THE PROPAGATION CHARACTERISTICS OF SOUND TUBES AND ACOUSTIC FILTERS

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ABSTRACT

The process of measuring the propagation characteristics of uniform sound tubes and acoustic filters is complicated by the reflections which may occur at the ends of these structures. The present paper applies some theoretical results on the effect of reflections, obtained in a previous paper,[1] to the measurement of the propagation characteristics of tubes and acoustic filters. In making these measurements the device to be measured is inserted in an acoustic transmission system, and the resulting changes in the magnitude and phase of the transmitted wave are measured. The actual observations are made in electrical circuits connected with the terminals of the acoustic system by loud speakers. The impedance of the acoustic system at the point of insertion is made an acoustic resistance. For measurements on straight tubes, the acoustic resistances used are of such a value as to prevent any appreciable reflections from the ends of the tubes, and as a result, the propagation characteristics of an infinite tube are obtained. The results of the measurements on straight tubes indicate that the Helmholtz-Kirchoff law is valid, while the results of measurements on acoustic filters are in good agreement with the theoretical results obtained previously.

I. INTRODUCTION

THE transmission of sound waves in straight tubes has received considerable attention from physicists. On the theoretical side, a number of workers—notably Helmholtz and Kirchhoff,—have determined the effect of viscosity and heat conduction dissipation on the propagation of a wave in a tube of infinite length. They have shown that the vector ratios of the pressures or linear velocities at any two points of an infinite tube can be expressed as the napierian base e raised to the power αl, where l is the distance between the two points, and α is a complex number depending on the condition considered.

Attempts to check experimentally the Helmholtz-Kirchhoff law on the propagation of a sound wave in an infinitely long tube are complicated by the fact that in practice we must measure the transmission in a tube of finite length and hence one in which reflection of the wave motion occurs at the ends. A neglect of these considerations has led many observers to conclude that the Helmholtz-Kirchhoff law was not valid.

The Helmholtz-Kirchhoff law has two measurable quantities to check, the attenuation constant of the tube, and the velocity of propagation of sound in the tube. A number of measurements of the sound velocity[2,3] have

[1] W. P. Mason, "A Study of the Regular Combination of Acoustic Elements with Applications to Recurrent Acoustic Filters, Tapered Acoustic Filters, and Horns," Bell System Tech. Journal, April, 1927.

[2] The derivation of the Helmholtz-Kirchoff law and the attempts to check this formula are described by I. B. Crandall, "Theory of Vibrating Systems and Sound," D. Van Nostrand, 1926, p. 229–241.

been made, but apparently no measurements of the sound attenuation have been made in the range in which the formula should hold.[4] The early measurements did not check the Helmholtz-Kirchhoff law, but the later measurements indicate that the form of this equation is correct. E. H. Stevens[3] finds a variation of velocity with frequency greater than that given by theory, while E. Grüneisen and E. Merkel,[3] find values less than that given by theory.

The laws of reflection and the methods for taking account of them in the electric line—which is the analogue of an acoustic tube—have been well understood since Heaviside's work on the propagation of electric waves. Heaviside found it necessary to introduce another parameter besides the propagation constant—which is the analogue of the exponent of the e in the Helmholtz-Kirchhoff law—namely, the characteristic impedance of the line which may be defined as the ratio of the electromotive force to the current at the input of an infinitely long line. With these parameters he could express the relations between the current at any point in the line and the applied electromotive force for any boundary conditions at the ends of the line.

The Heaviside impedance method was first applied to the study of acoustics by Webster.[5] In his paper Webster introduces the characteristic impedance of a dissipationless tube. In a previous paper,[1] the writer has extended this method to take account of dissipation as well. The present paper applies these theoretical results to eliminate the effect of reflections in a finite tube, and measurements have been made which show that the Helmholtz-Kirchhoff law is entirely valid for propagation of sound waves in a smooth tube.

In the theoretical paper,[1] the combination of straight tubes to form acoustic filters, was considered. The use of the formulas taking account of the wave motion removes the assumption introduced by Stewart,[6] in his theory of acoustic filters, that no wave motion need be considered in the elements. These wave formulas take account of dissipation and in addition the effects of the terminating conditions have been investigated. An expression was obtained for the insertion factor, giving the absolute values of the ratios of pressures or volume velocities in the termination of an acoustic system with the filter in, to these quantities with the filter out. Hence this factor represents the effect of inserting the filter in a given acoustic system. The combination of filters whose conducting tube areas increase in some regular manner was also investigated, and it was shown that in addition to the filtering action, a transformer action takes place. Horns are the limiting cases of tapered acoustic filters, and their equations can be derived from those of tapered acoustic filters.

