EFFECT OF FREQUENCY ON THE END CORRECTION OF PIPES

By S. Herbert Anderson and Floyd C. Ostensen

Abstract

Using Blaikley's method, the end corrections of "closed" pipes have been determined for pipes of brass, with walls 0.12 cm thick and the inside diameter 9.92 cm, 7.36 cm and 4.84 cm respectively, over a range of frequencies from 128 vps to 1715 vps, and over a range of λ/D (wave-length/diameter) from 27 to 4. An electrical detector was used for determining the resonance peaks, which makes this work ten times more accurate than any formerly reported. The highest frequency used with each pipe was the limiting frequency for which resonance could be obtained, and in each case was such that $\lambda/D=4$ approximately.

The end correction was found to be dependent upon the frequency, increasing with decreasing values of λ/D to $\lambda/D=6$ approximately, beyond which there was a decrease. The results indicate that when a pipe is used as a generator of a complex tone, (1) there is a limit to the number of partials which is determined by the value of λ/D for prime, (2) the higher proper tones will in some cases be sharp, but generally flat of the harmonic series, and that the corresponding partials will be of low intensity depending upon the amount of forcing required to bring them into the harmonic series.

Introduction

It is well known that narrow pipes used for the production of sound have a retinue of partials larger than wide pipes. One reason generally assigned for this difference is that the "end correction" apparently becomes smaller[1] as the wave-length approaches the diameter of the pipe, so that the higher proper tones are sharper than the corresponding harmonics of the fundamental tone. The bases for this statement, theoretical and experimental are not very convincing.

Theoretical treatment of the end correction of pipes has been given by Helmholtz[2] and by Rayleigh.[3] The former found a rigorous solution for a pipe that was *nearly* cylindrical. If R is the radius of the open end, the pipe is practically a cylinder of radius R throughout the greater part of its length, but bulges slightly at a distance $0.54R$ from the mouth. He also assumes, (1) that the mouth is fitted with an infinite flange, and (2) that the wave-length is large compared to R. He finds the end correction under these conditions to be $\pi R/4$. By a different method Rayleigh computed the end correction of a true cylinder, but subject to the same restrictions, and found it to be $0.82R$.

In practice both these restrictions are violated. Pipes are seldom, if ever, fitted with flanges, and while the diameter of the pipe is generally small

[1] Barton, "Textbook of Sound," Macmillan, 1908, p. 253; Crandall, "Theory of Vibrating Systems and Sound," Van Nostrand, 1926.

[2] Helmholz, Crelle-Borchardt, Journal f. Mathematik, 57, 1 (1860).

compared to the wave-length of the first mode of vibration, for higher modes the quarter-wave-length may approximate the diameter of the pipe. In this case the reflection from the open end must be quite different from that for longer wave-lengths.

As the unflanged pipe is not amenable to theory, we must depend alone on experiment for information of the end corrections of practical pipes. From his own work[4] and that of Bosanquet,[5] Rayleigh concludes that this is $0.6R$ for wave-lengths large compared to the diameter of the pipe. The effect of shorter wave-lengths was not tested. At an earlier date rather extensive work was done by Wertheim[6] and Zamminer.[7] The former found a mean value for open pipes of $0.663R$ and for closed (open at one end only) of $0.746R$, and not much variation with wave-length. On the other hand Zamminer found the end correction to decrease with decreasing values of λ/D. These results cannot be considered very reliable as in both cases the pipes were made to sound by blowing a jet of air across the open ends. Under such conditions the frequency obtained depends somewhat on the shape and velocity of the air jet and the reflection is quite different from that occurring at an undisturbed open end. Bosanquet[8] also found the correction to be dependent upon the wave-length, but the larger correction is for small values of λ/D. He considers his best values for the end correction to be $0.635R$ for $\lambda/D=6$ and $0.543R$ for $\lambda/D=15$ with open pipes. In his work the vibrations were started by a jet of air and the pitch determined after the jet was stopped as the sound died away. He estimates his error to be about 7 percent.

The best experimental work heretofore done was by Blaikley.[9] His method consisted in finding the shortest resonant length L_1 of a closed pipe when forced to vibrate by the action of a tuning fork, and the next longer resonant length L_2. Then the correction is given by

$$c=(L_2-L_1)/2-L_1 \tag{1}$$

His range of λ/D was small, from 13 to 26, and he found no great variation, the mean value being $0.576R$ with a brass tube 5.3 cm in diameter.

In view of the discrepancies of this earlier work it seemed wise to attempt another determination of the end correction over a wide range of wave-lengths and with some improvements in method.

