Second Series *August, 1927* *Vol. 30, No. 2*

THE

PHYSICAL REVIEW

TOTAL REFLECTION OF X-RAYS AND THE INDEX OF REFRACTION OF CERTAIN METALS

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Abstract

Total reflection of a beam of x-rays ($\lambda = 0.7078$A) is obtained from mirrors of glass, tin, silver, selenium, and zinc. The deviations of the reflected rays at the critical positions were measured on photographic films placed 312.5 cm from the mirrors. The index of refraction was calculated from the measured critical angle in each case, with the following results: crown glass $\delta \times 10^6 = 1.711$; tin 3.97; silver 5.78; selenium (vitreous) 2.67; and zinc 4.59. The experimental values of δ agree with those calculated by the Drude Lorentz dispersion formula to about 0.5%. It is shown in the case of the metals that there are undoubtedly in each atom 2 electrons which have the critical K absorption frequency.

THE specular reflection of x-rays from flat polished surfaces was shown by Compton[1] to take place provided the glancing angle of incidence was less than the critical angle. His observations were made on glass and speculum metal and were used to determine the index of refraction for several wavelengths. He also shows that these results are in accord with the usual electron theory of dispersion as first developed by Drude[2] and Lorentz.[3]

Other investigators have determined the index of refraction for a few substances. Webster and Clark[4] obtained a deviation of the K lines from rhodium by means of a rhodium prism. Stentrom[5] and Siegbahn[6] employed a method which depends upon the effect of refraction in producing a small deviation from Bragg's law. Von Nardroff,[7] Davis and Hatley[8] used crystal wedges which were ground and polished in such a way that the surface made a small angle with the reflecting planes. By this device they were able considerably to increase the amount of the refraction. Von Nardroff applied his results to a determination of the number of K electrons of iron and finds that for $K = 2$ the dispersion formula gives a close check upon his experimental determination of the index of refraction.

[1] Compton, Phil. Mag. **45,** 1121 (1923).
[2] Drude, "Theory of Optics" (transl.) p. 388.
[3] Lorentz, "The Theory of Electrons" 2nd. Ed. p. 149.
[4] Webster and Clark, Phys. Rev. **8,** 528 (1916).
[5] Stentrom, Dissertations, Lund (1919).
[6] Siegbahn, Comptes rendus **173,** 1350 (1921) and **174** (1922).
[7] Von Nardroff, Phys. Rev. **24,** 143 (1924).
[8] Davis and Hatley, Phys. Rev. **23,** 290 (1923) and **24,** 486 (1924).

Linnick and Lascharew[9] have described a visual method for observing the reflected beam from polished surfaces and made photographs for measuring the critical angles from glass, alum, iron, mica and quartz, using the K lines from copper ($\lambda = 1.537$Å). Their optical path was too short (about 40 cm) to permit accurate determinations of the index of refraction.

The present work makes use of the total reflection method and was undertaken with the purpose of determining (1) how closely the Drude-Lorentz dispersion formula for the index of refraction would check the experimental values, (2) whether the critical absorption K frequencies of some of the metals as determined otherwise might be used in the total reflection formulas and (3) whether the number of K electrons per atom having these frequencies was 2 or some other number.

Apparatus and Experimental Procedure

The $K\alpha$ radiation from a molybdenum tube was excited in the usual manner. No attempt was made to produce a monochromatic radiation, for the shorter rays from the tube fall off before the critical angle of the $K\alpha$ lines is reached. The longer rays, only, are reflected through angles larger than the critical angle of $K\alpha$, and these are relatively so weak that no difficulty was experienced by having them present.

The reflection apparatus consisted of a 2 inch (5.1 cm) brass tube about 600 cm long, near the center of which was placed the reflecting surface. The end of the tube nearer the x-ray tube carried a narrow lead slit. Another slit was placed very close to the mirror. The mirror was mounted on the

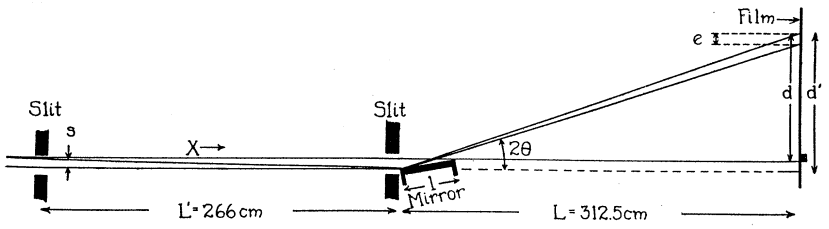

Fig. 1. Reflection of x-rays at critical position.

axis of a spectrometer table and could be rotated by a fine tangent screw through very small angles. The film was placed at the far end of the tube. Its distance from the front edge of the mirror was 312.5 cm. The whole apparatus could be operated at a reduced air pressure or filled with a light gas such as hydrogen, if desired, but it was found unnecessary to do either with the large amount of energy available. A satisfactory exposure from the best surfaces required only four minutes for a fixed position of the mirror. The approximate position of the critical angle was determined and then a longer exposure was made during which the mirror was turned successively by small amounts through the critical position.

[9] Linnik and Lascharew, Zeits. f. Physik. **38,** 659 (1926).

Measurements were made on the film from the edge of the shadow cast by the mirror in the path of the direct rays to the position in the reflected ray at which the intensity decreases abruptly. The measured distance is a little too large because the rays of the x-ray beam are not strictly parallel.

