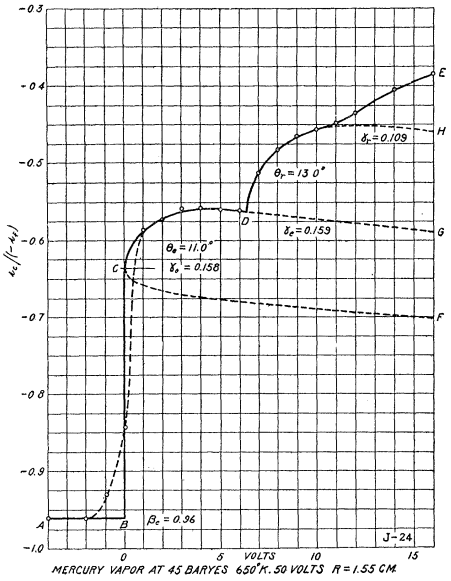

Fig. 10.

Under standard conditions (20°C and 1000 baryes) these probabilities become

$P = 59.0$ cm^{-1} $P_r = 11.0$ cm^{-1}

$P_e = 22.1$ cm^{-1} $P_i = 14.9$ cm^{-1}

$P_k = 36.9$ cm^{-1} $P_x = 11.0$ cm^{-1}.

The curve *ABCDH* in Fig. 10 is the theoretical curve corresponding to the foregoing values of the probabilities and the scattering angles. Although the resonance voltage (6.3) and the ionizing voltage (10.0), corresponding to the resonance potential (6.7) and the ionization potential (10.4) in mercury due to a shift of 0.4 volts at zero, are very different from those of nitrogen, the general agreement with the theory and the character of the results are much the same.

With mercury vapor also the experimental points begin to depart from the theoretical curve DH as soon as the voltage rises above the ionizing potential.

More than two hundred sets of volt-ampere characteristic curves for various gases, pressures and voltages have been analyzed and in all cases it has been found that by suitable choice of P, P_e, P_r, θ_e and θ_r the curve up to the ionizing voltage can be satisfactorily represented by the equations we have developed. The theory is particularly confirmed by the fact that the experimental values of all these constants are practically independent of the gas pressure. A summary of these results is given in Table VIII.

Discussion of the Accuracy of the Results

In fitting the theoretical curves in Fig. 7 to the experimental data we have 6 adjustable constants at our disposal, and it may therefore seem that the accuracy in the determination of all the constants cannot be high. By trial we find that the permissible limits in the choice of the values of γ_0 are rather narrow, so we estimate the probable error of γ_0 to be less than 0.01.

We find, however, that the curves such as AC in Fig. 9 can be displaced a considerable distance in the general direction of their length without greatly altering the agreement between the theoretical curve and the experimental points. Especially is this true with the resonance curve EFG which is more nearly straight and for which agreement with experiment is expected only for a rather limited range of voltage.

Trial shows for the data of Fig. 7 that the agreement becomes poor if E_E is altered by more than about 10 percent which corresponds to a variation of 5 percent in θ_E or θ_e. A 10 percent change in E_E involves about a 4 percent change in γ_e, but for any given value of E_E the probable error in γ_e would be not over 1 or 2 percent.

In the case of the resonance electrons the possible individual errors in γ_r or E_R become relatively large. In Table IV are given a series of pairs of values of γ_r and E_R corresponding to displacements of the curve EFG approximately parallel to the nearly straight line EF.

Table IV

Effect of displacement of curve EFG in Fig. 7

Displacement	γ_r	E_R	P_r	P_x	θ_R	θ_r	E_F
D_1	0.182	25.	6.3	−1.1	30.7°	29.4°	23.
D_2	0.152	19.8	5.3	0.0	26.8	25.7	18.
D_3	0.103	11.2	3.6	+1.7	20.6	18.4	17.
D_4	0.070	7.0	2.4	+3.8	16.3	13.3	14.7
D_5	0.033	2.3	1.2	+4.2	9.3	0.0	13.9

From the values of γ_r and E_R we can calculate P_r, P_x, etc., as given in the table. The displacement D_1 makes P_x negative and therefore must be excluded. The displacement D_2 has been chosen to make $P_x=0$. Thus the maximum possible values of P_r and θ_r are 5.3 and 25.7° respectively.

Displacement D_5 corresponds to the lowest possible value of P_r and the greatest possible value of P_x since any further displacement would make θ_r become imaginary.

For each different displacement, the point corresponding to F in Fig. 9, at which the observed points begin to depart from the theoretical curve, occurs at a different voltage E'. Adding 13.2 volts to these values we obtain the voltages E_F given in the last column of Table IV, and this voltage should presumably be approximately equal to the ionizing potential of the gas (17.0 volts for nitrogen). Thus we are led to choose D_3 as the most probable displacement.

In the analysis of the experimental data on the resonance electrons the displacement of the curve, which determines the value of E_r has usually been chosen so as to make E_F come at the ionizing potential. In other cases, especially when the voltage drop along the filament was not eliminated by using the commutator, the point E_F could not be determined with sufficient accuracy. In order to get approximate values of γ_r and P_r in these cases it was assumed that θ_r could be put equal to θ_e. From this, θ_R, and E_R were then calculated and a displacement was then chosen to give this value of E_R.

The values of P_r and θ_r found by such methods are of course only rough, but it is felt that our main conclusions regarding the rather small angles through which the electrons are scattered by the resonance collisions are not vitally affected thereby.

Estimation of temperature and pressure. To calculate the P's at standard pressure and temperature from the experimental data we must know the pressure and temperature of the gas in the cylinder C. The pressure outside the cylinder was measured by a McLeod gauge but because of the effect of thermal effusion the pressure inside the cylinder was usually considerably higher than that outside.