Method

The experimental procedure was essentially that of Blaikley. The pipe was mounted vertically with the open end at the top and the length of the vibrating air column was altered by letting water in or out at the bottom. A glass gauge tube 3.3 cm inside diameter was set parallel to the pipe and

[3] Rayleigh, Theory of Sound, vol. 2, 2nd Ed., Macmillan, App. A (1926).
[4] Rayleigh, Phil Mag. (5) **3**, 456 (1877).
[5] Bosanquet, Phil. Mag. (5) **4**, 219 (1877).
[6] Wertheim, Ann. d. chimie et d. physique (3) **31**, 394 (1851).
[7] Zamminer, Pogg. Ann. d. physik u. chemie **97**, 183 (1856).
[8] Bosanquet, Phil. Mag. (5) **4**, 219 (1877).
[9] Blaikley, Phil Mag. (5) **7**, 339 (1879).

connected at the bottom to the same water intake. The level of the water was read on a scale on the back side of the gauge tube. The water reservoir was raised and lowered by a windlass so designed that the level in the pipe could be changed by as small an amount as 0.05 cm. Three sizes of brass pipes were used, 9.92 cm, 7.36 cm and 4.84 cm inside diameter respectively. In all three pipes the thickness of the wall was 0.12 cm.

The tuning fork which was electrically driven was mounted 7 to 10 cm above the open end of the pipe. Three types of drives were used: (1) the door-bell type of interrupter for forks of frequencies 128 to 384 vps; (2) a microphone button interrupter for frequencies from 512 to 1152 vps; (3) a thermionic tube drive for frequencies above 1152 vps. The position of the fork (if more than 7 cm above the opening of the pipe) and the form of the drive were found to have no effect on the end correction with the two larger pipes, but did with the smaller as will be discussed later.

Since the ear is very insensitive to small changes of intensity of sound, an electrical detector was designed for determining the peaks of resonance in the pipes. A Koenig resonator, tuned to the frequency of the fork, was suspended so that its mouth was about 5 cm to one side of the opening and a little above it. This was connected by a piece of tubing to a Magnavox loud speaker, used as a generator. The e.m.f. from the loud speaker was stepped up by a transformer, the secondary of which was connected to the grid circuit of a C-301 thermionic tube, used for amplification. The plate circuit was connected to the primary of an audiofrequency transformer, and the secondary of the same was connected through a carborundum crystal rectifier to a galvanometer. With this device a change in intensity of sound due to a change in water level in the pipe of 0.05 cm in the vicinity of the maximum of resonance could be detected. With the ear alone a change in water level of 0.5 cm was necessary to produce a noticeable change in intensity.

Sources of Error

Blaikley in his work takes account of four sources of error: (1) position of the fork; (2) temperature changes; (3) capillary action of the water in the pipe; (4) humidity. In addition to these three other possible sources of error were considered by us: (5) the stationary wave-pattern in the room; (6) resonance in the walls of the pipe; (7) action of the Koenig resonator in loading the pipe.

1. *Position of the fork.* As noted above, with the two larger pipes the position of the fork had no effect on the end correction unless it was closer to the open end than 7 cm. Neither did the form of the drive have any effect. But with the smallest pipe it was found that for two different frequencies, 512 and 1152 vps respectively, the end correction was 0.15 cm greater with the microphone drive than with the thermionic tube drive. This was accounted for by the greater area of the framework of the former, which was about four times that of the latter. From these relations it was estimated that, if it were possible to obtain resonance with the fork at a very great distance from the mouth of the pipe, the end correction would be

less by 0.2 cm for the microphone drive and 0.05 cm for the other two which had about the same area. All these measurements were with the fork 7.8 cm from the opening. Increasing the distance somewhat reduced the end correction, but this was harder to determine accurately because at great distances resonance was not great enough. Blaikley found the effect of the fork in his work by blowing jets of air across two pipes of exactly the same length and diameter with the fork mounted over one of them. Under these conditions beats could be heard between the two pipes. This was tried in our work, but no beats could be detected with the fork at the distance used. If the fork was lowered to within 3 cm of the opening, slow beats were heard. Probably Blaikley mounted his fork at this distance or less.

2. *Temperature changes.* A temperature change occurring between the measurements of the two resonance lengths would affect the observations in two ways: (1) by causing a change in the velocity of sound, and (2) by causing a change in the frequency of the fork. But if the temperature change were no greater than 0.5°C, the error would be small compared to other observational errors. No data were retained of readings taken with greater temperature variations than this. To be sure, some error would be introduced if the observations were made at a temperature other than that at which the fork was rated, since the results show that the end correction is a function of the frequency. But a simple calculation shows this to be negligible for a temperature change of 0.5°C.