An inspection of the diagram (Fig. 1) will readily show how the following relations are obtained, from which the critical angle θ may be calculated. Since $e=sL/L'$, $d'=d+s$, $s=l \sin\theta=l\theta$, and $2\theta=(d'-e)/L$, it follows that $\theta=d/[2L-l(1-L/L')]$ when d is the deviation measured on the film and l is the length of the mirror. It is obvious that had L' been equal to L no correction would have been necessary. The second slit serves only to reduce the amount of secondary radiation produced in the long tube and hence lessens the undesirable fogging of the film.

Some of the photographic results are reproduced in Fig. 2. The sharp character of the reflected ray from an optically flat piece of glass is shown in (*a*). The mirror was moved but once in this photograph and then to another position which was slightly greater than the critical position. The decrease in intensity is prominent. This is also clearly shown in (*b*) in which case the glass mirror was turned about 3 seconds every minute.

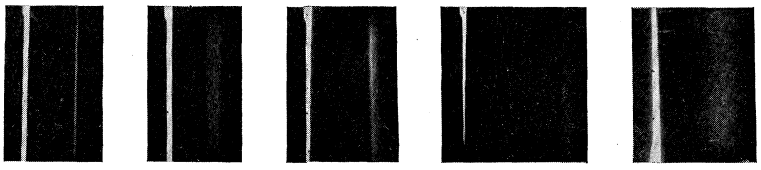

Fig. 2.

The selenium mirror was made by simply pouring hot melted selenium upon a thick piece of plate glass. The sudden cooling produces the vitreous form. The surface was very smooth except for a few small scattered pits due to gas pockets. The reproduction in Fig. 2*c* shows the trace of the reflected beam upon turning the mirror through a sufficient angle to pass through the critical position. A front surface silvered mirror was used for measuring the critical angle for silver with the result shown in Fig. 2*d*.

Considerable difficulty was encountered in producing satisfactory mirrors with tin and zinc. The final specimens were planed very carefully in a precision planing machine and then burnished or polished by hand. The surfaces obtained were not accurately flat but were highly polished. The critical edge in the reflection from tin (Fig. 2*e*) was not sharp enough to reproduce well but permitted the measurement of the deviation to a little better than 0.01 cm. The lack of sharpness is explained as due to a double reflection of some of the rays which would cause them to be reflected through angles greater than the critical angle. The final result from zinc was a little better than that from tin.

Experimental and Theoretical Results

In Table I the measured deviations are listed together with the critical angles. The index of refraction was determined from the measured critical

TABLE I

Measured values of the critical angles.

Substance	Density gm/cc	l (cm)	Measured deviation (cm)	Critical angle (rad.)
Glass (crown)	2.58	5.0	1.155 ±.003	.00185
Tin	7.30	4.5	1.76 ±.01	.00282
Silver	10.5	3.5	2.124 ±.003	.00340
Selenium (vitreous)	4.25	5.0	1.444 ±.005	.00231
Zinc	7.10	5.0	1.89 ±.01	.00303

angle by using the relation $\delta = 1 - \mu = \frac{1}{2}\theta^2$ where μ is the index of refraction. The values of δ are tabulated below in Table II.

Making use of the Drude-Lorentz dispersion formula

$$\delta = \frac{e^2 n}{2\pi m}\left[\frac{K}{\nu_0^2 - \nu_K^2} + \frac{Z-K}{\nu_0^2}\right]$$

we may calculate δ. Here e and m are the charge and mass of the electron, with n the number of atoms per cc, Z the atomic number, and K the number of K electrons per atom.

The K critical absorption wave-lengths as determined by Blake, Duane and Hu[10] were used for determining ν_K. The frequencies of all the other electrons are so small in comparison with that of the incident radiation that they may be neglected. In the case of glass the K frequencies of the constituent elements are also small enough to be neglected, except for those of barium, zinc and arsenic, which elements occur in relatively small quantities and hence the effects of their K electrons are negligible. The values of δ were calculated for $K=1$, 2 and 3. Each result is tabulated below.

TABLE II

Values of $\delta \times 10^6$.

Substance	Experimental value	Calculated values $K=1$	$K=2$	$K=3$
Glass	1.711	1.717	(K frequencies neglected)	
Tin	3.97	4.07	3.95	3.81
Silver	5.78	5.990	5.749	5.496
Selenium	2.67	2.573	2.654	2.734
Zinc	4.59	4.52	4.58	4.65

It is readily seen upon comparing these results with the experimental value of δ in the case of each of the metals that K is undoubtedly equal to 2. It is also to be concluded that the K critical absorption frequencies function in the phenomenon of reflection.

Appreciation of the skill and patience of Mr. Thomas Watson, University mechanic, in preparing the metal mirrors is gratefully acknowledged.

UNIVERSITY OF CALIFORNIA AT LOS ANGELES,
April 15, 1927.

[10] Blake, Duane and Hu, Phys. Rev., July and Dec. 1917; June 1918 and Dec. 1919.

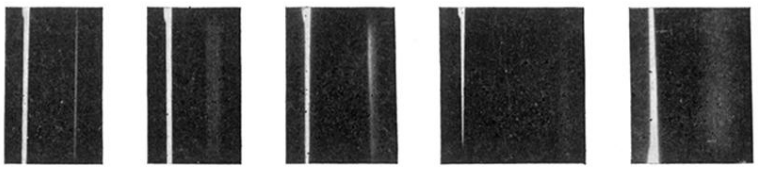

Fig. 2.