THE VIBRATION OF BELLS

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ABSTRACT

Relative frequencies, nodal meridians, and nodal circles of the bells of the Harkness Memorial Chime.—This is a study of the ten bells of the Harkness Memorial Chime at Yale University. The large size of these bells makes it possible to examine partial tones of higher order than could be studied on smaller bells. For the first ten partials the average values found for the relative frequencies, the numbers of nodal meridians, and the numbers of nodal circles are shown in the following table.

Partial	1	2	3	4	5	6	7	8	9	10
Relative Frequency	0.50	1.00	1.20	1.49	2.00	2.56	2.96	3.28	3.80	4.06
Nodal Meridians	4	4	6	6	8	8	10	8	10	12
Nodal Circles	0	1	1	1	1	2	2			

The nodal circles for the 2d, 3d, 4th, and 5th partials are at levels about 1/3, 1/2, 1/6, and 1/2 the height of the bell, those for the 6th partial are roughly at 1/5 and 3/5, for the 7th at 2/5 and 4/5 of the height.

Amplitudes of the partials.—Curves showing the vibration in the air near the bells were obtained by a microphone, amplifier, and oscillograph. There is first an irregular motion which may arise from noises in the mechanism by which the bells are struck. The clapper probably strikes after some 0.01 sec. to 0.03 sec., for after this time the amplitude is larger and the curve more regular. Since the partial tones of a bell are not harmonic a Fourier analysis is not directly applicable, but a *modified Fourier analysis* may be employed. An approximation to the amplitude of each component is obtained by proceeding as if a Fourier analysis were being made, the range of integration for each component being however independently chosen as a whole number of periods of that component. A method is given for obtaining corrections to the approximate amplitudes thus found.

The strike note of bells.—The curious phenomenon of the *strike note* is considered. The hypothesis that the pitch of the strike note is determined by that of the fifth partial, but that it is very generally judged an octave lower, is supported by several facts which have been already known and by the new fact that the fifth partial appears with surprising promptness and vigor. One possible cause for such a misjudging of the octave is the fact that the fifth, seventh, and tenth partials are all brought out well when a bell is struck on the soundbow, and that the relative frequencies of these partials are not far from 2:3:4, thus perhaps giving rise in the ear of the observer to the pitch of the strike note.

INTRODUCTION

THE vibrations of bells are not well understood. Even on so elementary a matter as the number and position of nodal lines the very meager published values are not in satisfactory agreement. Moreover the different partial tones of a single bell usually bear inharmonious relations to each other, and the relative frequencies of the partial tones in one bell often differ from the relative frequencies in other bells—even when the bells were cast by the same founder. As to the frequencies which it is desirable for the partials to have, there seems now to be more or less agreement[1] that in the best bells the first seven partials have frequencies in the ratio 1:2:2.4:3:4:5:6, although the third partial is sometimes represented by 2.5 instead of 2.4, being thus a major third above the second partial instead of a minor third above it.[2]

In addition to the partials of a bell certain other tones can be heard. The most important of these is called the "strike note." The strike note is the most prominent note heard when bells are sounded in fairly rapid succession. It is usually this note that it meant when the pitch of a bell is given. Griesbacher[3] states that the strike note is audible at distances at which none of the partial tones can be heard. In bells that are well tuned the strike note coincides with the second partial, but in most cases the two tones differ in pitch, often to the extent of a major second or more. If a tuning fork has nearly the pitch of a partial tone, beats are easily detected; but no beats can be obtained from a fork and the strike note. The strike note is not a combination tone, and Blessing[4] states that when sounding with other notes it is not capable of producing combination tones. The partial tones are produced, as we should expect, by division of the bell into various numbers of transversely vibrating segments separated by nodal lines. The strike note appears to be produced in some other way. The partial tones can be elicited by resonance, and when a bell has been struck they can be reinforced in the usual manner by Helmholtz resonators. The strike note is not reinforced by a resonator, and Biehle[5] has even called it an "imaginary tone." Neither is it easy to obtain the strike note by resonance. A tuning fork may be adjusted until its pitch is that of one of the partial tones of a bell. If the fork is then struck and its stem pressed against the bell the partial tone comes out very clearly. When the strike note has a pitch different from that of the second partial it is not easy to obtain any such response from the strike note. Griesbacher[6] has found however that by tuning a fork to the pitch of the strike note, setting the fork into vibration, and then pressing either