The radiation from the filament (ranging from 11 to 20 watts in the various experiments) heated the molybdenum cylinder to rather high temperatures. If the cylinder and end-plates had completely surrounded the filament so that the whole energy would be re-radiated from the outside surface, the temperature of the electrodes in the absence of gas, would have been 720° to 790°K. But from the small annular spaces about 2 mm wide between the end plates and cylinder the radiation was nearly that from a black body. Thus the radiative equilibria gave temperatures from 670 to 720°K. The effect of the heat conduction of various gases was calculated, taking into account the Smoluckowski temperature drop at the surfaces.[20] It was found that the cylinder temperature gradually decreased as the pressure rose. For example, argon at 100 baryes would lower the cylinder temperature about 70°. But the heat conduction from the filament tends to raise the gas temperature above that of the cylinder, by amounts that increase as the pressure rises. This effect nearly compensates for the cooling of the cylinder. Thus, the average temperature of the gas along the radius

[20] See I. Langmuir, J. Amer. Chem. Soc. **37**, 421 (1915).

was found to be approximately independent of the pressure and to lie between 650° and 700°K in various typical experiments. At pressures so low that the free-paths of the molecules are large compared to the openings into the cylinder thermal effusion makes the pressure p_c in the cylinder greater than that outside in the ratio of the square roots of the temperatures. The following semi-empirical equation was used in all the experiments to calculate the pressure in the cylinder

$$\frac{p_c}{p_0}=\frac{(T_c/T_0)^{1/2}+bp_0/\lambda_1}{1+bp_0/\lambda_1} \tag{64}$$

where p_0 and T_0 are the pressure and temperature outside the cylinder, while p_C and T_C are those inside; b is the width of the space between the cylinder and end-plates, while λ_1 is the mean free path of the molecules of gas at 1 barye pressure at the temperature of the cylinder.[21]

It will be appreciated that the accuracy in this estimation of the temperature of the gas and the pressure increases by effusion cannot be high. Thus in reducing the observed values P_1, P_e, P_k and P_r to standard conditions errors of perhaps 10 percent have been made. However, it should be noted that the effusion decreases the effect of an error in the estimation of the temperature since the pressure increases as the temperature rises and thus causes a slower variation in the density.

To avoid errors due to uncertainty in the temperature of the cylinder a large number of experiments were made with argon, and helium in tubes in which the "cylinder C" consisted in a vaporized magnesium deposit on the inside of the glass wall of the tube. By a blast of air the tube wall was kept at about 30°C as was measured by the resistance of a fine tungsten wire in close contact with the glass. The average temperature of the gas along the radius was usually about 50° above that of the tube wall, because of heat conducted from the filament.

The values of the probabilities P and P_K found with these tubes were practically the same as those found with the molybdenum cylinders, as will be seen in the summary of results (Table VIII). Thus we are justified in believing that the calculated temperatures of 650° to 700°K in the latter case are not seriously in error.

Small angle deflections. At voltages in the neighborhood of $E_C=0$ the observed currents, as indicated by the dotted line $B'C'$ (Fig. 7) do not rise as abruptly as they should according to the theoretical curve BC, but increase gradually over a range of about 2 or 3 volts, showing that the electrons which have not made collisions with molecules are not moving in strictly radial directions when they strike the cylinder. Possible causes of these small angular deflections from the radial direction are: (1) Initial velocities of electrons due to thermal agitation; (2) Non-axial position of the filament; (3) Transverse magnetic field (earth's field); (4) Scattering by the electric field of ions; (5) Scattering by the fields of molecules; (6) Scattering by

[21] The free-paths λ_1 at 650° were taken to be for Hg 13.1 cm; Ne 51; A 28; He 73; N_2 25 and H_2 47 cm. The slit width b was 2 mm in some cases and 2.5 in others.

energy transferred from other primary electrons. Let us consider briefly the magnitude of these effects.

1. *Initial velocities.* The average kinetic energy of the electrons which leave the filament is $2kT_F$, but the longitudinal component $\frac{1}{2}kT_F$ is not effective in causing them to move against a retarding field at the cylinder. The effective energy $(3/2)\ kT_F$, for a filament temperature of 2300°K corresponds to 0.30 volts, and can thus explain only about one tenth of the observed spread in voltage over which the primary electrons are collected.

2. *Displacement of filament.* If the filament is displaced a distance δ along a radius, the electrons which move along another radius at right angles to the first reach the cylinder with a transverse energy V_T given by

$$V_T = V(\delta/R)^2 \tag{65}$$

where V is the original radial energy. Since in the experiments the filaments were displaced from the center by not more than 1/20th of the radius, V_T for 100 volt primary electrons, cannot have been greater than 0.25 volts.

3. *Effect of earth's field.* The radius of curvature, r, of a V-volt electron in a transverse magnetic field of H gauss is

$$r = 3.36V^{1/2}/H \text{ cm}. \tag{66}$$

If the electron travels a short distance x (small compared to r) perpendicular to the field, the transverse energy, V_T, (in volts) given to the electron by the magnetic field is

$$V_T = 0.089H^2x^2 \tag{67}$$

this energy being independent of the velocity of the electrons. Thus in a tube of 2.6 cm radius (as for the data of Table III and Fig. 7) a transverse field of 0.5 gauss will give a transverse energy of 0.15 volts. Since the electrons which move parallel to the field are not deflected, the primary electrons are thus spread out over a small range. A longitudinal field, however, would give the same transverse energy to all the electrons so would not produce a spread but only a slight displacement of the current-voltage curve.

In our experiments a transverse field of 0.5 gauss may well have been present but this field can have caused only part of the spread in voltage indicated by the lines $B'C'$ in Figs. 7 and 10.