3. *Capillary action.* Capillary action is negligible in pipes of the size used in this experiment.

4. *Humidity.* Variations in humidity would affect the end correction in the same way as temperature changes, by causing a change in the wave-length. The change in wave-length due to a change in humidity of a most extreme character, from absolutely dry air to saturation at 20°C, is 0.5 percent. Unless such a change occurred between the observations of the two resonant lengths (which was not the case) the error in the end correction due to humidity was negligible.

5. *The stationary wave-pattern in the room.* The effect of this was tested by placing absorbing material on the wall of the room nearest the pipe. This caused a change in reflection and hence a change in the stationary wave-pattern. No difference in the end correction was observed under the two conditions.

6. *Resonance in the walls of the pipe.* This was tested with the smallest pipe by clamping a lead ring weighing 1.2 kg at the antinode. This was tried at frequencies of 512, 640, 1280 and 1408 vps, respectively, with negative results.

7. *Action of the Koenig resonator in loading the pipe.* The loading effect of the resonator was negligible as is shown by the fact that changing the position of the resonator with reference to the pipe had no noticeable effect on the end correction obtained, although of course it did influence the magnitude of the galvanometer deflection. Furthermore, in the course of the work the apparatus was dismantled and reassembled. At no time was an exact

measurement made of the position of the resonator, and in the second set-up it was put only *approximately* in the former position. Yet the values of the end correction taken on the reassembling of the apparatus agreed with the first results as well as the individual readings between each other.

Finally, the end corrections taken at different times show the order of accuracy of the work. For the pipe 9.92 cm in diameter, the maximum deviation[10] of any end correction from the mean was 0.06 cm and the average deviation was 0.022 cm; for the 7.36 cm pipe the maximum was 0.05 cm and the average 0.024 cm; for the 4.84 cm pipe the maximum was 0.05 cm and the average 0.025 cm.

TABLE I

Resonance lengths and end correcctions for brass pipes of various diameters.

	Pipe 9.92 cm in diameter					Pipe 7.36 cm in diameter					Pipe 4.84 cm in diameter					
n	L_1	L_2	c	Av. c	c/R	L_1	L_2	c	Av. c	c/R	L_1	L_2	c	Av. c	C[8]	C/R
128	64.12[1]	198.52	3.08	2.95	0.59											
	64.52	198.62	2.53													
	64.02	198.52	3.23													
256	30.78	98.30	2.98	2.99	0.60	31.45	98.70	2.18	2.19	0.60	32.42	100.34	1.54	1.53	1.48	0.61
	30.75	98.24	3.00			31.44	98.71	2.20			32.04	99.14	1.51			
384	19.30	64.04	3.07	3.03	0.61	20.26	65.18	2.20	2.21	0.60	20.91	65.79	1.53	1.51	1.46	0.60
	19.20	63.56	2.98			20.24	65.15	2.22			20.90	65.68	1.49			
512	13.70	47.20	3.05	3.03	0.61	14.36	47.43	2.18	2.23	0.61	15.08	48.64	1.70	1.70	1.50	0.62
	13.75	47.25	3.00			14.31	47.46				15.11	48.71	1.69			
640	10.34	37.35	3.17	3.16	0.63	11.04	37.90	2.39	2.39[2]	0.65	11.78	38.74	1.70	1.69	1.49	0.62
	10.37	37.39	3.14			11.03	37.91	2.41			11.82	38.80	1.67			
768	8.75	31.65	2.70	2.64	0.53	8.95	31.53	2.34	2.39[3]	0.62	9.52	31.97	1.71	1.70	1.50	0.62
	8.87	31.77	2.58			9.01	31.54	2.26			9.54	31.97	1.68			
832	7.75	28.63	2.69	2.69	0.54	8.12	28.83	2.24	2.22	0.60						
	7.75	28.63	2.69			8.16	28.87	2.20								
838	7.55	48.58[4]	2.71	2.70	0.54											
	28.08[5]	69.10[6]	2.69													
853.3	7.58	27.75	2.51	2.55	0.51	7.95	28.21	2.18	2.16	0.59						
	7.50	27.65	2.58			7.95	48.29[7]	2.14								
896	7.35	26.65	2.30	2.29	0.46	7.44	26.67	2.18	2.18	0.59	7.78	27.00	1.83	1.79	1.59	0.66
	7.35	26.60	2.28			7.45	26.69				7.83	26.98	1.75			
1024						6.27	23.18	2.19	2.16	0.59	6.59	23.42	1.83	1.80	1.60	0.66
						6.29	23.13	2.13			6.61	23.34	1.76			
1115						5.60	21.12	2.16	2.14	0.58						
						5.60	21.02	2.11								
1152						5.28	20.22	2.19	2.24	0.61	5.61	20.64	1.91	1.86	1.66	0.69
						5.24	20.27	2.28			5.67	20.63	1.81			
1280											4.91	18.28	1.78	1.80	1.75	0.72
											4.86	18.22	1.82			
1408											4.15	16.04	1.80	1.82	1.77	0.73
											4.14	16.07	1.83			
1536											3.72	14.60	1.72	1.71	1.66	0.69
											3.71	14.51	1.69			
1664											3.46	13.77	1.70	1.65	1.60	0.66
											3.54	13.81	1.60			
1715											3.27	13.24	1.72	1.75	1.70	0.70
											3.24	13.26	1.77			