[1] J. Biehle, Archiv für Musikwiss. **1,** 306 (1919); W. W. Starmer, Beiaardkunst, Handelingen van het Eerste Congress, p. 69, Mechelen (1922); William Gorham Rice, Carillons and Singing Towers of the Old World and the New, p. 226, Dodd, Mead, and Company (1925); P. Griesbacher, Glockenmusik, pp. 14, 23, Alfred Coppenrath (1927).

[2] It is possible to tune the partial tones of a bell to some extent by cutting out material from the inside of the bell at different levels. This fact was known in the seventeenth century to the two Hemony brothers, who cast bells that are still famous for the quality of their sound. But the art was lost and not rediscovered until some thirty years ago. The rediscovery was made independently by Canon Simpson (Pall Mall Magazine, Oct., 1895, and Sept., 1896) and more recently by Johannes Biehle. Simpson's method has been developed by John Taylor and Company of Loughborough, England, and Gillett and Johnston of Croydon, England. So far as I know, all of the dozen or more carillons that have been installed in this country and Canada during the past four or five years have been cast by one or the other of these two firms.

[3] Griesbacher, ref. 1, p. 40.

[4] P. J. Blessing, Phys. Zeits. **12,** 597 (1911).

[5] Biehle, ref. 1, p. 296.

[6] Griesbacher, ref. 1, pp. 53–55.

the stem or the lowest part of a prong against the lip, where the inner and outer surfaces of the bell meet, it is possible to obtain the strike note.[7]

Griesbacher appears to think[6] that in the production of the strike note the metal on the inner and outer sides of the soundbow, on which the clapper strikes, vibrates in opposite directions. That would seem to mean that the waves which produce the strike note are compressional rather than flexural. But this explanation seems to be ruled out[8] by work which I did[9] some years ago on the Dorothea Carlile Chime at Smith College. Moreover if the strike note were due to such compressional waves it would seem that it should be possible to pick it up by a Helmholtz resonator held just beyond the lip in the region where flexural vibrations of the bell would give a minimum of intensity. I do not find this to be the case.

In a number of bells which Rayleigh[10] examined he found that the strike note was an octave below the fifth partial.[11] Biehle[12] and Griesbacher,[13] both of whom have examined many bells, state that the strike note is almost always an exact octave below the fifth partial, but that in rare cases there are slight deviations from this relationship. In my study of the Dorothea Carlile Chime I found the strike note an octave below the fifth partial, and I suggested as a possible explanation of the strike note that the fifth partial may perhaps come out with especial promptness and prominence, the octave in which it lies being however very generally misjudged.[14]

[7] Griesbacher states that he has been able in this way to obtain the strike notes of hundreds of bells. His book did not reach me until most of the experimental work reported in this paper had been completed. I was skeptical about the possibility of getting the strike note to respond to a tuning fork, but I have succeeded on two bells in verifying his statement that it is possible. I have also succeeded in getting the strike note to respond by holding the fork at an oblique angle to the surface of the bell, and then touching the stem lightly to the soundbow. If my explanation of the strike note as a misjudged octave is correct, I do not see why this effect should be obtained.

[8] Suppose that we have a bell and also a ring made of bell metal. Then from measurements on particular bells it is not difficult to show (See ref. 9 and Lamb, Dynamical Theory of Sound, ed. of 1910, p. 136) that if the ring is to vibrate in the compressional mode which has the lowest frequency, and if the frequency of the ring is the same as the strike note of the bell, then the diameter of the ring must be nearly three times the diameter of the mouth of the bell. Now it is true that the normal modes of vibration for a ring differ from those for a bell, but it hardly seems likely that for compressional modes of the same frequency the dimensions would differ so widely.

[9] A. T. Jones, Phys. Rev. **16**, 247 (1920).

