FURTHER EXPERIMENTAL TESTS OF THEORIES OF THE ABSORPTION OF X-RAYS

By F. K. Richtmyer*

Abstract

When a beam of x-rays passes through a slab of absorbing material photo-electrons are in general expelled from all levels. If the beam is (1) monochromatic and (2) of wave-length shorter than the K absorption limit of the absorber, the ratio, R_L^K, of the number of photoelectrons coming from (K, L, $M\cdots$) levels to the number coming from (L, $M\cdots$) levels is given by the maximum value τ_{max} reached by the atomic absorption coefficient as the limit is approached from the short wave-length side to the minimum value τ_{min} reached as the limit is approached from the long wave-length side. Because of absence of data on scattering coefficients it is impossible to obtain accurate experimental values of R_L^K for purposes of comparing with experiment the theories of x-ray absorption proposed by Thomson, Compton, de Broglie and Kramers. A careful repetition of the measurements previously reported (Phys. Rev. **27**, 1 (1926) shows that, after making any reasonable assumption as to the magnitude of scattering coefficients, the experimental values of R_L^K are more nearly in agreement with Kramers' theory, excepting for a small observed variation of R_L^K with λ. However the absolute magnitude of the absorption discontinuity, i. e. $\tau_{max}-\tau_{min}$, is given very closely by de Broglie's theory.

Mass absorption coefficient of Sn in the wave-length range $\lambda_K/2<\lambda<\lambda_K$. A brief report is also made of some preliminary measurements of a redetermination of the variation of absorption with wave-length in which it is hoped to attain considerably higher precision than previously reported. It is shown that in the wave-length range $\lambda_K/2<\lambda<\lambda_K$ the observed values of mass absorption coefficient of Sn is a linear function of λ^3 and that the exponent of λ cannot differ from 3 by as much as 1 percent. There is no suggestion of a deviation from the λ^3 law as the absorption limit is approached from either side.

I. The Magnitude of the K Absorption Discontinuity

In a previous communication the writer[1] presented a critique, from the standpoint of experimental data, of the theories of the absorption of x-rays proposed, respectively, by J. J. Thomson,[2] A. H. Compton,[3] de Broglie,[4] and Kramers,[5] with particular reference to the magnitude of the K absorption discontinuity which these several theories predict. It was shown that the magnitude of this discontinuity, designated in the previous paper by

* The writer acknowledges with sincere appreciation his indebtedness to the Heckscher Research Council of Cornell University for the financial assistance which made these measurements possible. He also wishes to express his thanks to the Research Laboratory of the General Electric Company for the loan of certain important pieces of apparatus.

[1] Richtmyer, Phys. Rev. **27**, 1 (1926).

[2] Thomson, Conduction of Electricity through Gases, 2d ed., p. 321.

[3] A. H. Compton, Nat. Res. Council, Bul. **20**, 37 (1922).

[4] L. de Broglie, Journ. de Phys. et Rad. **3**, 33 (1922).

[5] Kramers, Phil. Mag. **46**, 836 (1923).

R_L^K,* as observed experimentally, is more nearly in agreement with Kramers' theory. Kramers' theory however predicts no variation of R_L^K with atomic number, whereas, experimentally, R_L^K *increases* with *decreasing* atomic number. The theories of Compton and de Broglie are in qualitative agreement with this latter experimental fact, but these theories predict a much larger variation of R_L^K with atomic number than is actually observed.

The advent of the Schrödinger wave theory has inspired new theories of the absorption of x-rays, by Wentzel[6] and Oppenheimer.[7] These later theories differ from the four earlier ones in that, while the latter agree with each other in giving the well known λ^3 law, the former yield more complicated relations for the variation of τ with λ ($\tau \equiv$ the atomic "fluorescent" absorption coefficient).

Any successful attempt to decide which, *if any*, of these six theories is correct must be based upon much more precise measurements of absorption coefficients, and *scattering coefficients*, than any yet reported by any investigator. I say *scattering coefficients*, since whereas theories give the true atomic absorption coefficient τ, experiment yields the *total* atomic absorption coefficient μ, which includes scattering; i.e., $\mu = \tau + \sigma$ where σ is the scattering coefficient. There are *theories* to predictt the value of σ as a function of λ, but there are no reliable *measurements*. To be sure, σ is, at least for the heavier elements and for not too short wave-lengths, small compared to τ. But σ, and its (probable) variation with λ cannot be neglected when, for example, one attempts to decide between the $\lambda^{2.92}$ law proposed (experimentally) by Allen[8] and the λ^3 law, proposed by the first four theories above mentioned, for the variation of absorption with wave-length.