[3] E. H. Stevens, Ann. d. Physik **7**, 285 (1902)
E. Grüneisen and E. Merkel, Ann. d. Physik **66**, 344 (1921).

[4] L. F. G. Simmon and F. C. Johansen, Phil. Mag. **50**, 53 (Sept. 1925) give measurements at very low frequencies.

[5] A. G. Webster, "Acoustic Impedance, and the Theory of Horns and of the Phonograph," Nat. Acad. of Science, **5**, 275 (1919).

[6] G. W. Stewart, Phys. Rev. **20**, 528 (1922); **23**, 520 (1924); **25**, 90 (1925).

To obtain an experimental check of the equations for acoustic filters, the same measuring circuit used to measure the straight tubes was employed, and a good agreement with theory was found.

II. Method of Measurement

The method employed here for measuring the transmission characteristics of acoustic devices is an adaptation of the ordinary electrical method for measuring insertion factors and insertion phase angle differences. The method consists primarily in transmitting energy, in electric or acoustic form, simultaneously over two parallel branches. One branch contains the structure under investigation, while the other contains an adjustable comparison circuit. The current reaching the termination of both can be compared both in magnitude and phase, by adjustments in the comparison branch and in the terminating circuit.

The knowledge ordinarily required about all electrical networks is what change will these networks produce when inserted in a given electrical

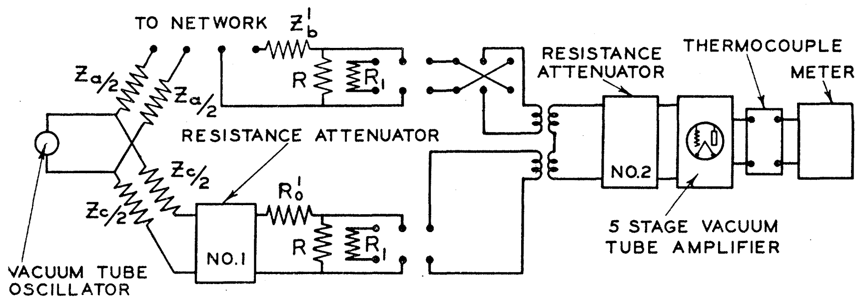

Fig. 1. Diagram of electric transmission measuring circuit.

system. That is, we need to know the ratio of the current in the termination of the system when the network is inserted to the current in the termination when the network is not in circuit. The absolute value of this ratio is called the insertion factor, while the difference in phase angle between the two currents is called the insertion phase angle difference.

The circuit employed for measuring these quantities is shown on Fig. 1. It consists in a source of sinusoidal voltage, here a vacuum tube oscillator, connected to two parallel branches, which in turn are connected to a terminating circuit by means of which the outputs from the two parallel branches can be compared.

The upper branch contains an input electrical impedance Z_a, and an output impedance Z_b. The terminating impedance Z_b is made up of an impedance Z_b' and a small resistance R. Wires from both sides of this resistance go to a double-pole double-throw switch, which on one side is connected to a resistance R_1, and on the other side to a reversing switch. From the reversing switch, wires go to one side of a three-winding transformer, with two equal input windings.

The lower branch contains an impedance Z_c in series with a resistance attenuator, which in turn is terminated in a resistance R_0. This resistance is made up to two parts, a resistance R_0' and a small resistance R. Wires from this resistance go to a double-pole double-throw switch and then to the other side of the transformer. The output of the transformer is connected to a second resistance attenuator, whose termination is connected to a vacuum tube amplifier and a thermocouple.

To measure the insertion factor and phase angle due to the unknown circuit, the impedance Z_c plus R_0 is set equal to Z_a plus Z_b. The resistance attenuators employed are combinations of resistances which have the property that when they are terminated in the resistance R_0, the impedance at the input terminals of the attenuator is always R_0. By turning the dials of the attenuator, various known ratios of output to input current can be obtained. Hence the current into the lower branch will have the value $e/(Z_a+Z_b)$. If the unknown network is not inserted in the upper branch and the impedances Z_a and Z_b are connected in series, the current in upper and lower branches will be identical both in phase and magnitude. The current in the output of the upper branch with the network inserted will obviously be the current which exists when the network is inserted between the impedances Z_a and Z_b and a source of e.m.f. e is placed in series with Z_a. Hence the ratio of the current in the impedance Z_c to the current in the impedance Z_b is the ratio of the current in Z_b with the network out of circuit to the current in Z_b with the network in the circuit.