[10] Rayleigh, Phil. Mag. **29**, 1 (1890), and Theory of Sound, ed. 2, vol. 1, p. 393.

[11] The statements in Rayleigh's paper in the Philosophical Magazine have been quoted (Winkelmann, Hdbuch d. Physik, ed. of 1909, vol. 2, p. 405) and seemed to indicate that the strike note was usually two octaves lower than the fifth partial, but in his treatment of the same question in the Theory of Sound he is careful to state definitely that "the nominal pitch, as given by the makers, is an octave below" the fifth partial.

[12] Biehle, ref. 1, pp. 299–300.

[13] Griesbacher, ref. 1, pp. 54, 57.

[14] Cases of error in the judgment of an octave are not uncommon. Helmholtz states (Sensations of Tone, tr. Ellis, ed. 4, p. 62) that when Tartini discovered combination tones he gave the pitch an octave too high, and that Henrici gave for some of the partial tones of tuning forks pitches an ocatve too low. Rayleigh states (ref. 10, p. 5) that "pure tones are often

The work described in this paper was undertaken in the hope that it would clear up some of the discrepancies as to the nodal lines of bells, and would also contribute to an understanding of the strike note. This latter purpose was to be attained by securing information as to the relative intensities of the different partials, and especially by finding out whether the fifth partial really is very prompt and vigorous. Most of the work was carried out on the bells of the Harkness Memorial Chime[15] at Yale University.

THE FREQUENCIES OF THE PARTIALS

The frequencies of the partials were determined by counting the beats which they made with notes from a vacuum tube oscillatory set. This set was calibrated by comparison with thirteen tuning forks made by König, and during the work on the bells the oscillator was compared frequently with a tuning fork of known frequency. The frequencies obtained are not as reliable as had been hoped. They can probably be trusted to one percent.

The results are summarised in Table I, which also includes a summary of the corresponding results from the earlier work on the Dorothea Carlile

TABLE I. *Relative frequencies of the partials.* The frequencies are calculated with reference to the fifth partial, and the frequency of the fifth partial is taken as 2, so as to make the frequency of the strike note equal to 1.

Partial	Ideal Ratios	Dorothea Carlile Chime No. of Bells Examined	Av.	Av. Dev.	Harkness Chime No. of Bells Examined	Av.	Av. Dev.
1	0.50	12	0.58	0.010	10	0.50	0.004
2	1.00	12	0.92	0.037	10	1.00	0.008
3	1.20	12	1.19	0.016	10	1.20	0.013
4	1.50	12	1.71	0.023	10	1.49	0.014
5	2.00	12	2.00		10	2.00	
6	2.50	6	2.74	0.069	10	2.56	0.045
7	3.00	6	3.00	0.028	10	2.96	0.022
8	3.33				7	3.28	0.036
9	3.75	2	3.77	0.177	7	3.80	0.113
10	4.00	1	4.07		7	4.06	0.052

Chime. It will be seen that the relative frequencies of the first five partials on all the bells of the Harkness Chime are very close to those indicated above as given by the best bells. The tenth partial is about an octave above the fifth, and the seventh, eighth, ninth, and tenth do not differ widely from the series *sol*, *la*, *ti*, *do*. If these values were precise, and we use a scale that is not tempered, the relative frequencies of the first ten partials of an ideal bell would be those given in the second column of the table.

estimated by musicians an octave too low." Obata (Frank. Inst. Journ. **203,** 659 (1927)) has recently found that the sounds from the striking of bricks and from xylophones "give a feeling of sounds about two octaves lower than those determined by the oscillographic records."

[15] The Harkness Memorial Chime consists of ten bells cast in 1921 by John Taylor and Company. The largest bell is $f\#_2$, with a strike note of about 182 cycles/sec. It weighs about 6300 kg, and has a maximum diameter of 219 cm. The smallest bell is g_3, of about 385 cycles/sec. This bell weighs about 680 kg, and has a maximum diameter of 104 cm.