4. *Scattering by ions.* H. A. Wilson[22] in considering the scattering of β-rays by matter has derived equations which are useful in calculating the scattering of electrons by the ions in a gas.

Consider a narrow beam of electrons of energy corresponding to the potential V, passing through a layer of ionized gas of thickness t which contains n univalent charges per unit volume (ions plus electrons). The mean square angular deflection produced by all charges which lie within a distance r of the electron beam is

$$\theta_0^2 = 8\pi nt(r^2/a^2)(1-\pi^2/8+\log a) \tag{68}$$

[22] H. A. Wilson, Proc. Roy. Soc. **A102,** 9 (1923).

where a is a parameter having the value

$$a=2rV/e. \tag{69}$$

In the derivation of Eq. (68) it was assumed that a is large compared to unity and the scattering is thus nearly all due to numerous small deflections, the close encounters which give large deflections being so rare that they contribute little to θ_0. Expressing V in volts, r and t in cm and n in ions per cm^3 we find

$$a=1.39\times10^7 rV$$

and from Eq. (68)

$$\theta_0=1.47\times10^{-6}((nt)^{1/2}/V)(1+0.142\log_{10}rV)^{1/2}. \tag{70}$$

We see that θ_0 becomes infinite if r is infinite so that according to this equation the forces due to the ions at large distances cannot be neglected.

It was assumed in this derivation that the ions were distributed throughout the gas at random. We know, however, from the work of Debye and Huckel[23] that the positive ions tend to accumulate around the negative and vice versa so that the field of a given ion falls to $1/\epsilon$ of the value it should have by Coulomb's law, in a distance λ, which for the case of univalent ions having a temperature T is

$$\lambda=6.92(T/n)^{1/2}\text{ cm}. \tag{71}$$

Thus the value of r in Eq. (70) should be placed equal to λ, since the field due to ions at greater distances cannot be important.

For the example we have considered in Table III and Fig. 7 the positive ion current density I_P corresponding to β_C is 5.3×10^{-5} amps per cm^2. From this by Eq. (1) placing $T_P=3000°$ we find $n_P=0.88\times10^{10}$ ions per cm^3. Since there must be an equal number of negative charges we may take $n=1.76\times10^{10}$. Assuming the temperature T in Eq. (71) (average for electrons and ions) to be 5000°, we find $\lambda=3.7\times10^{-3}$ cm. Taking this for the value of r in Eq. (70) and letting $t=2.6$ cm, the radius of the cylinder we find $\theta_0=0.003$ radians or 0.17° as the mean angular deflection of the electrons due to the electric field of the ions.

The mean transverse energy corresponding to this deflection is $V(\theta_0)^2$ or only 0.0009 volts which is entirely negligible as a cause of the spread of the voltage at which the primary electrons are collected.

5. *Scattering by the field of molecules.* In an electric field of intensity X a molecule becomes a dipole of moment αx where α can be calculated from the dielectric constant D of the gas by the equation

$$\alpha=(D-1)/4\pi n \tag{72}$$

where n is the number of gas molecules per unit volume.

[23] P. Debye and E. Huckel, Phys. Zeits. **24**, 185, 305 (1923).

Because of this polarization, an electron and a gas molecule at a distance r attract one another with the force

$$F = 2\alpha e^2/r^5. \tag{73}$$

An electron which passes a gas molecule with a velocity corresponding to the potential V is thus deflected through an angle θ given by[24]

$$\theta = (3\pi/8)\alpha e/Vr_0^4 \tag{74}$$

where r_0 is the distance of nearest approach. This equation is derived on the assumption that θ is a small angle. The transverse energy $V\theta_0^2$ acquired by the electron is thus

$$V_T = (3\pi/8)^2\alpha^2 e^2/Vr_0^8 \tag{75}$$

expressing V in volts and α and r_0 in terms of cm we find

$$V_T = 2.84\times 10^{-14}\alpha^2/Vr_0^8. \tag{76}$$

The dielectric constant of nitrogen at atmospheric pressure and 0°C is 1.000581, so by Eq. (72) we find for this gas $\alpha = 1.71 \times 10^{-24}$. We have found the probability of collision of electrons with nitrogen molecules at 1000 baryes and 20°C to be 20.0 cm^{-1}. The number of molecules per cm^3 is then 2.48×10^{16} so the target area per molecule is 8.05×10^{-16} cm^2 corresponding to a target radius of 1.60×10^{-8} cm. All 100 volt electrons which pass within this distance of the center of the molecule presumably make elastic or inelastic collisions and are thus acted on by forces which are not of the type we are now considering. We may assume, however, that beyond this distance the force F given by Eq. (73) is effective. Thus placing $r_0 = 1.60 \times 10^{-8}$ cm in Eq. (76) and inserting the value of α we find that the maximum transverse energy that a 100 volt electron can acquire from this force is 0.192 volt.

The transverse energy falls off with a very high power of the distance r_0 so that the effect of multiple scattering is negligible compared with single scattering.

We thus conclude that the scattering due to the forces acting between electrons and molecules resulting from the polarization of the latter, is very small.

6. *Scattering by energy transferred between primary electrons.* It has been shown[25] that with high primary electron current density and low gas pressures the longitudinal energy of some of the primary electrons actually increases as the electrons traverse the space between the cathode sheath and the collector sheath. These electrons may thus reach the collector even when it is many volts negative with respect to the cathode.

In some of the present experiments with hydrogen and argon this effect was studied by varying the electron emission of the cathode from 1 to 90 milliamperes.

[24] F. Zwicky, Physik. Zeitsch. **24,** 171–183 (1923).

[25] I. Langmuir, Phys. Rev. **26,** 585 (1925).

