DOUBLET SEPARATION AND FINE STRUCTURE OF THE BALMER LINES OF HYDROGEN[1]

By Norton A. Kent, Lucien B. Taylor and Hazel Pearson

Abstract

Using two optical trains, (1) two crossed Lummer plates, the larger of resolving power of about 670,000, and (2) an echelon of resolving power of about 660,000, the writers have determined the wave-length difference between the two well known components λ' and λ'' ($\lambda'>\lambda''$) of $H\alpha$, $H\beta$ and $H\gamma$ as 0.1370, 0.0791 and 0.0666A respectively. These values are in good agreement with those obtained by Houston with an interferometer, which latter are in our estimation the most reliable thus far obtained.

Further, microphotometer curves of enlargements of the original Lummer plate negatives reveal another component in λ' unresolved but unquestionably present, in $H\alpha$, $H\beta$ and $H\gamma$. Hansen noticed assymmetries here in $H\alpha$ and $H\beta$. There are also indications of other components in λ''. The *magnitudes* of these components agree well with the theoretical magnitudes given by the new quantum mechanics with the spinning electron.

Object of the Investigation

THE object of this study was two-fold: (I) to measure the doublet separation of $H\alpha$, $H\beta$ and $H\gamma$, and (II) to search for further structure, using large instruments, the dispersion of the large Lummer plate and the resolving powers of this plate and the echelon being in general about twice that of instruments used by others.

The importance of the investigation lies in its bearing upon the various theories of atomic structure. It was undertaken in an attempt to verify the old quantum theory, the new quantum mechanics, with the spinning electron of Goudsmit and Uhlenbeck[2] not having been advanced at that time.

Historical Survey

Critical analyses of the errors which may enter into the determination of the difference in wave-length of closely adjacent lines, particularly the Balmer series doublets, with the addition, in some cases, of new data, have been made by Oldenberg,[3] Lau,[4] Janicki[5] and others. There may enter:

(1) An instrumental error—the "shrinking effect," which occurs when the resolving power of the spectroscope is just sufficient to separate the lines. The curves then overlap and the intensity maxima approach each other.

[1] Preliminary reports of this work have appeared in Nature **119**, 163 (1927) and Phys. Rev. **29**, 748 (1927).

[2] S. Goudsmit and G. E. Uhlenbeck, Physica **6**, 273 (1926).

[3] Oldenberg, Ann. d. Physik. **67**, 253 (1922).

[4] Lau, Phys. Zeits. **25**, 60 (1924).

[5] Janicki, Ann. d. Physik. **76**, 561 (1925).

(2) A visual error, due to the fact that the eye locates the most intense blackening near the edge of the image which possesses the steepest intensity gradient.

(3) A photographic error—the maximum blackening of the plate appearing on the outer edges of narrow doublets, that is where the developer possesses the greater concentration.

Janicki measured by machine the published reproductions of the photographic plates taken by Schrum,[6] with a Lummer plate, and thus eliminated the visual error. Smaller values of $\Delta\lambda$ thus resulted. The two other errors (1) and (3) are least in the lower orders of a Lummer plate pattern as the images here are of less intensity and lie further apart.

Van Cittert[7] discusses mathematically the determination of $\Delta\lambda$ with a Lummer plate, criticizes this instrument severely and, while citing other sources of error, such as (a) the mist of diffused light and (b) the continuous background (these of course being possible with other instruments as well) emphasizes especially: (a) The effect of changes in the coëfficient of reflection within the plate at its surface, as one passes from the central region of the pattern. $\Delta\lambda$ is thus decreased but even in the lowest orders the error is small. (b) A change in the image which is formed because the Lummer plate modifies the intensity distribution and produces in the lower and weaker orders an approach of the components of a doublet.

Hansen,[8] giving results obtained with several Lummer plates, discusses in an extensive paper the intensity assymmetries of the hydrogen doublets, tabulates corrections to be made for the shrinking effect as a function of the "saddle height" between the maxima of the curves, cites the effect of decrease of the reflection coefficient as one passes from lower to higher orders, this causing a decrease in $\Delta\lambda$; and cites also a decrease in the sharpness of the interference as causing an increase in the relative saddle height. He further discusses the size of the illuminating slit of the registering photometer and the effect of the inequality of the curves traced by the machine, causing an increase in $\Delta\lambda$ as one passes to higher orders. These and other errors we summarize in Table I, together with methods of elimination.

The variations in the values obtained in previous investigations, some of the most recent of which are listed in Table II, are not at all surprising in view of these numerous errors which may enter, especially with the use of the Lummer plate and echelon. The influence of these errors is well illustrated in the values of $\Delta\lambda$ given in Tables IV to VII.

As to fine structure: Considerable attention has been given to the influence of the Stark effect in bringing up components which, on the old quantum theory, were ruled out by the principle of selection. Hansen feels that the presence of another component in λ', the shorter wave-length component of the doublet, has been proven by him by an observed assymmetry; and that λ'' is possibly double is shown by the variation in the relative intensities of the components of the main doublet with a variation of pressure.

[6] Schrum, Proc. Roy. Soc. London **A105,** 259 (1924).
[7] Van Cittert, Ann. d. Physik. **77,** 372 (1925).
[8] Hansen, Ann. d. Physik. **78,** 558 (1925).

TABLE I

Errors which may enter into the determination of the difference in wave-lengths of closely adjacent lines.

Nature of Error	Advancing to higher orders Δλ is:—	Lummer Plate: Magnitude	Lummer Plate: To eliminate	Echelon: Magnitude	Echelon: To eliminate	Fabry-Perot Interferometer: Magnitude	Fabry-Perot Interferometer: To eliminate
I. Instrumental							
a. shrinking	decreased	variable	use lowest orders	variable	use "faint" plates	variable	adjust mirror distance[4]
b. reflection coefficient[1]	decreased	very small	use lowest orders	negligible		negligible[3]	
c. intensity distribution[2]	increased	large in lowest orders	use higher orders	very large	Use microphotometer and correct errors. See IV b, below	negligible	
II. Visual	increased	This error may be large in all these instruments, if dense plates be used; but it may be avoided by using a microphotometer.					
III. Photographic a. Developer b. "Mist"[5] c. Continuous background	increased	Error *a* is probably small if not negligible and may be avoided by long development with weak developer and sufficient rocking of the plate. Errors *b* and *c* are variable in magnitude and may be eliminated by excluding extraneous light and exposing and developing properly.					
IV. Microphotometer a. Slit width	A slit sufficiently narrow to resolve the lines is necessary. If the slit be too narrow the plate grain may vitiate the curves. To remedy this, increase the slit length. This must not, however, be pushed too far with echelon, and especially interferometer, images, for they are curved. With the interferometer if a long slit be necessary the outer orders may be used as their radii of curvature are greater.						
b. Inequality of heights of curve	Error *b* is not strictly a microphotometer error but arises from I, *c*. It is variable in magnitude and is largest in the lowest orders of the Lummer plate pattern. An approximate correction may be made for it by measuring distances from the points of tangency of curves joining the peaks of the components.[6]						

[1] The decrease of the value of the reflection coefficient with increasing angle of incidence within, and against the surface of, the plate causes (1) a loss of light which decreases as we proceed toward higher orders, and (2) a decrease of the sharpness of the interference.