Nodal Lines

General considerations. Two systems of nodal lines are generally recognized in bells. One is a system of meridians which run up and down the bell at different azimuths, and the other is a system of circles which lie at different levels. Rayleigh[10] and Vas Nunes[16] examined a few of the nodal lines on a few bells. Biehle[12] has examined the partial tones of several hundred bells, and has published some brief statements about nodal lines. van der Elst[17] is developing a bar resonator which is intended in part for studying the nodal lines of bells, but I think he has as yet published nothing as to results obtained with it.

On the Harkness Chime I have examined the nodal lines in practically the same way that I described in connection with the work on the Dorothea Carlile Chime.[9] That is, I connected a Helmholtz resonator through a piece of rubber tubing to the binaurals of a stethoscope, tuned the resonator by covering the mouth to a greater or less extent with a finger, moved the resonator about near the surface of the bell, and listened for the positions of minimum intensity.

Nodal meridians. The numbers of nodal meridians which I found for the first ten partials on the bells of the Harkness Chime are shown in Table II.

Table II. *Nodal lines.* The positions of the nodal circles are indicated by fractions. The unit chosen is the distance measured along the outer surface of the bell from the lip to the shoulder. Numbers in parentheses give average deviations. The middle of the soundbow, on which the clapper strikes, is at 0.12 (0.007). Except where otherwise noted the results are from observations on all of the bells.

Partial	No. of Nodal Meridians	No. of Nodal Circles	Location of Nodal Circles	Remarks
1	4	0		
2	4	1	0.32 (0.037)	
3	6	1	0.53 (0.010)	Position of circle determined on all bells except two of the largest.[18]
4	6	1	{0.16 (0.026) 0.15 (0.010)	These two values were obtained by measuring along the outer and inner surfaces of the bells.
5	8	1	0.47 (0.045)	
6	8	2	{0.21 (0.022) 0.55 (0.021)	
7	10	2	{0.43 (0.031) 0.76 (0.067)	Locations of these two circles are avs. for all except the two smallest bells.
8	8			
9	10			
10	12			

[16] Abraham Vas Nunes, Experimenteel Onderzoek van Klokken van F. Hemony, pp. 95–97. Dissertation, Amsterdam (1909).

[17] W. van der Elst, Physica, **6**, 42 (1926).

[18] Most of the determinations of the location of the nodal circle for the third partial were made after the rest of the work was completed and ladders had been removed from the tower. See footnote 21.

These numbers agree with those which I found on the Dorothea Carlile Chime.[19] My results agree with those of Rayleigh up to the fifth partial. The largest bell that he used was considerably smaller than the smallest on the Harkness Chime, so that for higher partials he was not able to examine nodal meridians. Biehle gives numbers of nodal meridians for the first eight partials. For the first seven my results agree with his.[20] My results do not agree entirely with those of Vas Nunes. But not only did Vas Nunes work without resonators—which makes any determination of this sort very difficult—but the series of frequencies which he found is somewhat different from the series that I have found, and it may be that there are also differences in the nodal lines.

Nodal circles. The nodal circles are more difficult to locate than the meridians. My results for the first seven partials are given in Table II. On comparing with Table I in my former paper it will be seen that there is good agreement, and that on the Harkness Chime I have been able to locate circles for the fourth and seventh partials in regions of the bells where I was not able to detect those partials on the smaller bells of the Dorothea Carlile Chime. In the case of the fourth partial there is a nodal surface that cuts through the material of the bell in the neighborhood of the sound-bow. Mechanically it seems remarkable to have a surface of minimum transverse motion so close to a parallel free edge.

Biehle[12] gives the numbers of nodal circles for the first eight partials as 0, 1, 0, 1, 0, 1, 0, 1. It will be seen that except for the first, second, and fourth partials my numbers do not agree with his. For any one partial Vas Nunes[16] was not able to detect more than one nodal circle, although he supposed that the higher partials would have more. Since the partials which he found differ from those which I found, it is somewhat difficult to compare results. But in so far as I am able to compare tones that seem to correspond, the results for the nodal circles appear to agree. Rayleigh[10] gives results as to nodal circles for the first four partials. His results are definite for only the first two, but in no case is there disagreement between his results and mine.[21]

[19] Comparison with my former paper does not show agreement in the numbers of nodal meridians for the highest partials. In preliminary work on the Harkness Chime I missed the eighth partial, and it turns out that in the study of the Dorothea Carlile Chime I also missed it. On checking over the higher partials on the two largest bells of the Dorothea Carlile Chime—the only ones on which I was able to detect partials higher than the seventh—I have found this eighth partial, and I now find that the numbers of nodal meridians on the Dorothea Carlile Chime—so far as I am able to examine them—agree with those which I have found on the Harkness Chime.