Fig. 11 gives some typical data obtained with a tube having a molybdenum cylinder 5.0 cm in diameter with end plates 5.1 cm apart. The filament diameter was 0.18 mm. The full line curves were obtained with argon at a pressure of 13.4 baryes as indicated on the McLeod gauge, corresponding to 19.1 baryes in the cylinder at 650°K. The anode voltage was 60. Cathode currents, $-i_F$, of 3, 9, 29 and 90 milliamperes were used. To prevent overlapping the ordinates of 3 of the curves were increased by the amounts Δ indicated on the curves.

With a current of only 3 milliamperes the electrons begin to be collected in appreciable numbers when the collector is about 3 volts below the cathode, but as the current is increased to 9, 29 and 90, the voltages at which electrons are first collected become respectively -7, -19 and -32 volts. A more

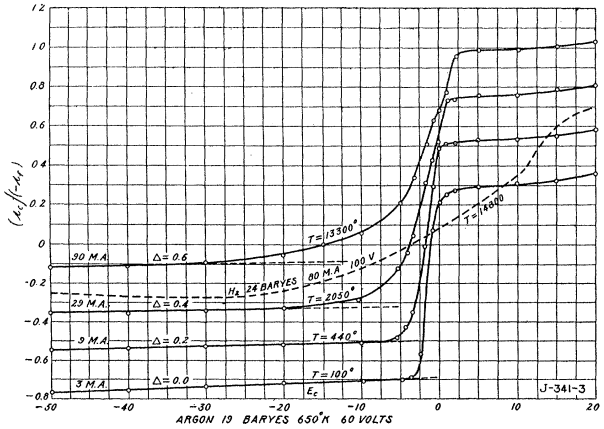

Fig. 11.

rational way of measuring the scattering is to determine[25] the slope of the curve obtained by plotting the electron current against the voltage on probability paper. The square of the reciprocal of this slope divided by 2900 V_0 (V_0 being the energy of the primary electrons in volts) gives the temperature corresponding to the random motion of the electrons which is assumed to be superposed on the translational motion of the beam. The temperatures calculated in this way are indicated on the curves of Fig. 11. The probability plots in these cases were straight only for negative voltages (below about -2 volts) so the slopes were taken from these straight portions.[26] With hydrogen similar effects were obtained as the current density was increased. The dotted curve in Fig. 11 was obtained with the same tube containing hydrogen at 16.4 baryes (24 baryes in the cylinder at 650°K) with a current of 80 milliamperes and an anode voltage of 100.

[26] Experiments have indicated that the deviations from the straight lines at the positive voltages are largely dependent on the fact that a collector is used which nearly completely surrounds the ionized gas. A small disk shaped collector in a discharge in a large spherical bulb gives a much more symmetrical curve which gives a nearly straight probability plot.

This effect was studied at several pressures of argon and hydrogen with various anode voltages. In each case the electron temperatures were calculated from probability plots. A summary of the results are given in Table V. The data given for argon at 19.1 and for hydrogen at 24 baryes are those illustrated in Fig. 11.

The data for argon show that the energy scattering decreases markedly as the pressure is raised, as was previously found for mercury vapor at pressures above 2 baryes[25].

TABLE V

Energy scattering at high currents

Gas	Pressure (baryes at 650°K)	Anode potential	Current $-i_F$	Electron temperatures
Argon	3.9	50 volts	3 ma.	500°K
			9	2000
			30	16000
			90	44000
	19.1	60	3	100
			9	440
			28	2050
			85	13300
	18.6	100.	9	300
			30	2100
			90	12000
	29.	100.	90	6700
Hydrogen	24	100	90	14800
	53	100	9	1200
	53	100	90	70000

On the other hand with hydrogen at a pressure of 53 baryes, a much greater scattering was found than at 24 baryes. With argon, as with mercury vapor, the scattering decreases slightly as the anode voltage is raised.

With 5 ma. emission from the cathode, which was the current usually employed in our studies of electron collisions, the energy scattering effect was always practically negligible.

Effects of current through cathode. In most of the experimental determinations of the current-voltage characteristics of the collector c which have been described, a commutator was used so that there was no current through the filament when the collector current was measured. To avoid the inconvenience of the commutator and the slight irregular variations in current that were sometimes caused by brush trouble, many characteristic curves were made without using the commutator. Two effects are thus introduced, the voltage drop along the filament (about 3-5 volts) caused the breaks in the curves, such as those at B and D in Fig. 7, to be less abrupt, and the magnetic field of the filament current deflects the primary electrons which leave the cathode with energy V_r in a radial direction, so that they acquire axial energy component V_a in the same direction as the motion of the electrons which constitute the heating current in the filament. From the

general equations given by Hull[27] for the motions of electrons near a conductor, we readily find, for the case that V_a is small compared to V_r that

$$V_a = 0.0188 A_F^2 [\log_{10} r/r_F)]^2 \tag{77}$$

where V_a is the axial energy in volts at a distance r from the axis of the filament, r_F is the radius of the filament and A_F is the filament heating current in amperes. The axial energy is thus independent of the anode voltage which determines V_r. The effect of the magnetic field is thus to lower the radial component of the energy by the amount V_a so that the break in the curve (B in Fig. 7) is displaced to the right by the voltage V_a. The experiments show this effect distinctly and in complete agreement with the theory.

The effect of the voltage drop along the filament can be calculated if we know the shape of the volt-ampere characteristic in the absence of this voltage drop. Let i be the current to the cylindrical collector when this is at a voltage E above the cathode and there is no voltage drop along the filament. Let i_M be the measured current when the collector is at a voltage E above the mid-point of the cathode, the voltage drop along the cathode being V_F. Then, if the collector is at a potential above that of the positive end of the filament, we find

$$i_M = (1/V_F) \int_{E-1/2V_F}^{E+1/2V_F} i \, d\, E$$

Expansion by Taylor's theorem and integration gives

$$i_M = i + (1/24) V_F^2 d^2 i/dE^2 + (1/1920) V_F^4 d^4 i/dE^4 + \cdots. \tag{78}$$

If the theoretical relation between i and E is not known, but experiment has given i_M as a function of E, we may use the inverted series,

$$i = i_M - (1/24) V_F^2 \, d^2 i_M/dE^2 + (7/5760) V_F^4 \, d^4 i_M/dE^4 + \cdots$$

and thus obtain the relation between the current and voltage that would exist if there were no voltage drop along the filament.