To measure this ratio, the voltage in the upper branch across the small resistance R is impressed upon the succeeding circuit and by adjusting the gain of the amplifier a convenient reading is obtained on the meter connected to the thermocouple. Then throwing the upper branch switch to the resistance R_1, which has the same value of impedance as the circuit on the opposite side of the switch, and throwing the voltage across the resistance R of the lower branch through the succeeding circuit, a second reading is obtained. The resistance attenuator No. 1 is then adjusted, until a reading equal to the first is obtained. Hence the voltages across R for both branches are the same, and therefore the output currents of the two branches are equal. The value of the insertion factor can then be read from the resistance attenuator. The square of the insertion factor gives the insertion power ratio, that is, the ratio of the powers in the termination of the system when the network is in the system and out of the system. In all curves given, ten times the logarithm to the base 10 of the power insertion factor has been plotted so that the results are shown in the standard unit of attenuation, TU.

The method for measuring the phase angle is to compare the absolute values of the vector sum and vector difference of the currents from the two branches. This will determine the value of the angle, except that it will not be known whether it is in the upper or lower quadrants. By starting with a sufficiently low frequency whose phase angle is known, its location can readily be determined. To obtain the vector sum, both double-pole double-throw switches are thrown on the amplifier side, and a reading is obtained which corresponds to the vector sum of the two equal currents, since an equality balance had previously been obtained. Throwing over the reversing switch, a reading corresponding to the vector difference of the two currents is obtained. By setting the resistance attenuator for no attenuation for the smaller of these two readings and varying it until an equal reading for the vector sum and vector difference is obtained, the ratio of the vector sum to the vector difference can be measured. The phase angle Θ and this ratio r are related by the formula

$$\cos \Theta = (r^2 - 1)/(r^2 + 1)$$

In applying this method for acoustic measurements, use is made of the fact that insertion factors and insertion phase angle differences, are respectively multiplicative and additive. That is, the insertion factor for a complete network, between the impedances Z_a and Z_b, is equal to the product of the insertion factor for a portion of the network, measured between the impedances Z_a and Z_b, and the insertion factor for the remainder of the network, measured between the impedances at the insertion junction looking toward and away from the source. Hence, conversely, the insertion factor of the portion of the network inserted last, measured between the impedances at the insertion junction, is equal to the ratio of the insertion factor of the total network to that of the first portion of the network, both measured between Z_a and Z_b. A similar result holds for the insertion phase angle differences except that these are additive rather than multiplicative.

To adapt this circuit for acoustic measurements the complete network was so constructed that the energy entering it was converted to acoustic form, traversed an acoustic path, was reconverted and emerged as electric energy. For this purpose two #555 Western Electric loud speaker units were utilized. A point in the acoustic path was taken as the junction at which the portion of the network, here the acoustic device under investigation, was inserted. The measurements gave the insertion effects of the device between the acoustic impedances of the path at the point of insertion. One problem then was to provide the proper acoustic impedances at this point of insertion. This has been accomplished by means of some acoustic resistances, which will be described below. The acoustic system used for these measurements consisted of a loud speaker unit, which was connected to an acoustic resistance, this in turn being connected to an exponential horn. The device to be measured was then inserted, and the same three units in opposite order were used to terminate the device and connect back to the electrical circuit.

The measuring process consisted in measuring the insertion factor and phase angle with the acoustic device in circuit; then taking the acoustic device out, that is, connecting the horns directly together, and measuring the insertion factor and phase angle again. In the same manner as in the electrical case, the ratio of the two measurements represents the insertion factor, while the difference between the phase angles represents the insertion phase angle difference, both measured between acoustic impedances looking in each direction at the insertion junction.

In order to determine what these impedances should be in the ideal case, and how much departure from the ideal may be permitted in practice, we must examine the relations between the propagation constant and the insertion ratio as a function of these impedances.

III. Determination of the Reflection Effects for Straight Tubes and Acoustic Filters

In measuring the reduction of volume velocity or the phase change due to any acoustic structure, we have always a given input and a given output boundary condition to satisfy. The knowledge of the acoustic structure generally required is what change will the structure cause when it is inserted in a given acoustic system. We define the volume velocity insertion factor of a given structure in a given system as the ratio of the volume velocity in the termination of the system when the structure is in the system to the volume velocity when the structure is out of the system. A similar factor can be defined for the pressure, but in symmetrical devices such as we are considering, the same insertion factor holds for both quantities.