[2] This error enters strongly in the lowest orders and is especially great in a plate of large dispersion.

[3] If the interferometer be adjusted properly this error does not exist: the rings are of equal intensity although the dispersion decreases from the center outward.

[4] Adjust the distance between the mirrors so that, with a doublet such as $H\alpha$, the successive images of one component lie quite accurately between those of the other.

[5] See Van Cittert, Ann. d. Physik **77**, 4, Nt. 12.

[6] See Hansen, Ann. d. Physik **78**, 6, Nt. 22, Fig. 7 of p. 571.

TABLE II

Certain recent determinations of Δλ in Angstroms.

	McLennan and Lowe[9] 1912	Gehrcke and Lau[10] 1922	Geddes[11] 1922	Oldenberg[3] 1922	Schrum[6] 1924	Schrum and Janicki[12] 1925	Hansen[8] 1925	Janicki and Lau[13] 1925	Houston[14] 1926
H_α	.154	.126	.146	.140	.141	.130	.135–38	.132	.1358
$H\beta$	.085	.071	.078		.085	.076	.075	.072	.0782
$H\gamma$	.062	.056			.070	.060	.062	.059	.0665

Janicki and Lau remeasured with a microphotometer certain of their plates and obtained results which pointed to the presence of five components, which structure would confirm the old quantum theory with a modified selection principle.

[9] McLennan and Lowe, Proc. Roy. Soc. London **A100**, 217 (1921).

[10] Gehercke and Lau, Ann. d. Physik. **67**, 388 (1922).

[11] Geddes, Proc. Roy. Soc. Edin. **43**, **1**, 37 (1922).

[12] Schrum and Janicki, Ann. d. Physik. **76**, 561 (1925).

[13] Janicki and Lau, Zeits. f. Physik. **35**, 1 (1925).

[14] Houston, Astrophys. J. **64**, 81 (1926).

Apparatus and Experimental Procedure

The spectroscopic systems are sketched in Fig. 1. The crossed Lummer plate system A consisted of (a) a Hilger constant deviation spectroscope fitted to hold the small Lummer plate, L_1, dispersing vertically and (b) a second spectroscope for the large plate, L_2, dispersing horizontally. The echelon system B contained the echelon[15] and constant deviation prism in

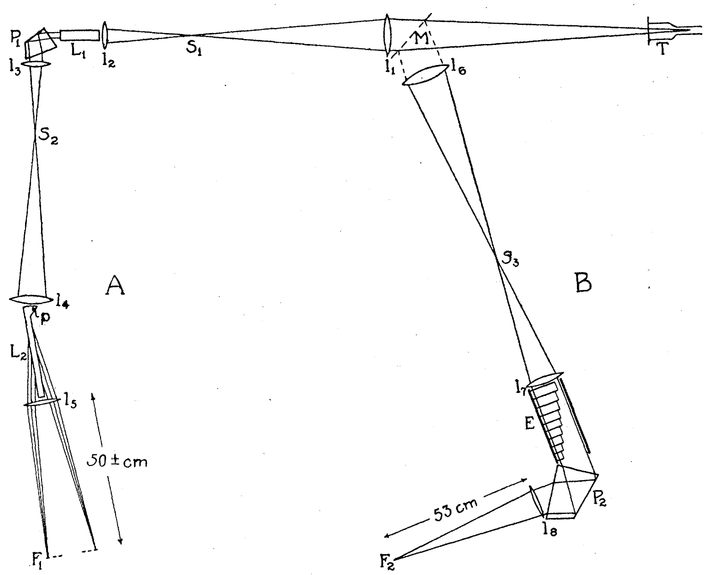


Fig. 1. Arrangement of spectroscopes. T, vacuum tube; M, plane mirror; $l_1 \cdots l_8$, lenses; L_1, L_2, small and large Lummer plates; P_1, P_2, prisms; F_1, F_2, focal planes of l_5 and l_8; S_1, S_2, S_3, slits; p, rotation point of large plate.

tandem in one instrument. The light for B was taken off by a mirror, M, which caught the light that would otherwise pass through the lower half of lens l_1, and be thrown away at the front end of plate L_1. The constants of the Lummer plates are shown in Table III.

Table III

Constants of the Lummer plates.

	Small plate	Large plate
Thickness	0.4827 cm	0.9963 cm
Length	13.1	30.0
Width	1.45	3.98

		Small Plate			Large Plate		
Line	λ(A)	μ	$d\mu/d\lambda(A^{-1})$	$\Delta\lambda_m$(A)	μ	$d\mu/d\lambda(A^{-1})$	$\Delta\lambda_m$(A)
H_α	6563	1.50746	− 315.436	−0.38606	1.57187	− 513.607	−0.17204
$H\beta$	4861	1.51560	− 725.535	−0.20645	1.58581	−1301.47	−0.09038
$H\gamma$	4341	1.52025	−1023.67	−0.16210	1.59417	−1941.01	−0.07006

[15] Described in Prod. Amer. Acad. Arts and Sciences **57,** 1 (1921).

The maximum theoretical resolving powers for Hα are, approximately, for the small plate, 300,000; for the large plate, 670,000; and for the echelon 660,000.

Lens l_5 was a compound monochromat, made by Bausch and Lomb, the field being flat to within a small fraction of a mm over a central region 5 cm in diameter.

The source was a Pyrex glass tube of the Wood[16] form (see Fig. 2). The capillary could be cooled by running water or by liquid air.

The discharge was obtained by a transformer kindly lent us by Professor Pierce of the Cruft Laboratory at Harvard. The current in the secondary circuit was maintained constant during an exposure and the pressure was held at such a value that the positive column striations were clearly marked.