For the electrons that make elastic collisions, the relation between i and E is given by Eqs. (54) and (53). Eq. (78) can be written

$$\gamma_M = \gamma + (1/24) V_F^2 d^2\gamma/dE^2$$

and by inserting the value of $d^2\gamma/dE^2$ from Eqs. (54) and (53) we find

$$(\gamma = \gamma_M + (1/48) V_F^2 \gamma_e/(V_0^2 \theta_0^4) \epsilon^{-x} (1 + 1/x) \tag{79}$$

where x is defined by Eq. (53). Since approximate values of θ_0 and γ_e can be obtained from the experimental data by neglecting V_F (or by experiments with a commutator) Eq. (79) can be used to correct the values of γ obtained

[27] A. W. Hull, Phys. Rev. **25**, 652 (1925).

without the use of the commutator. In many cases these corrections have been applied, but usually the corrections are so small as to be unimportant.

Analysis of Current Voltage Curves at Higher Voltages

We have seen from the data of Fig. 3 that in mercury vapor the end plates do not remove any appreciable number of electrons having normal components of energy as large as 80 volts when the primaries have 100 volts energy. We have concluded from this that very few, if any, of the primary electrons are turned through angles as great as 90° by elastic or resonance collisions.

Let us now consider the electrons which begin to be collected when the end plate potential is raised to +20 volts or more. We may make a rough analysis of these groups of electrons by assuming that they consist largely of electrons having a Maxwellian distribution of velocities.

Mutual effect of electron collectors. Let a, b, c and f be the currents from electrodes A, B, C, and F into space, i.e.,

$$a+b+c+f=0. \tag{80}$$

Let a_e and a_p be the electron and positive ion parts of the current from a, so that $a=a_e+a_p$, $b=b_e+b_p$, etc. Let A_a and B_a be the areas of A and B. Then we may write

$$a=A_a I_e \epsilon^{-e(E_s-E_a)/kT}+a_p \tag{81}$$

$$b=B_a I_e \epsilon^{-e(E_s-E_b)/kT}+b_p. \tag{82}$$

Here E_s is the potential of the space which varies with E_a and E_b and is unknown. Eliminating E_s and I_e between Eqs. (81) and (82) we obtain

$$log(b_e/a_e)+log(A_a/B_a)=e(E_B-E_A)/kT \tag{83}$$

We thus plot the electron current to an electrode divided by the electron current to the anode on semi-logarithmic paper against the differences between the collector and the anode voltages. The slope of the straight line through the points gives the temperature or average energy of the electrons, and the ordinate at the space potential divided by the collector area gives the current per unit area of these electrons in space.

We have analyzed the curve marked B in Fig. 3 for the groups of electrons having a Maxwellian distribution of velocities. The curve B represents the current to the end plates A and B when they are in parallel and the potential is varied. The cylinder C was anode in this case.

The electron current to the end plates divided by the electron current to the anode was plotted on semi-logarithmic paper against the differences between the collector and anode voltages. The points lie on a straight line between 30 and 88 volts. The slope of this straight line gives a temperature of 358000 degrees for this group of electrons which we designate the first Maxwellian group.

The experimental points begin to deviate upward from the straight line above 88 volts. By plotting the current differences between these points and the extrapolated straight line, against the differences between the collector and anode voltages on a semi-logarithmic scale we find that the points lie on a straight line between 90 and 96 volts. This straight line has a slope corresponding to a temperature of 30500 degrees for a second Maxwellian group. By again plotting the differences in current between the experimental points and the extrapolated straight line of the second Maxwellian group on a semi-logarithmic scale, we obtain points which lie on a straight line between 96 and 100 volts. The slope of this line corresponds to a temperature of 7150 degrees for this low velocity group of electrons. We designate this group of electrons as the "Ultimate Electrons" since they are always present in any type of electron discharge in gases, carry practically the entire current and neutralize the space charge created by the relatively low velocity positive ions produced as a result of ionization of the gas molecules.

The curve marked B in Fig. 2 has also been analyzed into three Maxwellian groups of electrons. The first Maxwellian group is measurable when the potential of the cylinder C is raised above $+20$ volts. The temperature of this group is 350000 degrees. The second group appears above 88 volts on the collector and has a temperature of 31000°. The third Maxwellian group or the "ultimate electrons" are collected at potentials above 96 volts on the collector and have a temperature of 6900 degrees.

The area of the end plates A and B are 7.0 cm^2 each and the area of the cylinder C is 37.0 cm^2 in the experiments shown in curve B, Figs. 2 and 3.

The positive ion current collected on $A+B=4.46\times10^{-4}$ amps or 3.18×10^{-5} amps. cm^{-2} at $+100$ volts potential. By extrapolation we find that in curve B, Fig. 3, 0.4×10^{-5} and 0.44×10^{-5} amps. cm^{-2} of Maxwellian groups 1 and 2 are collected by $A+B$ at $+100$ volts. Similarly the total electron current collected by $A+B$ at 100 volts is 8.18×10^{-5} amps. cm^{-2}. By difference there is 7.34×10^{-5} amps. cm^{-2} of ultimate electrons collected at 100 volts. Hence there are 2.31 electrons for every positive ion collected at 100 volts. At the potential of the space according to Eq. (1), there should be $(T_e m_p/(T_p m))^{1/2}$ or 855 electrons collected to 1 positive ion if $T_p=(1/2)T_e$.