To obtain this factor for a straight tube, use is made of the equations derived in the preceding paper.[1]

$$\begin{aligned} p &= p_1 \cosh \alpha L - (V_1 Z_L / S) \sinh \alpha L \\ V &= V_1 \cosh \alpha L - (p_1 S / Z_L) \sinh \alpha L \end{aligned} \tag{1}$$

where p is the excess pressure at the distance L from the beginning of the tube, p_1 the initial excess pressure at the beginning of the tube, S the cross sectional area of the tube, V the volume velocity at the distance L from the beginning of the tube, V_1 the volume velocity at the beginning of the tube, α the propagation constant and Z_L the specific characteristic impedance of the tube, *i.e.* the vector ratio per sq. cm of the pressure to the volume velocity for an infinite tube. α and Z_L for the frequencies of interest here are given by the formulas

$$\alpha = a + ib = \left[\frac{P\gamma' \omega^{1/2}}{2CS(2\rho)^{1/2}} + \frac{i\omega}{C} \left\{ 1 + \frac{P\gamma'}{2S(2\omega\rho)^{1/2}} \right\} \right] \tag{2}$$

$$Z_L = R + iX = (P_0 \gamma \rho)^{1/2} \left[1 + \frac{P\gamma'}{2S(2\omega\rho)^{1/2}} - i \frac{P\gamma'}{2S(2\omega\rho)^{1/2}} \right] \tag{3}$$

where P is the perimeter of the tube, ρ the density of the medium, $\omega = 2\pi$ times the frequency, f, P_0 the average atmospheric pressure, γ the ratio of the specific heats of the medium, $C = (P_0 \gamma / \rho)^{1/2}$ the velocity of sound in a dissipationless medium, $\gamma' = \mu^{1/2}[1 + (5/2)^{1/2} (\gamma^{1/2} - \gamma^{-1/2})]$, μ the coefficient of viscosity of the medium.

As shown in standard books, $e^{-\alpha L}$ is the ratio of the volume velocities, or pressures, at points of a tube of infinite length which are a distance L apart. bL, the phase constant, represents the phase rotation (in radians) between these two points. The velocity of propagation in a tube is C' where

$$C'=\frac{\omega}{b}=C\left\{1-\frac{P\gamma'}{2S(2\omega\rho)^{1/2}}\right\} \tag{4}$$

Assume now that the tube is terminated in an impedance Z_B/S, while at the input a source of simple harmonic pressure p_0, is placed whose internal impedance is Z_A/S. Then substituting in equation (1), the values

$$p/V=Z_B/S \text{ and } p_1=p_0-V_1Z_A/S$$

we obtain for V in terms of p_0, and the system parameters

$$V=\frac{p_0S}{Z_A+Z_B}\times\left[\frac{Z_A+Z_B}{2Z_B}\times\frac{2Z_L}{Z_L+Z_A}\times\frac{2Z_B}{Z_L+Z_B}\times e^{-\alpha L}\times\left\{\frac{1}{1-e^{-2\alpha L}\frac{(Z_L-Z_A)}{(Z_L+Z_A)}\frac{(Z_L-Z_B)}{(Z_L+Z_B)}}\right\}\right] \tag{5}$$

Now the volume velocity in the termination of the acoustic system if the tube were not present is obviously $p_0S/(Z_A+Z_B)$. Hence the ratio of the volume velocities of the system with the tube in and with the tube out will be the part of equation (5) in brackets. It can be shown that this factor will result for any structure which can be represented by a characteristic impedance and a propagation constant if these quantities are substituted respectively for Z_L and αL.