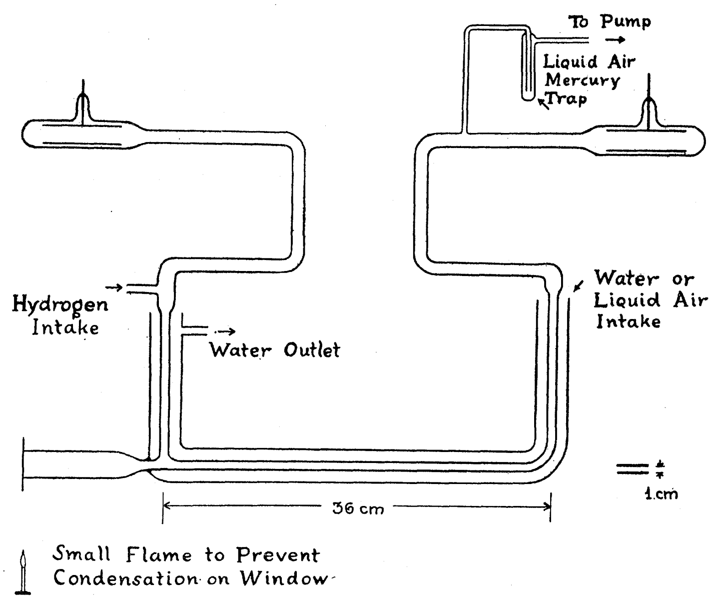


Fig. 2. Vacuum tube used as source.

Visual observation showed that the width of the lines decreased with current density. We therefore used generally but about 25 ma./cm². During exposures the temperature seldom changed more than 0.2°C and usually much less. The hydrogen was led into a reservoir containing water and thence passed to the discharge tube through a long capillary.

With the conditions under which we worked the preliminary formation of an ice film within the discharge tube, as suggested by Schrum,[6] was found unnecessary. The secondary spectrum could be almost eliminated, even at liquid air temperature, by obtaining proper conditions of pressure and amount of water vapor.

An attempt was made to increase the electron emission in the discharge tube by means of tungsten filaments. Three of these, kindly presented to

[16] Wood, Proc. Roy. Soc. London **A97**, 455 (1920).

us by Dr. Coolidge of the General Electric Co., were mounted, one each, near the two terminals and one near the capillary of the tube. With the conditions under which we were obliged to work, notably with much water vapor present, no enhancement of the Balmer series was observed, although the three filaments were used singly, in pairs, and all at once. They were therefore removed.

The dispersion of our 30 cm Lummer plate was such that the image of the nth order of the longer wave-length component of each of the three doublets was almost coincident with the $(n+1)$th order of the component of shorter wave-length: hence the use of the smaller plate to separate these components. The resulting cross pattern had disadvantages from the standpoint of the quantitative evaluation of the wave-length differences

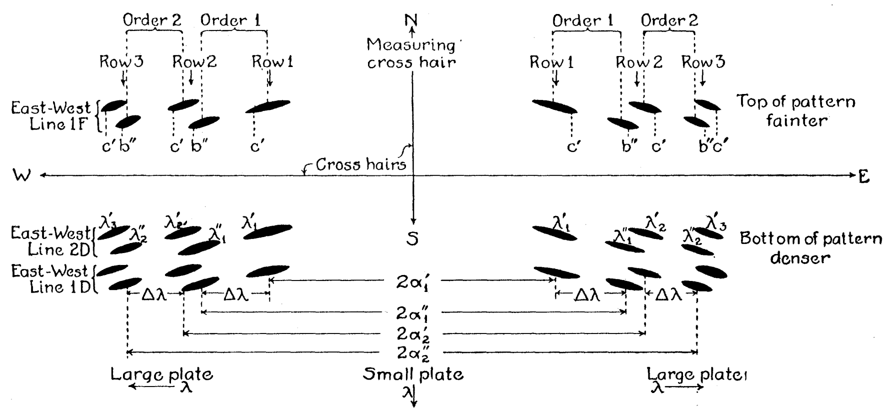


Fig. 3. Crossed Lummer plate pattern of Hα. Line of motion of plate in being measured is East-West. E-W line $1F$ contains the faintest orders of λ' and λ'' given by the small Lummer plate; E-W line $1D$, the densest. Rows "1" (on left and right sides of the pattern) contain only λ', the shorter wave-length component of the doublet. Rows "2" contain the second orders of λ' and the first of λ''. The theoretical positions of the components c' of λ' and b'' of λ'' are indicated on the E-W line $1F$ above the letters c' and b''.

between the components, for the pattern is composed of slanting lines as shown in Plate I and schematically in Fig. 3; but the combination of the two plates increased the chances of observing the presence of other than the well-known pair of components.

The equation used in determining $\Delta\lambda$, or the difference in wave-length between the two components of these doublets, as given by the Lummer plate system, was that of McLennan and McLeod[17] namely

$$\Delta\lambda = \frac{\alpha_s^2 - \alpha_n^2}{\alpha_{n+1}^2 - \alpha_n^2} \cdot \Delta\lambda_m$$

where n is the number of the order as seen on the photographic plate; α_n, α_s are the distances of the images of the nth order of lines of wave-length λ,

[17] McLennan and McLeod, Proc. Roy. Soc. London **A90**, 246 (1914).

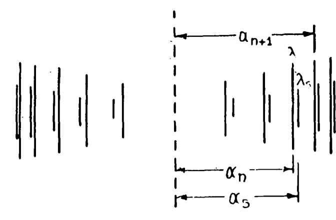


Fig. 4. Single Lummer plate pattern.

and λ_s (see Fig. 4) from the center of the pattern; $\Delta\lambda_m$ is that change in wave-length which would cause the image of the nth order of a pattern given by a wave-length of $(\lambda+\Delta\lambda)$ to coincide with the image of the $(n+1)$th order of the pattern for λ.

$\Delta\lambda_m$ may be calculated from the constants of the Lummer plate and the various values of α may be measured on the original negatives or machine tracings made therefrom. Although approximations are made in its development, this formula is accurate except for the very highest orders. $\Delta\lambda$ calculated from the 20th order is affected by only a few parts in the fourth decimal place of Angstroms.

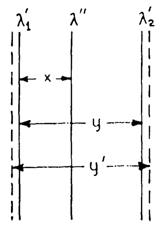


Fig. 5. Echelon pattern.

In obtaining $\Delta\lambda$ with the echelon, the axis of the instrument was set as nearly parallel to the incident light as was possible, but so that at least one component of the doublet was in double order condition, e.g., λ_1' and λ_2' as in Fig. 5. From the known constants of the instrument the distance, Δ_o, between these two images, was calculated. The values of Δ_o in Angstroms, after small negative corrections for prism dispersion are: for Hα, 0.3055; for Hβ, 0.1568; and for Hγ, 0.1214. Then $\Delta\lambda = x\Delta_o/y$.