Further, the values of R_L^K determined by the writer[1] and others are rendered ambiguous by the uncertainty of the value of σ at the wave-length λ_K. Various values of R_L^K can be obtained from the experimental data, depending on which particular theory of the variation of σ with λ is used. In the computation of R_L^K in the previous paper, it was assumed (1) that σ does not vary with λ and (2) that the value of σ is given, *to a first approximation*, by the intercept on the τ/ρ axis of the curve plotted between τ/ρ and λ^3. Assumption (1) is in agreement with the classical (Thomson's) theory of scattering, but assumption (2) differs from Thomson's theory in that the value of σ is determined from experiment.

On the hypothesis that the (true) absorption of x-rays is a quantum, photo-electric process, the ratio R_L^K gives the ratio of the number of $(K+L+M+\cdots)$ photo-electrons to the number of $(L+M+\cdots)$ photo-electrons emitted by an absorber when radiated by x-rays shorter than, but near to,

* R_L^K is defined as the ratio of the magnitude of the *true* atomic absorption coefficient τ_{max} on the short wave-length side of, but infinitely near to, the K absorption discontinuity, to the value of this coefficient τ_{min} on the long wave-length side of, but infinitely near to, that discontinuity.

[6] Wentzel, Zeits. f. Physik **38,** 518 (1926).

[7] Oppenheimer, Zeits. f. Physik **41,** 268 (1927).

[8] Allen, Phys. Rev. **24,** 1 (1924).

its K limit. The determination of this ratio is therefore a matter of considerable importance in connection with any theory of radiation and its interaction with matter.

As pointed out above, a precise determination of this ratio $R_L{}^K$ is impossible until reliable data on scattering coefficients are available. Nevertheless, it seemed not undesirable to proceed as far as possible in obtaining precise data on the *absolute* magnitude of the K absorption discontinuity for several elements.

The method of measurement was similar to that previously described and involved nothing essentially novel, other than the precautions to reach maximum precision. Among these may be mentioned the following: (1) Measurements on a given element were made on one sample only in order to eliminate possible differences in samples. (2) The a.c. generator for supplying power was run from a *regulated* d.c. supply, but to secure still greater constancy manual regulation was used to supplement the automatic regulation. In this way the potential supplied to the x-ray tube, and also the current through the tube, were maintained constant to about 0.05 percent. (3) Read's "balance" method,[9] slightly modified, was used for measuring x-ray intensities. This method is particularly advantageous for making accurate measurements of ionization currents which cover a wide range of magnitudes since the electrometer is used as a zero instrument. Relative *rates of drift* of an electrometer, as frequently used, are not dependable unless the rates of drift which are compared are all of the same *order* of magnitude. Read's balance method, however, is quite independent of this source of error.

It has been shown by the writer[10] that, after correcting for the finite width of the spectrometer slits, (1) the width of the K absorption discontinuity of Mo, As and elements of the higher atomic number is zero within the limits of error of measurement; and (2) there is no deviation from the λ^3 law in the immediate neighborhood of the discontinuity. To measure the absolute magnitude of the discontinuity, it is only necessary therefore to obtain values of the absorption coefficient for a short range of wave-lengths on both sides of the K limit, plot the absorption coefficient as a function of λ^3, and project the curves from each side up to $\lambda_K{}^3$, as is shown in Fig. 1, which gives the absorption discontinuity for tin.* (The ordinates are mass absorption coefficients, μ/ρ, in c.g.s. units. The abscissas are the cubes of the wave-length in angstroms.) The two graphs, one on each side of the limit, are straight lines *with no suggestion of a deviation from linearity*

[9] H. S. Read, Phys. Rev. **28,** 898 (1926).

[10] Richtmyer, Phys. Rev. **26,** 724 (1925). See also Richtmyer and Bishop, Phys. Rev. **27,** 794 (1926).

* Parenthetically, this method yields a very precise determination for λ_K. Points a and b, which mark the *apparent* beginning of the discontinuity can be easily located with relatively high accuracy. The writer has shown (reference [10]) that the *observed* width of the discontinuity is exactly equal to the slit width. The wave-length of the discontinuity, therefore, is exactly half way between the wave-length corresponding to point a and that corresponding to point b.

as the limit is approached. Projecting these straight lines to the line λ_K^3 one finds that μ/ρ_{max} is 46.6; and μ/ρ_{min} is 7.98.

In this way the maximum and minimum values of μ/ρ at λ_K were determined for Mo, Ag, Sn and Au, as shown in the columns two and three of Table I.