In measuring straight tubes, in order to check the Helmholtz-Kirchhoff law, we are interested in measuring the term $e^{-\alpha L}$. Hence, all factors except this must be eliminated or else corrected for. We notice that if $Z_A=Z_L$ or $Z_B=Z_L$, then the expression reduces to $e^{-\alpha L}$, and hence the insertion factor reduces to the propagation factor. Now the characteristic impedance of a tube, given by equation (3) is very nearly a pure resistance, it having in addition a small amount of negative reactance, which in general is less than two or three percent of the resistance for the frequencies of interest. Hence, if Z_A or Z_B are made pure resistances equal in value to Z_L, all terms but $e^{-\alpha L}$ are eliminated. Since it is difficult to make Z_A or Z_B exactly equal to Z_L the most accurate results will be obtained by letting $Z_A=Z_B$ and making them both as near Z_L as possible. So, if Z_A does not differ from Z_L by more than twenty percent in resistance, and if its reactance is not more than twenty percent of the absolute value of Z_L, the error caused by the first and third terms will not be more than 5 percent in power ratio or more than 0.1 radians in phase angle. This is about the accuracy of measurement of the power ratio, but somewhat less than the accuracy that can be obtained for the phase angle, which is about 0.02 radians. Hence if the measuring circuit is such that it gives the ratio of the volume velocities or pressures in the termination of an acoustic system with the tube in and out, and if impedance terminations are obtained which do not differ by more than 20 percent from Z_L, a system is obtained which will measure $e^{-\alpha L}$ accurately.

IV. Method for Obtaining an Acoustic Resistance

Equations (1), (2) and (3) show one method for obtaining an acoustic resistance, namely by using a long tube. The impedance of an infinitely long tube is Z_L. The impedance of a finite tube terminated in an impedance Z_d is by equation (1), substituting $p = VZ_d/S$ and solving for $p_1/V_1 = Z_1/S$

$$\frac{Z_1}{S} = \frac{Z_L}{S}\left[\frac{(Z_d/S)\cosh \alpha L + (Z_L/S)\sinh \alpha L}{(Z_L/S)\cosh \alpha L + (Z_d/S)\sinh \alpha L}\right] \tag{6}$$

The impedances Z_d for which Z_1 will show the greatest variation from Z_L are zero and infinity. For these values, Z_1 is respectively

$$Z_L \tanh \alpha L \text{ and } Z_L \coth \alpha L \tag{7}$$

If the attenuation constant aL, of the propagation constant αL is made quite large, Z_1 will not depart far from Z_L. If for example the value of aL in equation (2) is 1.5, then the resistance component cannot vary by more than 10 percent from that of Z_L, and the reactance component will not be more than 10 percent of the resistance component. Hence if a long tube is put on each end of the acoustic device to be measured, a good termination is obtained, which satisfies the above requirements. The length of tube required, however, would be quite great. For example for the largest size tube measured below, the length of tube required for each end would be 34 meters if we wish to measure down to 200 cycles.

This length of tube is rather long for ordinary use, and hence some work has been done on methods for decreasing this length. One method for accomplishing this result is to build up the end termination out of smaller tubes, the sum of whose inside area equals that of the device we wish to terminate. With the smallest hexagonal tubes available, it was found that the length required on each end would be reduced to 6 meters in this way without causing the characteristic impedance to deviate by more than seven percent from the characteristic impedance of a long tube. This length being still rather long, two other methods were devised for reducing the tube length.

The first method employed was to use half of the tubes closed and half open. Since the tubes are all in parallel in the electrical sense, the total impedance of an open tube and a closed tube will be

$$\frac{Z}{S} = \frac{(Z_L/S_1)\tanh \alpha_L)(Z_L/S_1)\coth \alpha_L)}{(Z_L/S_1)\tanh \alpha L + (Z_L/S_1)\coth \alpha L} = \frac{Z_L}{2S_1}\tanh 2\alpha L \tag{8}$$

where S_1 is the area of a single tube. Hence an open and closed tube in parallel are equivalent to an open ended tube of twice the area and twice the length of either. By employing combinations of open and closed tubes the length can be decreased to one-half its former value.

The second method employed is to combine tubes which have the same characteristic impedance, but whose lengths differ among themselves. Starting with the shortest, the length from tube to tube is increased by an equal amount in each case. To show what sort of an impedance such a combination will give, equation (6) is written in a different form

$$\frac{Z_1}{S_1}=\frac{Z_L}{S_1}\left[\frac{1-\dfrac{(Z_L-Z_d)}{(Z_L+Z_d)}e^{-2\alpha L}}{1+\dfrac{(Z_L-Z_d)}{(Z_L+Z_d)}e^{-2\alpha L}}\right]=\frac{Z_L}{S_1}\left[\frac{1-Ke^{-2\alpha L}}{1+Ke^{-2\alpha L}}\right] \tag{9}$$

where K is the volume velocity reflection factor $(Z_L-Z_d)/(Z_L+Z_d)$.