This process may be criticized on the ground that, as λ_1' and λ_2' are not extremely narrow and also lie near the outer regions of the intensity curve of the instrument, the measured distance, y, would be too small. λ'', however, lies so nearly midway between λ_1' and λ_2' that both x and y would suffer nearly equal percentage changes, and thus $\Delta\lambda$ would remain unchanged. This reasoning holds for Hα, Hβ, and Hγ. It relieves us of the necessity of determining the true distance, y', at which narrow lines would lie.

Methods of Measurement

The plates obtained with the crossed Lummer plate system, indicated by the letter L were measured:

(a) Visually, by a Gaertner comparator. Settings were made—on the center of the slanting images in any given East-West line—by the North-South cross hair—set, for instance, as indicated in Fig. 3, on "Line 1 D."

(b) Mechanically by the Moll recording microphotometer of the Harvard Observatory, kindly put at our disposal by its Director, Professor Shapley. To obtain these curves, the plate was moved past the luminous rectangle R of Fig. 6 so that there were exposed on the first run only the images of the shorter wave-length component, e.g., λ_1', λ_2', λ_3', etc. lying aslant on any one horizontal line of Fig. 3. The plate case was then moved up in its recess so as to unmask the corresponding East-West line of the longer wave-length images λ_1'', λ_2'', λ_3'', etc., and a second run was made. As the vertical height of the photometer slit was but 0.2 mm, the intensity of illumination was low.

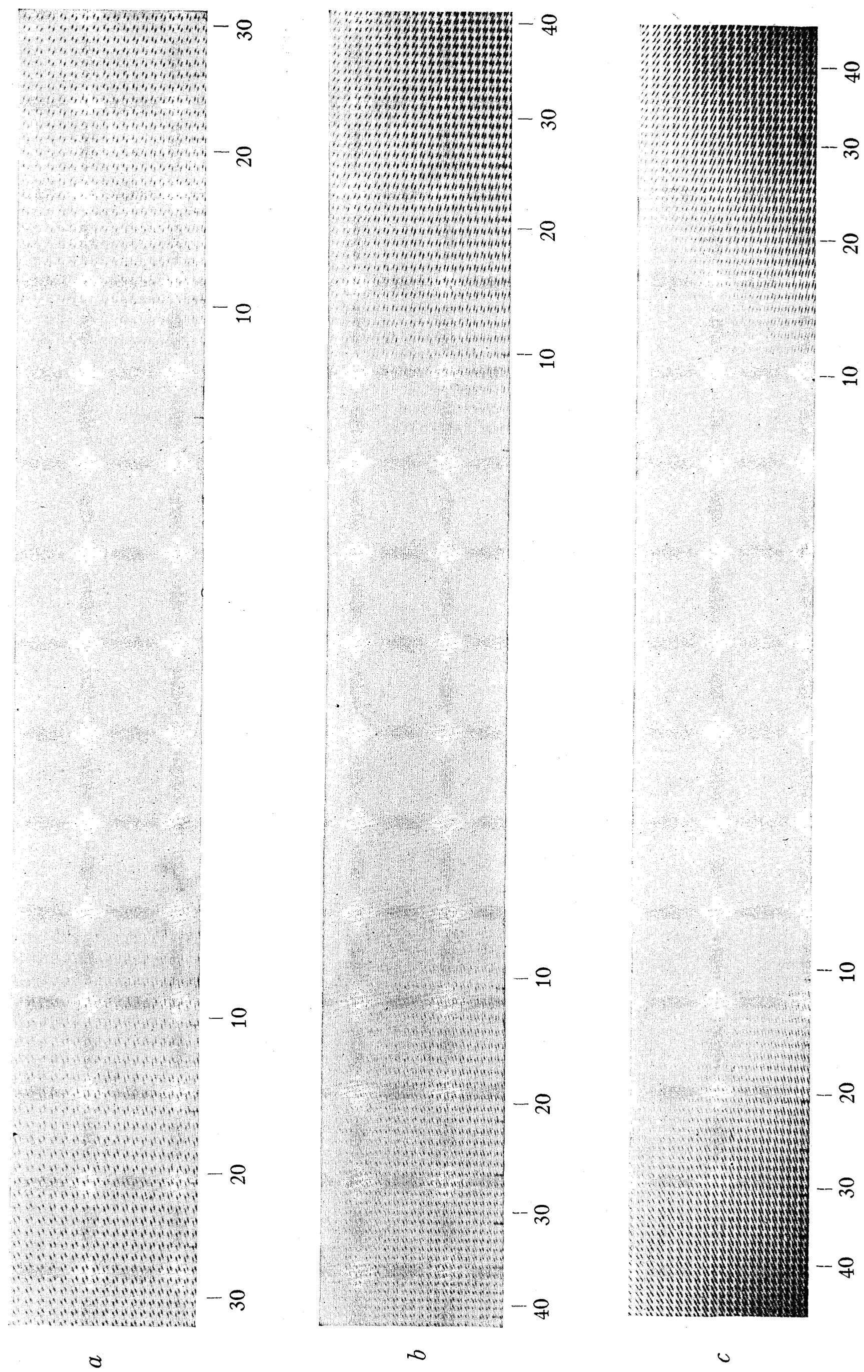
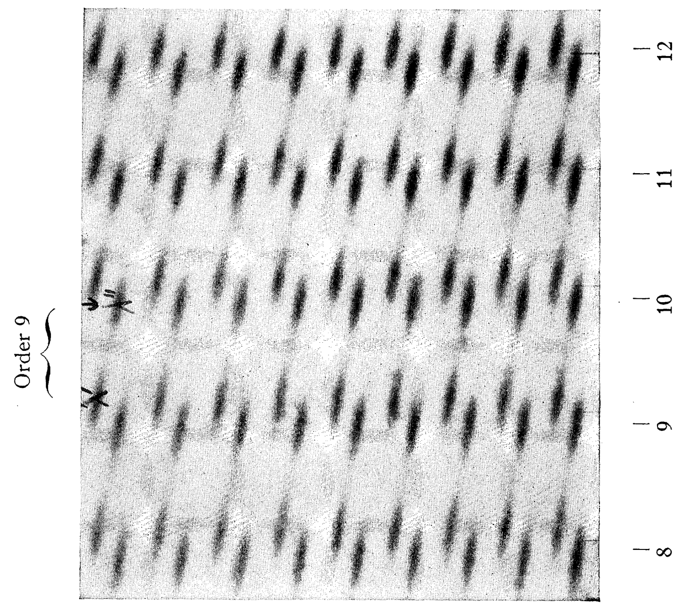
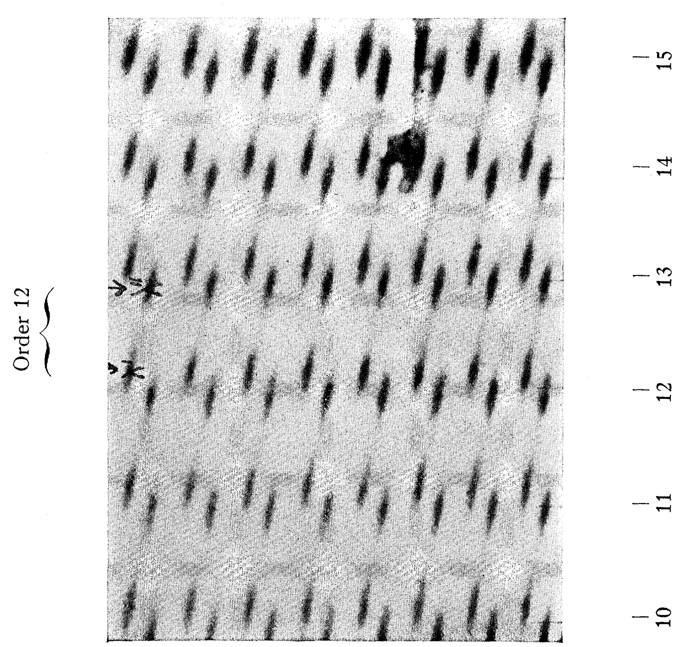