[20] For the eighth partial Biehle gives ten nodal meridians—the same number that I published for the Dorothea Carlile Chime when I thought that the ninth partial was the eighth.

[21] In the case of the third partial Rayleigh gives a result for one bell. He says, "There is no well-defined nodal circle. The sound is indeed very faint, when the tap is much removed from the sound-bow; it was thought to fall to a minimum when the tap was about halfway up." In my paper on the Dorothea Carlile Chime I indicated that the third partial has one nodal circle. On the Harkness Chime I had great difficulty in deciding whether this partial really has a nodal circle or not. There was no doubt that the note heard in a resonator tuned to this partial disappeared and then reappeared when the resonator was moved up and down along the

I have not found the nodal circles for partials higher than the seventh. But I may note that the first, second, third, fifth, seventh, and tenth partials are well brought out by tapping on the soundbow, whereas the fourth, sixth, eighth, and ninth are better obtained by tapping higher up. The eighth and ninth are separated by tapping at slightly different levels on the waist of the bell. These facts may perhaps mean that when a bell is struck by the clapper the fourth, sixth, eighth, and ninth partials do not come out as strongly as the others, so that in tuning a bell especial care should be given to the first, second, third, fifth, and seventh partials.[22]

The Amplitudes of the Partials

Experimental arrangement. Curves representing the sounds from the different bells were obtained by using a high quality microphone, an amplifier, and an oscillograph with recording camera. The accuracy of transmission was tested by holding successively near the microphone three tuning forks which differed considerably in pitch, each provided with the proper resonator. The distortion proved not to be important. The speed at which the photographic film moved was obtained by records from current in a shunt around an electrically driven tuning fork. But as the oscillograph which was available had only a single vibrator, curves from the fork and a bell could not be taken at the same time, and this casts some doubt on the accuracy of the timing.

Analysis of the curves. Since the periods of the partial tones of a bell do not form a harmonic series, a Fourier analysis of the curves is not directly applicable. The method finally adopted was the following. Some fifty to eighty equally spaced ordinates were measured and then multiplied by $\sin nt$ and $\cos nt$ respectively, where n stands for 2π times the frequency of some chosen partial tone. By adding these products approximations were obtained to the values of the integrals $\int y \sin nt\, dt$ and $\int y \cos nt\, dt$, the integration extending over a whole number of periods of the chosen partial. The amplitude was calculated as if a Fourier analysis were being made. The process was repeated for other sets of ordinates, so that from two to five values for the amplitude were obtained. The results were then averaged. The values obtained are in error, partly because of experimental inaccuracies

waist of a bell. But I came to think that this effect was due to the second partial overpowering the third in this region, and consequently stated, in a preliminary report on a part of this work (Phys. Rev. **29**, 616 (1927)), that the third partial has no nodal circle. When I attempted to check this over again on the Dorothea Carlile Chime I did not feel so sure, and after trying several methods that did not lead to the desired certainty I went back to Rayleigh's method of holding a resonator at one point while I tapped up and down the bell. This method seemed to show rather definitely a nodal circle for the third partial, and after checking over again several of the bells of the Harkness Chime by this method I now feel fairly sure that the third partial does have a nodal circle after all.

[22] On the bells of the Harkness Chime the first few partials have been tuned, on the bells of the Dorothea Carlile Chime they have not. And it is interesting to see from Table I that on the Dorothea Carlile Chime the average ratios of the frequencies for the third, fifth, and seventh partials are close to the ideal values, whereas the fourth and sixth partials deviate widely from those values.

in determining the speed of the film and the frequencies of the partials, partly because of the slight distortion in the transmission from microphone to oscillograph, partly because of the approximation involved in finding the integrals, partly because the range of integration is not a whole number of periods for partials other than the one in question, and probably partly because of an irregular motion to be mentioned shortly.