The impedance Z/S of n of these tubes in parallel, when the tube lengths are increased by a given amount Δ, from tube to tube, is given by the expression

$$\frac{S}{Z}=\frac{S_1}{Z_L}\left[\frac{1+Ke^{-2\alpha L}}{1-Ke^{-2\alpha L}}+\frac{1+Ke^{-2\alpha(L+\Delta)}}{1-Ke^{-2\alpha(L+\Delta)}}+\cdots+\frac{1+Ke^{-2\alpha(L+(n-1)\Delta)}}{1-Ke^{-2\alpha(L+(n-1)\Delta)}}\right] \tag{10}$$

Expandingthe expression and summing up the series, we obtain the infinite series

$$\frac{S}{Z}=\frac{S_1}{Z_L}\left[n+2Ke^{-2\alpha L}\left\{\frac{1-e^{-2n\Delta\alpha}}{1-e^{-2\Delta\alpha}}\right\}+\cdots+2K^ie^{-2i\alpha L}\times\left\{\frac{1-e^{-2ni\Delta\alpha}}{1-e^{-2i\Delta\alpha}}\right\}+\cdots\right] \tag{11}$$

Now the value of $(1-e^{-2n\Delta\alpha})/(1-e^{-2\Delta\alpha})$, if no dissipation is assumed, is shown on Fig. 2, and is equivalent to the expression for the light from

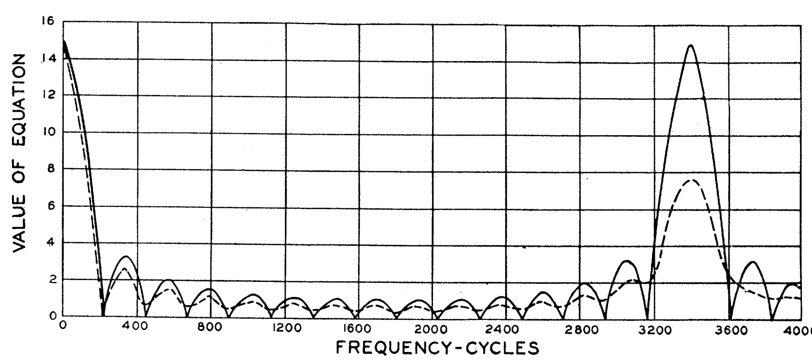

Fig. 2. Typical plot of equation $(1-e^{-2N\Delta\alpha})/(1-e^{2-\Delta\alpha})$. (Solid line gives non-dissipative case, dotted line shows effect of dissipation)

a diffraction grating between two successive images. It generally is around the value 1 except near the upper and lower end of the curve. The effect of dissipation is to smooth out this curve, as shown in Fig. 2, which gives the case actually used here. This expression is multiplied by $e^{-2\alpha L}$ which generally is a small number. For the case considered here it has the value

.25 at 1000 cycles. Hence above this frequency, all terms except the first two can be neglected, since they contain powers of $e^{-2\alpha L}$, and since the value of $(1-e^{-2ni\Delta\alpha})/(1-e^{-2i\Delta\alpha})$ is generally around 1. When the second term approaches the value 1, it will affect the absolute value of the impedance by less than the factor $(n-1)/n$ and hence if n is large, the total impedance will be nearly

$$Z/S = Z_L/nS_1$$

The lowest frequency for which this occurs will be when the length $2n\Delta$ is one wave-length.

Fig. 3, shows the acoustic resistance actually used for these measurements. It consists of 37 hexagonal brass tubes fitted together into a hexagonal form, leaving no space between the tubes. Thirty of these are grouped in pairs, one tube of each pair having an open end and the other tube a closed end. The lengths of these pairs vary from 38.1 cm to 73.6 cm in equal

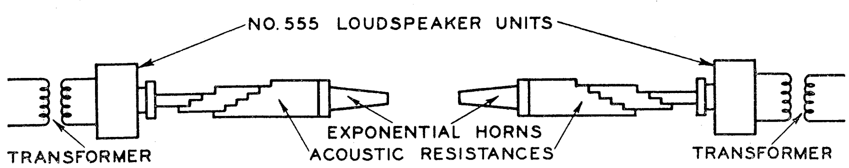

Fig. 3. Acoustic section of transmission measuring circuit.

steps. From wall to wall the diameter of these tubes is 0.32 cm. The seven tubes in the center are somewhat longer than the others, each of these being 78.8 cm, and are connected to the #555 loud speaker. They are used to conduct sound into the system, or to take off a small amount of sound at the receiving end. These tubes are only a small part of the total, and their length is such that no matter what their termination is, the impedance that they contribute to the total terminating impedance cannot vary by more than forty percent. As their area is a small fraction of the total, their effect on the terminating impedance is small.