Plate I. Pattern a is Plate 4 for $H\alpha$; b is plate 10 for $H\beta$; c is plate 25 for $H\gamma$. The magnification is 4.2 fold. See Fig. 3 and Plate II as aids in studying the pattern. The numbers refer to rows.



b



a

Plate II. Pattern *a* is plate *L*2 for Hα, magnification 32 fold; pattern *b* is plate *L*7 for Hα, magnification 28 fold. Observe the persistent assymetry toward the right in λ'.

The plates obtained with the echelon, indicated by the letter E, were measured: (a) Visually by the Gaertner comparator. (b) Mechanically, by the Moll microphotometer and later, through the kindness of Dr. Adams and Dr. Pettit of the Mt. Wilson Observatory by the microphotometer of that laboratory.

Results

I. *Doublet Separation.* The values of $\Delta\lambda$ obtained from the patterns given by the crossed Lummer plates are shown in Tables IV to VI. There appear therein none obtained from the microphotometer curves. The latter values are almost without exception from 6 to 11 percent higher than the corresponding values determined visually. The reason for this is as follows:

$\Delta\lambda$ depends upon the East and West separation of the density maxima of λ' and λ'' (see Fig. 6). The background of the plate is more dense in regions D than d and, as the plate passes along so that the small rectangle of

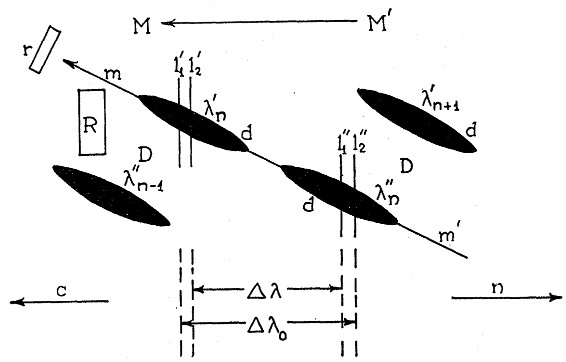


Fig. 6. To illustrate method of measuring plates with the microphotometer. R, r, luminous rectangles; M, M' and m, m', direction of motion of the plate past the luminous rectangles R, r in the study of the doublet separation and fine structure, respectively; $\Delta\lambda$, true doublet separation; $\Delta\lambda_0$ separation recorded by the microphotometer; c, n, direction of center of pattern and higher orders, respectively; D, regions of background of plate denser than regions d.

light falls upon the images of λ', the galvanometer registers a maximum density for them too near the center of the pattern, i.e., at l_1' instead of l_2', while on the following run, when the plate is moved up and the λ'' images are unmasked, the density maxima are registered as at l_2'' instead of l_1''.

Thus the λ' maxima are shoved *toward*, and the λ'' maxima *away from* the center of the pattern. Both these displacements increase $\Delta\lambda$ and the error increases with the density of the images. This error affects less the *visual values* of $\Delta\lambda$ because the eye does not integrate the density over a rectangular region, as does the machine, but locates the maximum blackening along the slanting line of the image. These curves then are of little value in the quantitative determination of $\Delta\lambda$. We are limited, therefore, to the visual measurements.

Of the errors summarized in Table I, the shrinking effect enters in only one plate $L21$ for $H\gamma$ for the images generally used, those of the orders below the 20th, are well separated. The reflection coefficient error is negligible. The intensity distribution error is prominent. The images of the shorter wave-length component lie so much nearer the center of the pattern than those of the longer that their maxima are displaced outward more and $\Delta\lambda$ is thereby decreased. It will be noticed, in Tables IV and V, that in all cases the low orders give low values. It is here, however, that the microphotometer curves do aid us, for from them we can determine how many of the low orders should be rejected because of too great a change in intensity from one order to the next. We thus use only those values which are given in bold faced type. Further, in the dense plate $L15$ for Hβ (Table V) there