To obtain the ratio R_L^K it is necessary first to subtract from the observed maximum and minimum values of μ/ρ the value of the mass scattering

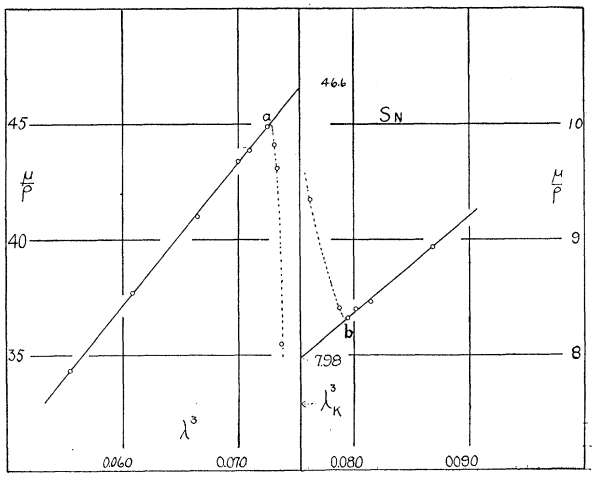

Fig. 1. The mass absorption coefficients of Sn in the neighborhood of the K absorption limit.

coefficient, σ/ρ, for the element concerned at its λ_K. These values of σ/ρ are unknown. They probably lie somewhere between 0.2 which is the value of σ/ρ given by Thomson's classical theory of scattering and, say, 1.0. Taking σ/ρ in turn as 0.0, 0.2 or 1.0, the values of R_L^K are given in columns four, five and six of the Table I. For comparison, the values of R_L^K as given

TABLE I

Observed and computed values of R_L^K.

Element	μ/ρ at K limit max.	min.	R_K^L observed $\sigma/\rho=0$	$=0.2$	$=1.0$	R_K^L calculated Kramers	de Broglie
Mo 42	81.2	12.4	6.55	6.63	7.02	5.5	14.4
Ag 47	54.7	9.08	6.05	6.12	6.65	5.5	13.3
Sn 50	46.6	7.98	5.86	5.98	6.56	5.5	12.4
Au 79	8.15	2.1	3.9	4.2	6.5	5.5	8.2

by the theories* of Kramers and of de Broglie are shown in the last two columns of the table. In any reasonable assumption for the value of σ/ρ, Kramers' theory is more nearly in *numerical* agreement with the observed values of R_L^K than is that of de Broglie.

* The theories of Thomson and of Compton give the same values of R_L^K as does de Broglie's theory. For further details see reference 1.

The situation is somewhat different, however, as regards the absolute magnitude of the discontinuity, i.e., the difference between columns two and three of the table. This difference (1) gives that part of the absorption due to the K electrons; and (2) is determined by experiment quite independently of scattering. An unambiguous comparison with theory is therefore possible. Fig. 2 shows such a comparison. The ordinates, τ_K, are the magnitudes of

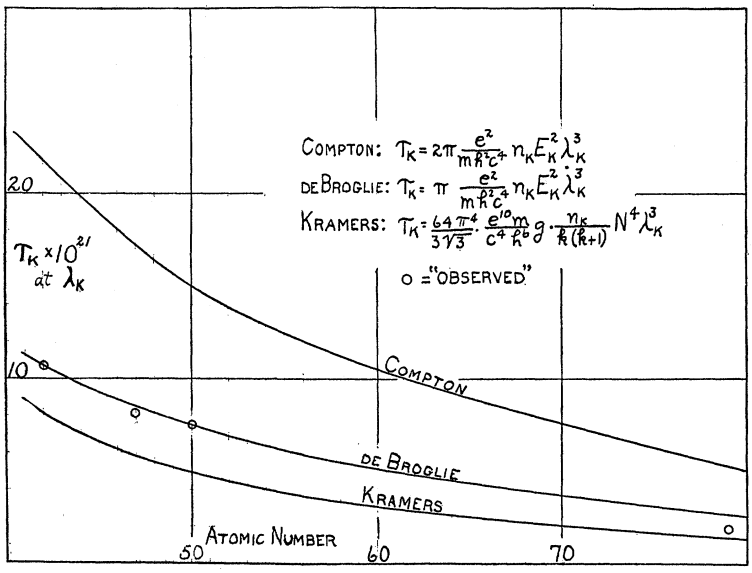

Fig. 2. The atomic absorption coefficients due to the K electrons, τ^K as a function of atomic number: Comparison of experiment with theory.

the discontinuities reduced to atomic (instead of mass) absorption coefficients. The full lines give the predictions of Compton, de Broglie and Kramers respectively. The circles give the observations, which are seen to be in excellent agreement with de Broglie's theory.