From the Boltzmann Eq. (3), $\log_e n_1/n_0 = Ve/kT$ we have $\log_e(855/2.31) = Ve/kT$, and thus since $T=7150°$, $V=3.65$ volts.

Therefore there is a positive ion sheath with a potential of 3.65 volts when the plates $A+B$ are at the anode potential, i.e., 100 volts positive with respect to the cathode. Thus there is a negative anode drop of 3.65 volts. Each collector in the discharge is then receiving all positive ions headed toward it and repelling all electrons which are not able to penetrate through the 3.65 volt sheath.

From Eq. (1) we have $n_p=(4.03\times10^{13}\times3.18\times10^{-5}\times606)/(3570)^{1/2}$ $=1.30\times10^{10}$ positive ions per cc per ma. of emission current.

Similarly the density of electrons in the various groups in space are $n_{e(1)}=2.9\times10^5$/cc ma.; $n_{e(2)}=4.1\times10^6$/cc ma.; $n_{e(\text{ult})}=1.30\times10^{10}$/cc ma. Thus

we observe that the ultimate electrons neutralize the positive ion space charge and practically the entire current in the discharge is carried by them.

The positive ion current to the collector C is equal to 5.13×10^{-5} amps. cm^{-2} at 100 volts on C in curve B, Fig. 2. The total electron current to C at 100 volts is 2.90×10^{-3} amps. Of this current 5.4×10^{-5} and 4.5×10^{-5} amps. belong to Maxwellian groups 1 and 2 respectively, leaving 2.8×10^{-3} amps. of ultimate electrons, which is equivalent to 7.57×10^{-5} amps. cm^{-2}. Hence there are 1.48 electrons to 1 positive ion collected at 100 volts.

Therefore assuming $T_p=(1/2)T_e$, we have $\epsilon^{Ve/kT}=574$, or $V=3.78$ as the retarding potential for electrons in the sheath when the collector is 100 volts positive with respect to the cathode.

From Eq. (1) we have $n_p=2.10\times10^{10}$/cc ma. emission. Similarly the density of electrons in the various groups in space are $n_{e(1)}=1.8\times10^{5}$/cc ma. emission; $n_{e(2)}=9.1\times10^{6}$/cc ma. emission; $n_{e(\text{ult})}=1.75\times10^{10}$/cc ma. emission.

We have repeatedly checked this method of calculating the negative anode drop in our experiments by means of the probe wire method of Langmuir and Mott Smith.[5]

For example in a typical experiment in mercury vapor at 17.3 baryes, the end plate A and the cylinder C were placed at 100 volts positive with respect to the cathode. A plot of the logarithm of the electron current against the voltage of the probe wire which was at a distance of one cm from the cathode gives us the average temperature of the ultimate electrons. Thus $T_e=3900°K$ in this case and E_s the potential of the space about the probe is 1.25 volts positive with respect to the anode. We find also that $I_e=0.51$ ma. of ultimate electrons per cm^2 per ma. emission, and $n_{e(\text{ult})}/=3.3\times10^{8}$ electrons per cc at the probe wire per ma. emission from the cathode.

The average current density of ultimate electrons reaching the anode is 0.013 ma. cm^{-2} per ma. emission from the cathode. Hence from Eq. (3) the anode is 1.07 volts negative with respect to the space near the collector. This agrees well with the probe wire measurement and also with the results calculated from the volt-ampere curves of the end plates and cylinder.

From the results obtained by analysis of the volt-ampere characteristics of both end plates and cylinder we may draw certain general conclusions. For example, when an electron having 100 volts energy strikes a mercury atom in such a manner that the atom gives up an electron and becomes a positive ion, there is, of course, a 10.4 volt loss by the incident electron in producing an ion. The remaining 90 volts energy is distributed between the incident electron and the electron given up by the mercury atom. The incident electrons and the electrons given up by the atoms give rise to the high velocity Maxwellian group of electrons which become random in direction and in the cases which we have considered have an average energy of about 45 volts.

Additional evidence of the formation of this high velocity Maxwellian group is obtained from the detailed analyses of the curves shown in Figs. 7 and 10. From these curves we see that the experimental curve deviates

gradually from the theoretical curve starting at the ionizing potential in each case.

Fig. 12 shows typical experimental curves in each of the different gases studied. The collector potential, E_c, is plotted against $i_c/-i_f$ in each case.

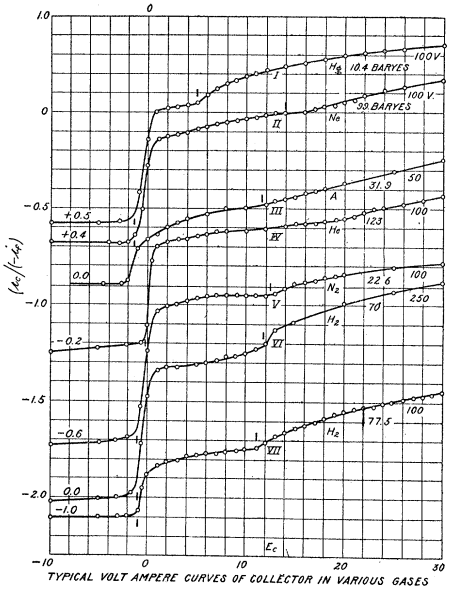

Fig. 12.

The curves have been shifted vertically by the amount indicated at the left end of each curve to prevent over-lapping. For example, curve 1 gives the results obtained in mercury vapor at 10.4 baryes pressure with the anode A at $+100$ volts potential, the curve having been shifted up 0.5 along the ordinate axis.