The entire acoustic circuit is shown on Fig. 3. As we wish to measure several sizes of tubes, and it is inconvenient to use more than one size of acoustic resistance, some logarithmic horns were used to taper down from the acoustic resistance to the size of tube to be measured. It has been shown previously,[1] that the characteristic impedance per sq. cm of a logarithmic horn remains unchanged throughout its length and hence the horn acts as a transformer. Furthermore, if we are interested in frequencies which are large compared to its cut-off frequency, the characteristic impedance is nearly that of a straight tube. The horn used here was one having a cut-off frequency of 55 cycles. At frequencies above 400 cycles, the irregularities introduced by the horn were less than 10 percent. Corrections for the horn distortion have been made on the measurements of the two smallest tubes up to 400 cycles. All other measurements given are the insertion factor measurements as made using the above circuit.

V. Experimental Results

Measurements have been made of the attenuation constant of four tubes of different diameters. The phase constant of the smallest tube has also been measured. The radii of the tubes measured were 0.851 cm, 0.687 cm, 0.525 cm, and 0.370 cm, while the corresponding lengths were 622 cm, 488.5 cm,

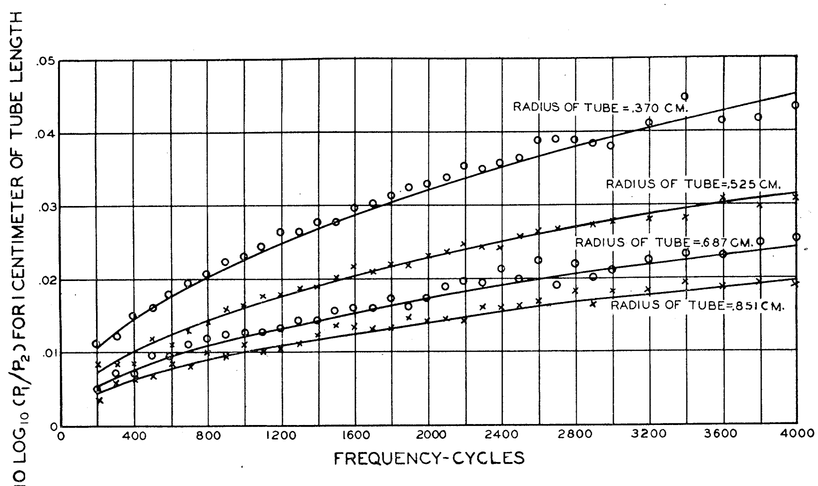

Fig. 4. Measurements of attenuation characteristics of four tubes.

426 cm and 763.4 cm. The thickness of the brass tube walls was about 1.02 mm. All measurements were made at room temperature, which averaged about 23.5°C. The measured values of the insertion power ratios are given by the points on Fig. 4. All values have been expressed per centimeter of tube length. The solid lines of this figure are a plot of the theoretical values

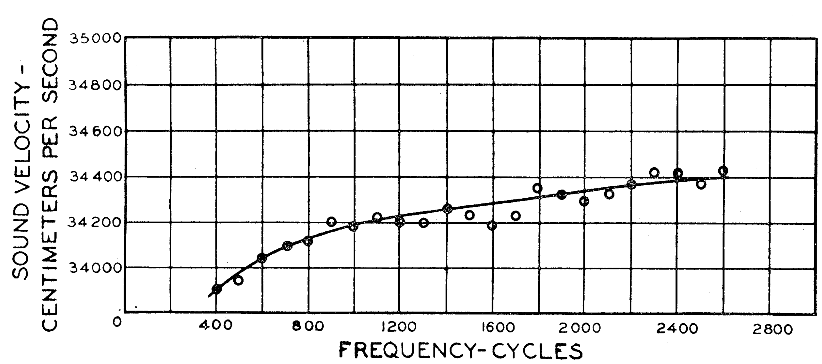

Fig. 5. Measurement of the velocity of sound in a small tube.

given by equation (2), assuming a value of μ of 1.86×10^{-4}. It is evident that the measured and theoretical values are in good agreement, and hence it appears that the Helmholtz-Kirchhoff law holds very well for the tubes measured, for frequencies from 200 to 4000 cycles.