TABLE IV

	Hα						Hβ			
Plate No.	L4	L8	L2	L3	L7	L30	L10	L13	L14	L21
Current (ma/cm²)	24	24	24	24	24	13	24	24	24	24
Exposure (min.)	20	22	25	45	55	70	30	15	19	30
Character	Faint	Faint	Medium	Medium	Medium	Medium	Faint	Medium	Medium	Medium
E-W line No.	10	10	10	10	10	10	12D	12D	12D	11D
Order No. 3					.131					
4				.132	.133				.077	.078
5		.133	.132	.132	.134			**.075**	.078	.079
6		.135	.134	.134	.135	.133		**.077**	**.080**	.079
7	.134	.136	.134	**.135**	**.137**	.135		**.076**	**.078**	.080
8	**.137**	**.136**	.135	**.134**	**.135**	**.138**	**.076**	**.075**	**.079**	**.081**
9	**.137**	**.138**	(.129)	**.136**	**.136**	**.140**	**.077**	**.079**	**.081**	**.081**
10	**.136**	**.137**	**.137**	**.136**	**.138**	**.140**	**.077**	**.079**	**.081**	**.081**
11	**.136**		**.135**	**.137**	**.138**	**.139**	(.073)	**.080**	**.079**	**.081**
12	**.136**		**.137**	**.138**		**.140**	**.077**		**.080**	**.081**
13	**.135**		**.137**	**.137**		**.140**	**.078**		**.081**	**.081**
14	**.136**		**.134**	(.130)		**.139**	**.076**		**.080**	**.080**
15	**.140**		**.140**			**.139**	**.078**		**.079**	**.081**
16	**.136**		**.139**			**.137**			**.079**	
17	**.139**					**.138**			**.078**	
18	**.140**					**.138**			**.080**	
19	**.139**					**.140**			**.082**	
20	**.138**					**.143**			**.080**	
21	**.143**					**.141**			**.079**	
22	**.137**					**.139**			**.080**	
23						**.138**			**.081**	
24						**.138**			**.082**	
25						**.136**			**.081**	
26						(.117)				
27						**.140**				
28						**.138**				
29						**.140**				
30						**.140**				
Mean $\Delta\lambda$	$.137_7$	$.137_0$	$.137_0$	$.136_1$	$.136_8$	$.139_1$	$.077_2$	$.077_7$	$.080_0$	$.080_9$
Weighted mean of means			$.137_1$						$.079_4$	

exists a small progressive increase of $\Delta\lambda$ with increase in density of the images whether we advance along any of the arrows a to f. Precisely the same effect enters in a dense Hα plate, $L25$ (not shown). This is probably a visual error, much resembling that which vitiated the results of the microphotometer tracings.[18] In $L21$ for Hγ, a very dense plate, the shrinking effect also enters and prevails. We thus rejected $L25$ and $L15$ entire and saved only the central values in $L21$ as it seemed probable that the two errors here offset each other.

By methods similar to those used in Table IV we obtain for Hγ mean values of $\Delta\lambda$ (plates $L18$, $L19$, and $L21$) 0.064_8, 0.065_3 and 0.067_3, respectively, which yield a weighted mean of means of 0.066_4. The weighted mean

[18] The increase cannot be due to the coming out of component c'' of Fig. 7 which would displace the center of gravity of the λ'' system at the most but 0.001A.

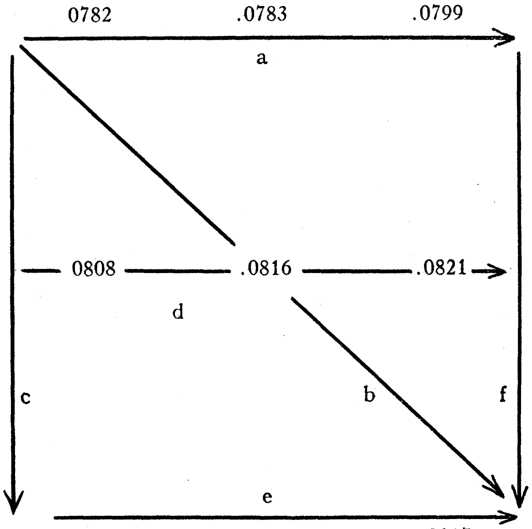
is determined in all cases by giving each mean a weight equal to the number of orders used. In Table IV *L*30 gives, at the very low current density of 13 ma/cm^2, a value for $\Delta\lambda$ of 0.139_1. We shall return to this point later.

The doublet separations as determined by the echelon are given in Tables VI and VII. All the plates taken which were suitable for visual measurement were used (the densest plates being rejected). From the best of these plates microphotometer tracings were made.

TABLE V

Plate No. L 15 for Hβ

Line No. / Order No.		2F and 3F Faint	13D and 12D Medium	3D and 2D Dense
4	Faint	0782	.0783	.0799
14	Medium	0808	.0816	.0821
24	Dense	0824	.0823	.0827



The chief errors entering with the echelon are: First, the shrinking effect. This could not be avoided in the visual measurements, but in the case of the microphotometer curves the saddle heights were obtained and corrections made using Hansen's table for *symmetrical* doublets. In using these we assume symmetry which is not strictly the case; but the error must be small. Second, the visual error. This is present in all the measurements but was avoided as much as possible by the rejection of the densest images. Third, the intensity distribution error. This fortunately enters to a very small extent in this particular case, as the image of one component in single order condition lies very nearly between the two images of the other in double order condition, as mentioned on page 274.

The visual and microphotometer values are listed in Tables VI and VII respectively. No good tracings could be obtained for Hγ as the images were poor. The weighted means are calculated as in Table IV.

TABLE VI

Visual measurements of $\Delta\lambda$ from echelon plates.

Line	Plate No.	Current ma/cm²	Character of image	$\Delta\lambda$* (Angstroms)	Means	Weighted means
H_α	E 8a	24	faint	0.1338		
	E 8b	24	faint	.1354	0.1367	
	E 6b	24	medium	.1386		
	E 7a	24	medium	.1388		0.136₈
	E35a	25	faint	.1305	0.1353	
			medium	.1400		
	E21a	23	faint	.1349	0.1365	
			medium	.1381		
$H\beta$	E22	23	medium	0.0771		
	E10b	24	medium	.0772	0.0774	
	E11b	24	medium	.0779		
	E 9b	24	dense	.0794	0.0794	0.079_4
	E38b	25	dense	.0818		
				.0826	0.0813	
				.0799		
$H\gamma$	E12b	25	faint	.0644	0.0637	
	E35b	25	faint	.0630		0.067_2
	E38b	25	medium	.0713	0.0707	
			medium	.0701		

* The agreement between different determinations of $\Delta\lambda$ from the same plate is such that four decimals may be saved.

TABLE VII

Microphotometer measurements of $\Delta\lambda$ from echelon plates.

Line	Tracing No.	Plate No.	Character of image	$\Delta\lambda$(A) (meas.)	Saddle height	Correc- tion	True $\Delta\lambda$	Means	Weighted means
H_α	1(1)	E21a	faint	0.1332	57	1.45	0.1351		
	(2)		faint	.1310	51	.95	.1322	0.1338	
	(3)		faint	.1325	55	1.25	.1342		
	2(8)		medium	.1321	71	4.1	.1375	0.1371	
	(10)		medium	.1324	67	3.1	.1367		
	2(5)	E35a	medium	.1297	82.8	9.6	.1422		0.137_3
	(6)		medium	.1306	80.9	8.3	.1414	0.1413	
	(7)		medium	.1297	79.9	7.7	.1404		
	II*(a)	E 8a	faint	.1356	65.5	2.8	.1394		
	(b)		faint	.1330	63.3	2.3	.1361	0.1269	
	(c)		faint	.1310	66.8	3.1	.1351		
$H\beta$	IV†(d)	E38(b)	dense	0.0749	86.8	15	0.0799		
					81.2	8.5		0.0785	0.078_5
	(e)		dense	.0744	86.7	14.7	.0771		
					84.6	11.2			

* Mount Wilson microphotometer: very reliable tracings.
† As both saddle heights were high, but unequal, two corrections were made and their difference taken.