The error due to inharmonic relations between the partials may be corrected as follows.[23] Suppose at first that there are only two partials, and that the curve is represented by

$$y = a_1 \cos n_1 t + b_1 \sin n_1 t + a_2 \cos n_2 t + b_2 \sin n_2 t. \tag{1}$$

Let s be an integer, and let p be a quantity defined by the relation $sT_2 = pT_1$, where $T_1 = 2\pi/n_1$ and $T_2 = 2\pi/n_2$. Then it is not difficult to show that

$$a_2 = \frac{2}{sT_2}\int_0^{sT_2} y \cos n_2 t dt - \frac{p}{\pi(p^2 - s^2)}\left\{a_1 \sin 2\pi p + b_1(1 - \cos 2\pi p)\right\} \tag{2}$$

and

$$b_2 = \frac{2}{sT_2}\int_0^{sT_2} y \sin n_2 t dt + \frac{s}{\pi(p^2 - s^2)}\left\{a_1(1 - \cos 2\pi p) - b_1 \sin 2\pi p\right\}. \tag{3}$$

From (2) and (3) it will be seen that if p is an integer the correction terms to the right of the integrals vanish, that if p is close to an integer these terms are small, and that if p and s are very different, i.e. if n_1 and n_2 are very different, these terms are small. When (1) contains more than two components each new component adds to (2) and (3) terms of the same form as the correction terms that have been written.

A first approximation is obtained by neglecting the correction terms in (2) and (3), a second approximation can be obtained by putting into the correction terms in the extended expressions which take the place of (2) and (3) the values found in the first approximation, and further approximations might be obtained by repeating the process. Results of applying these corrections are shown in Figs. 1 and 2. In Fig. 1 no important change is produced. In Fig. 2 the importance of the third partial is brought out more strongly, but the uncorrected values already indicated that it had the largest amplitude. The calculation of these corrections takes considerable time, and it has not been carried through except in these two cases.

Results. On curves which showed the beginning of the sound there was at first an irregular motion that lasted for from 0.01 sec. to 0.03 sec., and was followed by a considerable increase in amplitude and the development of a more regular type of vibration. In the initial irregular motion there was no evident periodicity, but there were suggestions of frequencies in the neighborhood of 1000 to 2000 or more cycles/sec. Such frequencies are higher than the tenth partials of the bells that were concerned, but not nearly high enough to arise from waves running back and forth through the ball of the

[23] I find that this method of correction is not new.

clapper. It is entirely possible that they may be due to noises in the mechanism by which the bells are struck. To what extent these irregular motions continued to be superposed on the more regular part of the curves I do not know.

Only a small number of satisfactory films were obtained, and the amplitudes have been calculated for only six. Three of these were exposed about the time a bell was struck, and in all three the analysis—beginning after the initial irregular motion—shows that the fifth partial has a considerably larger amplitude than the others. Fig. 1 shows the analysis of one of these films. On a few other films that were exposed when a bell was struck there was evident without analysis a prominent vibration which had a frequency about that of the fifth partial.

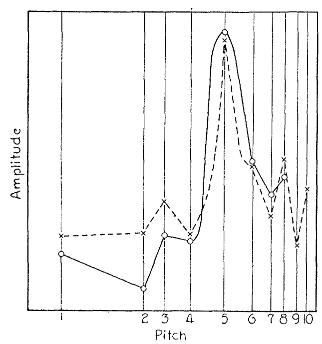

Fig. 1. Amplitudes of partials of a bell very soon after it was struck.

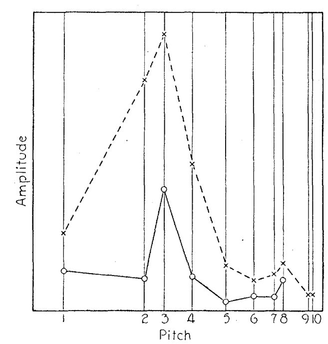

Fig. 2. Amplitudes of partials of another bell about 3 sec after it was struck.