II. THE λ^3 LAW

It was pointed out by the writer[11] that the λ^3 law is only a close approximation to the truth; that if measurements of μ/ρ made for various wave-lengths far below the K limit be extrapolated to the K limit, the values of μ/ρ predicted by such extrapolation are somewhat larger than the observed values. Accordingly Allen[8] has proposed that n, the coefficient of λ, is 2.92, instead of 3.00 in the experimental formula

$$\mu/\rho = k\lambda^n + \sigma/\rho$$

In view of these experimental facts which show a slight departure from the λ^3 law, and of the theories of Wentzel[6] and of Oppenheimer[7] above mentioned, it seems desirable, with the improved technique now available, to make a

[11] Richtmyer, Phys. Rev. **18**, 13 (1921).

complete redetermination of absorption coefficients. The writer has started such a redetermination, with the hope of attaining a precision of 0.2 percent, or better if possible. It is not difficult to make measurements of absorption coefficients with a precision of the order of 1 or 2 percent; but to increase the precision several fold is quite another matter. A coming interruption of many months seems to make it desirable to report briefly some preliminary measurements on Sn covering the wave-length range $0.19 < \lambda < 0.42$A. These measurements are shown graphically in the curve *abc*, Fig. 3, in which μ/ρ is plotted as a function of λ^3. *Over the range bc the graph is seen to be a straight line within the limits of error of measurement.* Point *c* is very close to the *K* discontinuity. The writer is not prepared to state, without further investigation, that the part of the curve indicated by the dotted line *ab* makes a sharp break at *b* with the line *bc*. It is certain, however, that an extrapolation of the line *bc* toward shorter wave-lengths gives values of μ/ρ higher than

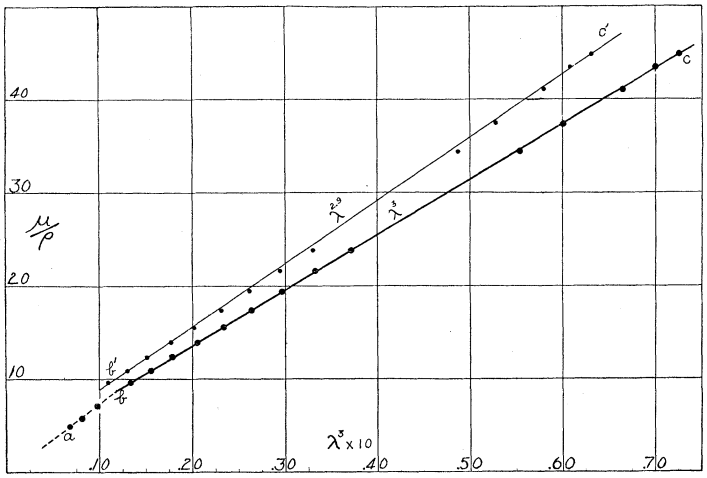

Fig. 3. The absorption coefficients of Sn in the range $\lambda_K/2 < \lambda < \lambda_K$.

observed by amounts much above the limits of precision. Incidentally, the *same* sample of tin was used for all these measurements; so that the apparent break at *b* in the graph *abc* is not due to the use of different samples of the absorber.

If, instead of λ^3, $\lambda^{2.9}$ is plotted as a function of μ/ρ (the $\lambda^{2.9}$ scale is not shown) the curve $b'c'$ results. This curve deviates from a straight line by amounts considerably in excess of experimental error. In other words *the experimentally observed (total) absorption coefficient of Sn in the wave-length range given approximately by* $\lambda_K/2 < \lambda < \lambda_K$ *is a linear function of* λ^3 *within experimental error.* The exponent of λ, if it does differ from 3, differs by less than one percent. *This law holds right up to the absorption limit,* as is seen by considering Figs. 1 and 3, in connection with the results reported in the paper[11] previously referred to, in which it was shown that even within the very narrow region at the absorption limit, a few thousandths of an angstrom

wide, where on account of the necessity of making observations with finite slit widths spectral regions from both sides of the limit overlap, observations taken in small steps through the limit were in complete agreement with the assumption that *there is no departure, within this narrow region, from the law of absorption which holds just outside the region*—which law is the λ^3 law.*

The writer hopes to resume these measurements in due course with improved apparatus. He wishes to take this opportunity to express his appreciation of the invaluable services of Mr. L. S. Taylor who assisted in making many of the measurements upon which this paper is based. (See abstracts of papers presented by Richtmyer and Taylor at meetings of the American Physical Society, Phys. Rev. **29**, 353, 606, 1927.)

CORNELL UNIVERSITY,
August, 1927.

* Of course, it is quite possible that if *accurate* ionization-chamber measurements could be made through an absorption limit, with sufficiently narrow slits one might find an indication of the *structure* of the limit such as has been observed in some instances by photographic means. Since, with a beam limited by two slits of equal width, the energy transmitted through the slits is proportional to the *square* of the slit width, such measurements present grave difficulties because of the low intensities involved. No such "structure" of an absorption limit has ever been observed by the writer.