TABLE VI

Ionization and resonance potentials of various gases used

Gas	Obs. Res. Pot.	Ion. Pot.
Hg	6.7 volts	10.4 volts
Ne	18.5	21.5
A	13.0	15.3
He	21.1	24.5
N_2	13.0	16.8
H_2	12.8	16.1

The resonance potentials at which the kinks appear on the curves for the various gases have been determined from these curves and also from the large scale plots used in the detailed analysis of the groups suffering elastic and inelastic collisions. These potentials are given in Table VI and are probably correct to ± 0.5 volt.

Fig. 13 shows the experimental curves obtained with nitrogen at a series of different pressures. The anode (A) voltage was $+75$ volts in each case. The curves have been raised along the ordinate by the amounts indicated

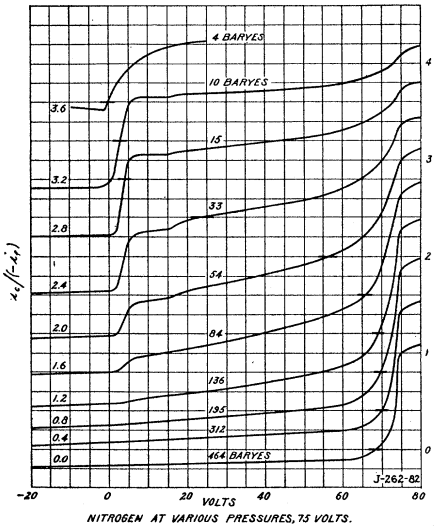

Fig. 13.

on the left side of the curves, the ordinate scale being at the right. The curves at the higher pressures show that no primary electrons reach the cylinder C without having suffered energy losses. As the pressure is decreased, the

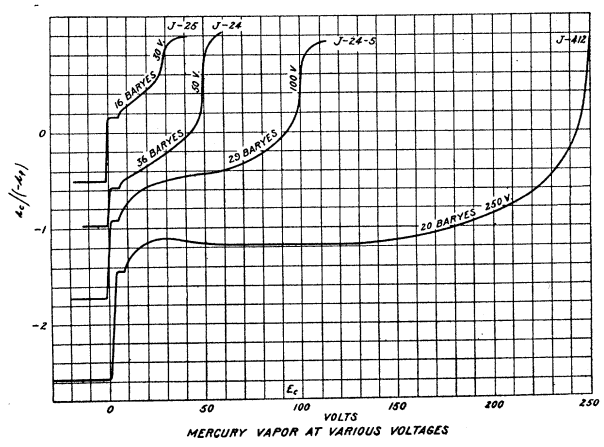

Fig. 14.

primary electrons begin to reach C without loss of energy until finally at 4 baryes pressure the intensity of ionization falls off rapidly at positive voltages on the collector because of the removal of nearly all the primary

electrons before they form ions and thus the positive ion sheaths around the cathode and cylinder over-lap, the current from the cathode drops suddenly to the "space charge" value and our theory no longer applies.

Fig. 14 shows the curves obtained in mercury vapor at a series of different anode voltages. The decrease in current values between +30 and +70 volts on the 250 volt, 20 barye curve is due to the fact that the reflection coefficient of the collector for electrons increases with the electron velocity. The constancy of the positive ion currents collected at negative voltages on the collector, the sharpness of the break at the resonance potential (6.7

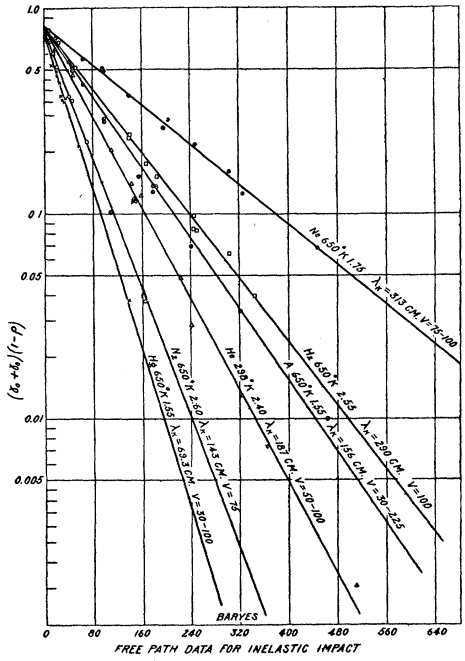

Fig. 15.

volts) and the relatively large number of electrons which suffer a 6.7 volt energy loss show up particularly well in these curves.

Electron Free Paths

The experimental data on the mean free path of high velocity electrons (above the ionizing potential) in the various gases give results in good agreement with the kinetic theory value calculated from viscosity measurements (i. e., $4\cdot(2)^{1/2}$ times the free paths of the gas molecules.)

We have plotted $(\gamma_0+\gamma_e)(1-\rho)$ against the gas pressure in baryes on a semi-logarithmic scale in Fig. 15. This quantity is the fraction of the primary electrons which are collectable by C without having made *inelastic* collisions with molecules and is thus given directly by the experiments when collector voltages just below the resonance potential are used.

Since the free path is inversely proportional to the pressure, we see readily from Eq. (32) that the slope $d \log_e y/dx$ of the lines of the semi-logarithmic plot of Fig. 15 should be R/λ_1, where R is the effective radius of the collector C and λ_1 is the free path of the electrons for inelastic collisions at unit pressure. The intercept of these lines on the vertical axis which corresponds to $1-\rho$ ranges from 0.77 to 0.83. Hence, ρ, the reflection coefficient of the collector for electrons striking with a velocity corresponding to ten to twenty volts varies from 0.17 to 0.23 in good agreement with the values derived from Eq. (58).