In calculating the final means from the values obtained by the Lummer plate system and the echelon we feel, first, that we are justified in rejecting entirely, for reasons previously stated, the microphotometer results of the crossed Lummer plate system; and, second, that the relative weights we should assign are: 3 to the Lummer plate visual measurements, *although visual* (both because of the greater dispersion of the instrument and the greater number of determinations), 1 to the visual and 2 to the microphotometer values obtained with the echelon. These weighted means are given in Table VIII.

TABLE VIII

Final results for the doublet separations of H_a, $H\beta$, $H\gamma$. Weighted means of $\Delta\lambda$ and the corresponding values of $\Delta\nu$ obtained under the conditions of tube diameter, of pressure, and of temperature specified in the text.

Line	Mean values given in Table No.:	Lummer plate system visual	Echelon visual	Echelon microphotometer	Weighted means of $\Delta\lambda$(A)	$\Delta\nu$ (cm^{-1})
$H\alpha$	IV	0.1371				
	VI		0.1363		$0.137_0+0.001$	0.318 ± 0.002
	VII			0.1373		
$H\beta$	IV	0.0794				
	VI		0.0794		$0.079_1\pm0.001$	0.335 ± 0.002
	VII			0.0785		
$H\gamma$	(in text)	0.0664				
	VI		0.0672		$0.066_6\pm0.002$	0.353 ± 0.004

The theoretical values of $\Delta\nu$ on the old quantum theory are given in Table IX. With them are also tabulated the values predicted by the new quantum mechanics, with the spinning electron, based on the intensities given by Sommerfeld and Unsöld.[19] To these are added the experimental results obtained by Houston and the writers. The observations agree better with the new theory. Houston's results we regard as the most accurate hitherto obtained. Our values are somewhat larger. He, however, used a large current density. A change of $\Delta\lambda$ with current density would explain the result given by plate $L30$ (see Table IV) taken at but 13 ma./sq.cm. and giving, for $H\alpha$, $\Delta\lambda=0.1391$A and $\Delta\nu=0.323$ cm^{-1}.

TABLE IX

Doublet separations, $\Delta\nu$, theoretical and observed, of H_a, $H\beta$, and $H\gamma$.

		H_a	$H\beta$	$H\gamma$
Theoretical	Old quantum theory*	0.328	0.350	0.357
	New quantum mechanics, with spinning electron	.320	.345	.354
Observed	Houston	.315	.331	.353
	Kent, Pearson and Taylor	.318 (.323)†	.335	.354

* These values follow on the Bohr-Kramers' theory without an electric field and neglecting component c'' of Fig. 7. See Birge, Phys. Rev. **17**, 594 (1921).

† At low current density.

It will be noted that the theoretical value for $H\beta$ on the new theory is greater than those observed by both Houston and us by an amount greater than the experimental error.

[19] A. Sommerfeld and A. Unsöld, Zeits. f. Physik. 237 (1926).

II. *Fine structure.* The new quantum mechanics with the spinning electron gives, at the hand of Sommerfeld and Unsöld, for Hα, Hβ and Hγ, structure and relative intensities as shown in Fig. 7.

The Lummer plate images of Hα, on the original negatives and under the proper photographic exposure conditions, show repeatedly a consider-

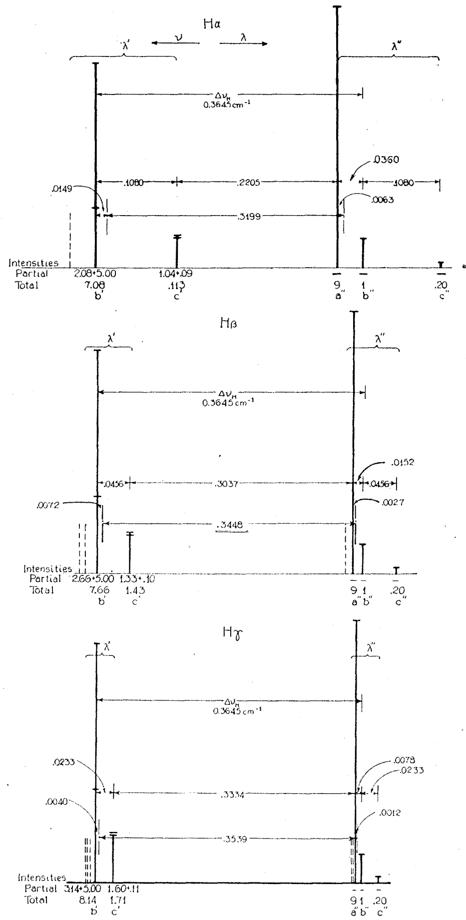


Fig. 7

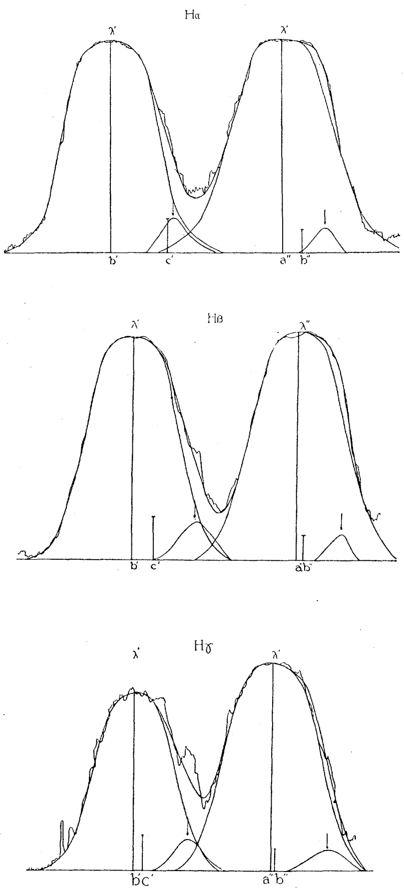


Fig. 8

Fig. 7. Theoretical structure according to the new quantum mechanics, with the spinning electron. The intensities given for Hγ are approximate.