A plot of the measured phase constant is shown on Fig. 5. What is plotted is ω/b, which is the velocity of propagation. By means of equation (4) and the known constants of the tube, we can calculate the theoretical value of the velocity for this tube. The result is shown by the full line of Fig. 5.

Measurements have been made of the insertion power ratio of two acoustic filters. The filter used, shown in Fig. 6, consists of a main conducting

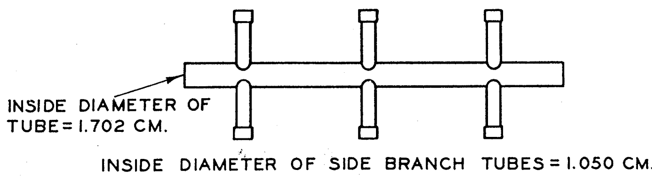

Fig. 6. A typical acoustic filter.

tube and two side branches at each junction point. By putting on or taking off caps, the side branches could be either closed or opened on the end. The first filter, a low-pass type, was obtained by putting all the caps on the side branches. The second, a high-pass type, was obtained by removing all of the caps. Since the side branch is small compared with the main conducting tube, we take the end correction for the side branch to be 0.82 R.[7] With this

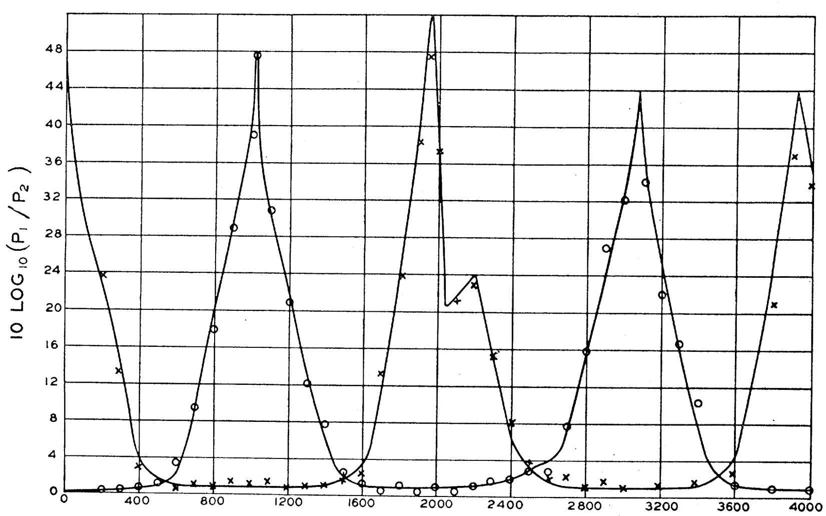

Fig. 7. Measurements of the insertion losses of two acoustic filters.

correction, the length of the side branch closed is 8.49 cm, and the length open is 8.79 cm. The distance between successive side branches is 16.7 cm. The total number of sections employed is three.

[7] Reference 1, page 263.

The measurements of these filters are shown by the points on Fig. 7. Calculated values of the insertion power ratio, making use of equations (25) and (31) of the preceding paper,[1] to give the filter parameters to be inserted in equation (5) of this paper, are given by the solid lines of Fig. 7. In making these calculations, the terminating impedances Z_A and Z_B have been taken as resistances whose values were $(P_0\gamma\rho)^{1/2}$ per sq. cm. The degree of agreement is shown by the figure.

VI. Discussion of Results

The results obtained for the attenuation measurements of straight tubes, which are little affected by temperature changes, indicate that the sound energy attenuation obeys closely the Helmholtz-Kirchhoff law. Since no means were available for preserving a constant temperature during the experiment, no great accuracy can be claimed for the deviation of velocity with frequency measurements. The results obtained agree with the Helmholtz-Kirchhoff law within the accuracy of measurement. The results obtained for the acoustic filter measurements agree well with the theoretical values and indicate that all assumptions made are valid.

The measuring method and circuit gives results accurate to about 5 percent in power ratio and about .02 radians in phase shift. In order to utilize the phase accuracy, it would be necessary to maintain a constant temperature and to have acoustic resistance terminations which do not vary by more than 5 percent from the characteristic impedances of the tubes. This last requirement can be met by eliminating the logarithmic horns, and by using acoustic resistances with longer tubes than were employed here.

In conclusion the writer wishes to express his thanks to Professors Wills and Webb of Columbia University for their interest in the investigation.

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