1 Resonance lengths and corrections expressed in cm.
2 Average of six measurements.
3 Average of three measurements.
4 Third resonant length.
5 Second resonant length.
6 Fourth resonant length.
7 Third resonant length.
8 Correction with fork at great distance from opening.

10 Except for frequency of 128 vps for which resonance was weak.

Results

The data taken and the end corrections computed are shown in Table 1. The variation of the end correction with frequency is well shown in the curves of Fig. 1. In plotting these curves the points were weighted according to the deviations between the values averaged for a given point and the curves drawn as smooth as was consistent with this weighting. The range of frequencies used extended from the lowest frequency which is generally employed with an actual organ pipe of the same diameter as the brass pipe used in the experiment up to the highest with which a distinct resonance could be obtained. It will be observed that for each of the three pipes there is a progressive increase in the end correction from the lowest frequency up to a certain point beyond which there is a decrease. This decrease is greatest with the largest pipe and becomes smaller with a decrease in the diameter of

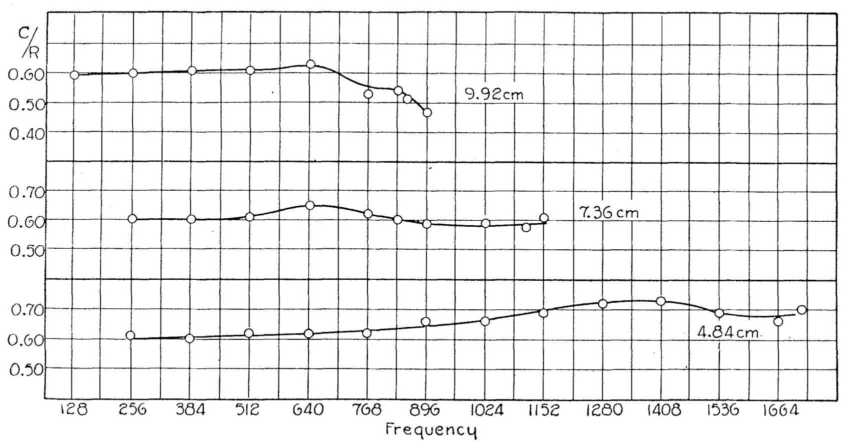

Fig. 1

the pipe, so that for the smallest one this minimum is still greater than the end correction at the beginning of the curve. The maximum correction occurs at such a frequency that λ/D is about 6. This is better shown in Fig. 2 in which the end corrections are plotted against D/λ. A scale of λ/D is laid off along the axis of abscissas.

With each pipe the highest frequency at which resonance could be obtained, and hence the end correction determined, was such that λ/D was about 4. This fact has an important bearing on the maximum number of partials that may be expected from a pipe of given *scaling* (D/λ for the prime) when used as a generator of a complex tone. If an organ pipe were designed with the diameter of the largest pipe used in this experiment, 9.92 cm, the frequency of the prime tone would be about 128 vps for a fairly large scaling of 1/27. If, as this work indicates, no proper tone can be produced whose frequency is higher than that corresponding to $\lambda/D=4$, then the partial of highest frequency will be the seventh, frequency 896 vps. Hence no more than seven partials would be expected with an open pipe of such scaling, and with a closed pipe the seventh would be the highest of the odd-numbered partials.