The values of λ_1 in cm calculated from the slopes of the lines are given in the 5th column of Table VII. These correspond to a pressure of 1 barye and the temperatures given in the second column. The electron free paths at the same temperature and pressure calculated from the viscosities are given in the 6th column. It is seen that the two sets of values agree rather closely.

In the last column of Table VII are the values of P_k, the probability of inelastic collision per cm at 20°C and 1000 baryes pressure, calculated from λ_1 by assuming that the probability is proportional to the density.

TABLE VII

Free paths for inelastic collisions.

Gas	T°K	R	Anode Voltage	λ_1	Kinetic Theory	P_k
Ne	650	1.75cm	75 and 100	313	288	7.1
H_2	650	2.55	100	290	266	7.6
A	650	1.55	30 50 100	156	160	1.4
He	298	2.40	50 100	187	166	5.4
N_2	650	2.60	75	143	146	1.5
Hg	650	1.55	30 50 100	69.3	72.9	32

We have summarized the results of the detailed analyses of more than two hundred sets of volt ampere characteristic curves for the various gases studied. The final results, expressed in each case at 1000 baryes pressure and 293°K are given in Table VIII. The various quantities in this table have all been defined with the exception of β (max.) The values of θ_r, the mean deflection for a single resonance collison, which appears in brackets, have been assumed to be the same as the mean deflection for a single elastic collision in order to obtain a reasonable value for P_r.

The value β (max.) gives the maximum number of ions which can be produced by an electron having a velocity corresponding to the potential given in the second column. The quantity β is independent of the pressure over a very wide range of pressures. At very low pressures, however, β decreases due to the fact that the sheaths around the cathode and the collector become so thick due to the low intensity of ionization that the electrons travel appreciable distances (compared to the tube radius) in the sheaths, (both in being decelerated and in being accelerated) in which the electrons will not have sufficient energy to produce an ion. If an inelastic collision

occurs under these conditions the electron will suffer a resonance energy loss instead of producing an ion. Then also at low pressures a larger proportion of the electrons will reach the anode while they still retain enough energy to cause ionization.

At pressures above the range in which β remains constant, β again decreases in value. From the theory of the diffusion of ions in ionized gases

TABLE VIII

Summary of data on electron impacts in gases

Gas	Volts	P	P_k	P_e	P_r	P_i	P_x	θ_e	θ_r	E_e	β_{max}
He	50	7.4	5.8	1.6	2.1	0.7	3.0	25	[25]	9.5	1.2
	100	5.6	4.5	1.1	0.5	1.22	2.78	19	16	11	2.9
Ne	75	7.9	7.0	0.9	1.1	1.38	4.52	21	21]	9.8	1.25
	100	7.6	6.9	0.7	1.0	1.80	4.1	19	[19]	11	2.0
A	30	32.4	14.3	18.1	4.4	3.5	6.4	24	[24]	5.3	0.45
	50	29.4	13.8	15.6	1.4	7.1	5.3	18	[18]	5.2	0.9
	100	24.9	14.0	10.9	1.3	8.55	4.15	12	[12]	4.7	1.65
	150	23.4	13.8	9.6	2.0	8.55	3.25	10	[10]	4.5	2.2
Hg	30	61.	36.	25.	13.	10.2	12.8	17	[17]	2.5	1.1
	50	59	36.9	22.1	11.0	14.9	11.0	11	13	3.1	1.4
	100	57.2	37.8	19.4	12.5	16.3	9.0	10	12	3.3	2.7
	250	40	24.	16.	5.0	15.3	3.7	6	[6]	3.2	5.3
H_2	100	10.9	6.9	4.0	2.6	2.9	1.4	5	16	7.3	1.45
	250	10.1	5.4	4.7	1.6	2.7	1.1	9	5	6.7	2.8
N_2	75	26.4	15.2	11.2	4.0	7.0	4.2	16	[16]	5.9	1.32
	100	20.0	12.2	7.8	3.6	7.7	0.9	14	18	5.8	1.65

due to Schottky and Issendorff[8] we find that, where the free paths of the electrons become small compared to the radius of the tube, the cathode may receive as many, or more ions than the collector. The current (i_F) must then be corrected for the large number of ions flowing to the cathode to obtain the true electron emission. When this correction is made and the pressure is not such that the electrons suffer an appreciable number of collisions in the sheaths we find that β agrees approximately with β(max).

In some experiments to test the constancy of β, the electron emission from the cathode ($-i_F$) was varied over a range of five hundred fold without changing the value of β. β being independent of the current density of primary electrons indicates that ionization is a single stage process in our experiments and that no two stage processes are effective in producing any measurable number of ions. It also indicates that recombination of electrons and ions is negligible under the conditions of these experiments up to 1000 baryes pressure.

The authors wish to take this opportunity to correct an erroneous statement made in a preliminary article in Science.[3] In this article it was stated that 60 or even 70 percent of the primary electrons lose nearly all of their energy on their first collisions with atoms producing a highly excited

atom or ion which subsequently caused the ionization of several other atoms. This incorrect statement resulted from our having neglected to consider the change in the reflection coefficient of electrons from the collector with voltage. As a result of our detailed analyses of the experimental curves with the reflection coefficient for each group of electrons treated separately we may safely say, that, if any electrons lose all of their energy in the first collision with an atom, the number suffering such energy loss is small.

The reflection of electrons from the collectors in our experiments is important because the ion sheaths which cover all electrodes (even the anode) act as *accelerating* grids to return the electrons to the field-free space where after reflection and *acceleration* through the positive ion sheath, it may have sufficient energy to cause ionization, resonance, etc. This is not true, however, under negative space charge conditions where the field is positive right up to the collector and any reflected electrons are trapped and collected. Thus, under negative space charge conditions in good vacuum, the three halves power law holds over a wide range of voltages.

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