In both these figures the abscissas are proportional to the logarithms of the frequencies, and the ordinates are proportional to the amplitudes. The numbers marked along the axes of abscissas indicate the partials. The scales for the amplitudes are arbitrary, and are different for the two figures. The points indicated by crosses and joined by dotted lines show the uncorrected amplitudes, and the points indicated by circles and joined by full lines show the corrected amplitudes.

A number of films were exposed about three seconds after the bells were struck. The general appearance of these films differs according to the size of the bell. For the three or four largest bells they show that several partials were still important. For the smaller bells they show that most of the partials had died down a good deal, leaving only one that had much amplitude. An examination of these films showed that the frequency of the tone that was now most prominent is about that of the third partial. Analyses were made of three of the films that were exposed about three seconds after the bells were struck. Two of these were cases where the curve showed only one prominent frequency, and in both of these cases the third partial does turn out to have a larger amplitude than any of the others. Fig. 2 shows one of these cases. The third curve was from one of the larger bells, and the analysis indicates that the third partial is prominent but does not have the largest amplitude.

This prominence of the third partial surprised me. I had supposed that the higher partials died out more quickly than the lower, and that the tone which persisted longest was always the first partial. I still suppose that in most bells the first partial lasts the longest, but in the cases of a number of bells to which I have recently listened I find that the third partial becomes very prominent soon after the bell is struck.

Conclusion

The work described in this paper extends our knowledge of the nodal lines of bells, and shows that, although the relative frequencies of the partial tones in the Taylor bells which form the Harkness Chime differ from those of the Meneely bells which form the Dorothea Carlile Chime, nevertheless the nodal lines of both chimes are very similar.

Although the work is not sufficiently accurate to give quantitative values for the relative amplitudes of the different partials, it does seem to show rather definitely that in the bells which have been studied the fifth partial really comes out with surprising strength in the very early stages of the vibration. Just why the fifth partial should be so prominent at first is not entirely clear. It seems to be true however that most of the vibration that gives rise to the fifth partial occurs in the neighborhood of the soundbow. One evidence of this is the fact that the surface of minimum intensity which extends outward from the nodal circle for this partial does not pass directly away from the bell, but runs upward not far from the surface.

Whatever the cause for the early prominence of the fifth partial, the fact that it is at first so prominent supports the suggestion that the strike note is determined by the fifth partial, the octave in which it lies being however misjudged. Another fact which supports this hypothesis is the short time that the strike note lasts. If the proposed hypothesis is correct it would be expected that the strike note should disappear as soon as other partials become as prominent as the fifth. Still another bit of evidence for the hypothesis is a statement by Blessing[4] that if the soundbow of a bell is turned thinner and thinner the strike note decreases in intensity, and that by the time the soundbow is cut off to such an extent that the inner and outer surfaces are parallel the strike note has disappeared. It would be interesting to know what happens to the fifth partial while this process is carried out.

If the above hypothesis is correct, why is the octave so generally misjudged? This question is really one for the psychologist, but in my former paper I hazarded two suggestions, and I may now point out further that the frequencies of the fifth, seventh, and tenth partials—all of which come out strongly from the soundbow—are nearly in the ratio 2:3:4. Now Fletcher[24] has found that when three pure tones with frequencies in the ratio 2:3:4 reach the ear together a note of frequency 1 may also be heard. This would be the frequency required for the strike note, but whether the frequencies of the fifth, seventh, and tenth partials are close enough to the ratio 2:3:4 to produce this effect I am somewhat doubtful.

[24] Harvey Fletcher, Phys. Rev. **23,** 427 (1924).

Against the proposed hypothesis we have Griesbacher's observation[6] that it is possible to get the strike note to respond to a tuning fork. Whether there is some way of reconciling this fact with the hypothesis I do not yet feel sure. The fact that the strike note will not beat with other tones of nearly the same pitch seems incompatible with any actual vibration that has the frequency of the strike note. The best hypothesis that I see at present is that of the misjudged octave.

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