The effect of the variation of the end correction with frequency and with the size of the pipe should also be considered. Taking the largest pipe again as an example, we may compute the length of a closed pipe to sound 128 vps at 20°C by

$$128 = 34,400/4(L_1 + 2.95) \tag{2}$$

The frequency of the seventh proper tone would be 905 vps. This is *sharper* than the seventh harmonic by 9 vps. Hence in order to have the seventh partial present in the complex tone, this proper tone would have to be "forced" into the harmonic series. This accounts for the fact that with pipes of this scaling the seventh partial is lacking entirely or is of very low intensity. The next lower proper tone, the fifth, would be flat by 2 vps, since the end correction is larger in the neighborhood of this frequency than at 128 vps.

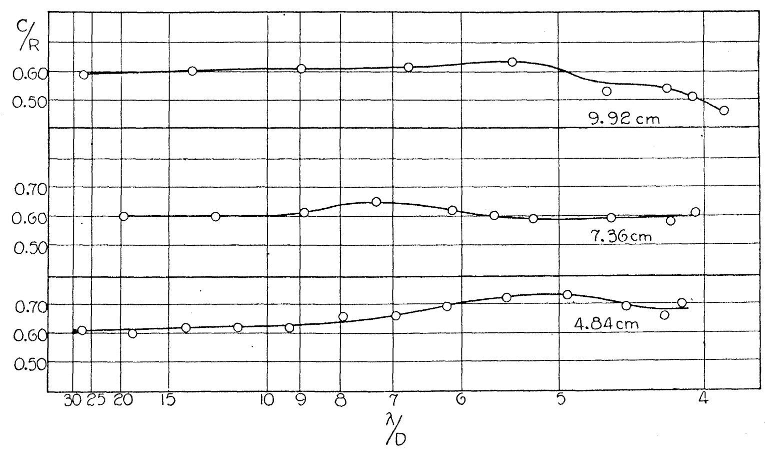

Fig. 2

Similarly the third proper tone would be flat by 1 vps. Hence the third partial is easily produced and the fifth with more difficulty. This in agreement with observations of the relative intensity of these partials in a stopped pipe of wide scaling. Hemholtz' statement[11] that, "For the wider stopped pipes, as for the wide open pipes, the next adjacent proper tones of the mass of air are distinctly higher than the corresponding upper partials of the prime and consequently these upper partials are very slightly, if at all, reinforced," is explained in terms of the variation of the end correction with frequency *only* for the seventh proper tone, since the others are flat instead of sharp.

A similar situation holds for open pipes. The second, third, fourth, and fifth proper tones will be flat of the harmonic series, the departure increasing with the number of the proper tone, while the sixth and seventh will be sharp.

If the 4.84 cm pipe were used with the same scaling, 1/27, the prime would have a frequency of about 256 vps and the highest partial possible

[11] Helmholtz, "Sensations of Tone," Longmans, Green and Co., 3rd Ed. (1895), p. 94.

would be the seventh. However the effect of the end corrections would be to make all the higher proper tones *flat* of the harmonic series, the greatest departure being for the fifth, viz., 11 vps. This in the neighborhood of 1280 vps is almost exactly the same difference in pitch as 9 vps in the neighnorhood of 896 vps. Hence, if the seventh partial is lacking in the former case, the fifth would be here. It would thus appear that, if the same scaling used in a set of pipes with a view of obtaining uniform quality of tone, *the higher partials are fewer and less prominent in the higher pitched pipes than in the lower pitched.* This point should be tested by analysis of organ pipe tones.

If the 4.84 cm pipe were used with a scaling of 1/54, as is used in the Gamba stop for example, the frequency of the prime would be about 128 vps. Under such conditions the ninth proper tone would be flat by only 2 vps and so readily forced into the harmonic series. This is in agreement with observations of small scaled pipes, such having a large retinue of partials.

Conclusions

It is recognized, of course, that there are other factors affecting the relative intensity of the partials tones of pipes, such as the material of the walls, size and shape of the mouth or embouchre, and the wind pressure of blowing. This paper considers only one factor, the effect of the end correction.

This work seems to verify the general idea that the difference in the number of partials in wide and narrow pipes is in part due to a variation of the end correction with frequency, but does not verify Hemholtz' statement that the higher proper tones of wide pipes are *sharper* than the corresponding partials of the harmonic series. Our work indicates that in general they should be *flat*. Furthermore, the end corrections are relatively larger for narrow pipes than for wide, so that for pipes of the same scaling the higher pitched ones will have fewer partials than the low pitched.

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Department of Physics,
University of Washington,
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