Fig. 8. Microphotometer curves obtained from *enlargements* of the original negatives. For Hα, plate *L7*, order 12, E-W line *7D*; for Hβ, plate *L12*, order 10, E-W line *7D*; for Hγ, plate *L21*, order 3, E-W line *6F* (a dense plate). Not much emphasis should be placed upon b'' in Hγ as constructed. λ'' is nearly symmetrical.

able extension of the long wave-length end of λ'. This is visible to the unaided eye in enlargements (see Plate II) and is undoubtedly due to the fine structure component, c', of Fig. 7—clearly present although unresolved. An homologous extension exists in Hβ and Hγ.

On this theory we need not have recourse to the Stark effect to explain the presence of this component. λ' has indeed been shown to be partially polarized, but the new theory has not yet advanced sufficiently to justify any definite statement as to the cause of this polarization.

In the microphotometer study of the structure, to obtain sufficient light, enlargements of the original, negatives were first used. The negative was run in the direction *mm'* of Fig. 6. The curves of Fig. 8 were obtained as follows: Graph paper was placed over the microphotometer negative and the original curve traced in, free hand. From the upper portion of λ', the λ' axis was determined and the longer wave-length slope of the curve constructed. With λ'' the top of the curve is unsymmetrical and the axis was here determined from a region about two-thirds the way down. The longer wave-length slope of λ'' was then drawn. By subtracting ordinates the curves for c' and b'' were obtained.

It will be noticed, however, that the peaks of these three curves are all flattened at the top, that the weaker doublet component is stronger than theory demands (see Table X for theoretical ratio) and in all cases, except that of c' in Hα, the observed satellite lies much too far from the main line.

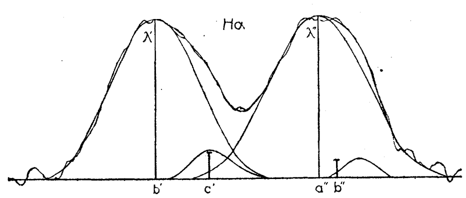


Fig. 9. Microphotometer curve from an original negative of Hα, plate *L*7, order 12, E-W line 1*D* (a rather dense image).

Over-exposure would produce just these three effects. The curves are, however, reproduced as an illustration of the different types of assymmetry in λ' and λ'' and as an indication of the probable existence of fine structure. It may be shown that at liquid air temperature the theoretical half-widths of b' and c' in Hα are about equal to their separation. Therefore in even this most favorable case resolution would be impossible at that temperature.

As it was thought that the processes of enlargement produced the enhancement, tracings were made of adjacent images on the original negative. Fig. 9, however, shows that the photographic distortion enters somewhat even here. We were thus forced to use lower and fainter orders despite the effect of plate grain.

These final curves for Hα and Hβ are shown in Fig. 10. As less photographic distortion was here present, the following more accurate method of drawing the curves could be employed: By trial the height, half-width and position of the curve for b' was determined. Using the same half-width for a'', its height and position were obtained. The horizontal scale between b' and a'' was thus found, $\Delta\nu$ being, as in Fig. 7, 0.3285 and 0.3493 for Hα and Hβ respectively. The theoretical positions of c', b'' and c'' were then determined. The theoretical intensity ratios, $b':a''$ (see Table X), could not be used because of the intensity distortion produced by the Lummer plate, but *within* each component of the doublet the theoretical relative intensities *were* assigned.

From the equation $I = I_o e^{-x^2}$ experimental curves for the five components were then plotted and the sums of the ordinates taken. The resultant curve fits the irregular microphotometer trace fairly well for both Hα and Hβ.

The theoretical half-widths for Hα and Hβ, at liquid air temperature, are 0.051 and 0.038; those used by us are 0.079 and 0.058A respectively. This indicates 175° absolute, instead of 78°, as the effective temperature in the tube, which is probably not far from the truth, as the tube was run continuously and the capillary was of rather large (1 cm) diameter. Furthermore the scale of the negative is not strictly normal as assumed in our construction, and finally the microphotometer curves do not indicate true intensities. Despite these facts, we feel justified in stating that with Hα and Hβ the resultant curves are not inconsistent with the presence of the five components given by the new quantum mechanics with the spinning electron. Of the presence of c' we are certain; we are reasonably sure of b''; and, lastly, the resultant curves fit the case better *with* c'' than without it.

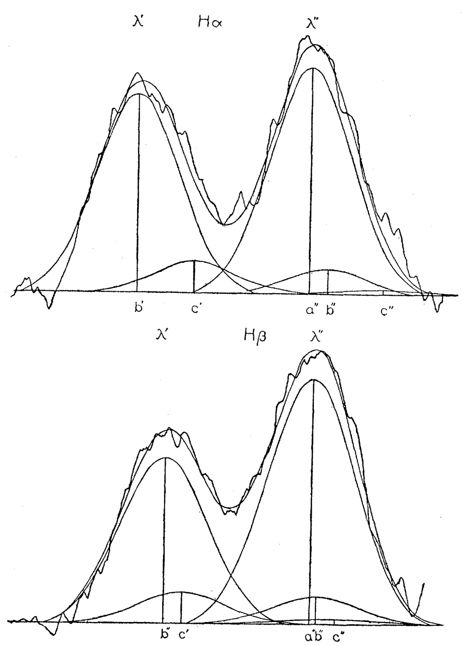


Fig. 10. Microphotometer curves from original negatives, faint images. For Hα, plate L 7 order 4, E-W line $6D$; for Hβ, plate L 12, order 4, E-W line $13D$.

To Dr. Houston of the California Institute of Technology we are indebted for the facts concerning the interferometer presented in Table I, for valuable

suggestions in connection with corrections made in evaluating our plates, for the calculation of the intensity relations given in Fig. 7 and for his cordial and able assistance, generously given. We must record also our great indebtedness to our various assistants, especially Messrs. Handy, Hartwell and Ireland.

TABLE X

Intensity ratios, $I_{\lambda''}$ $I_{\lambda'}$, *of the components of the doublets*—illustrating the effects of not only the intensity distribution of the Lummer plate but also over-exposure.

Line	Theoretical ratio	Observed Ratio: Denser images or higher orders	From figure:—	Fainter images or lower Orders	From figure:—
H_α	1.24:1	1.01:1	9	1.07:1	10
		1.01:1	10		
$H\beta$	1.12:1	1.02:1	9	1.43:1	10
$H\gamma$	1.03:1	—	—	1.02:1	8

This research was made possible by grants from the Rumford Committee of the American Academy of Arts and Sciences; this aid we most gratefully acknowledge. The work was begun at Boston University and completed at Mount Wilson Observatory and the California Institute of Technology in March 1927. To the last two institutions we are deeply indebted for the great facilities placed at our disposal.

BOSTON UNIVERSITY,
April. 1927.

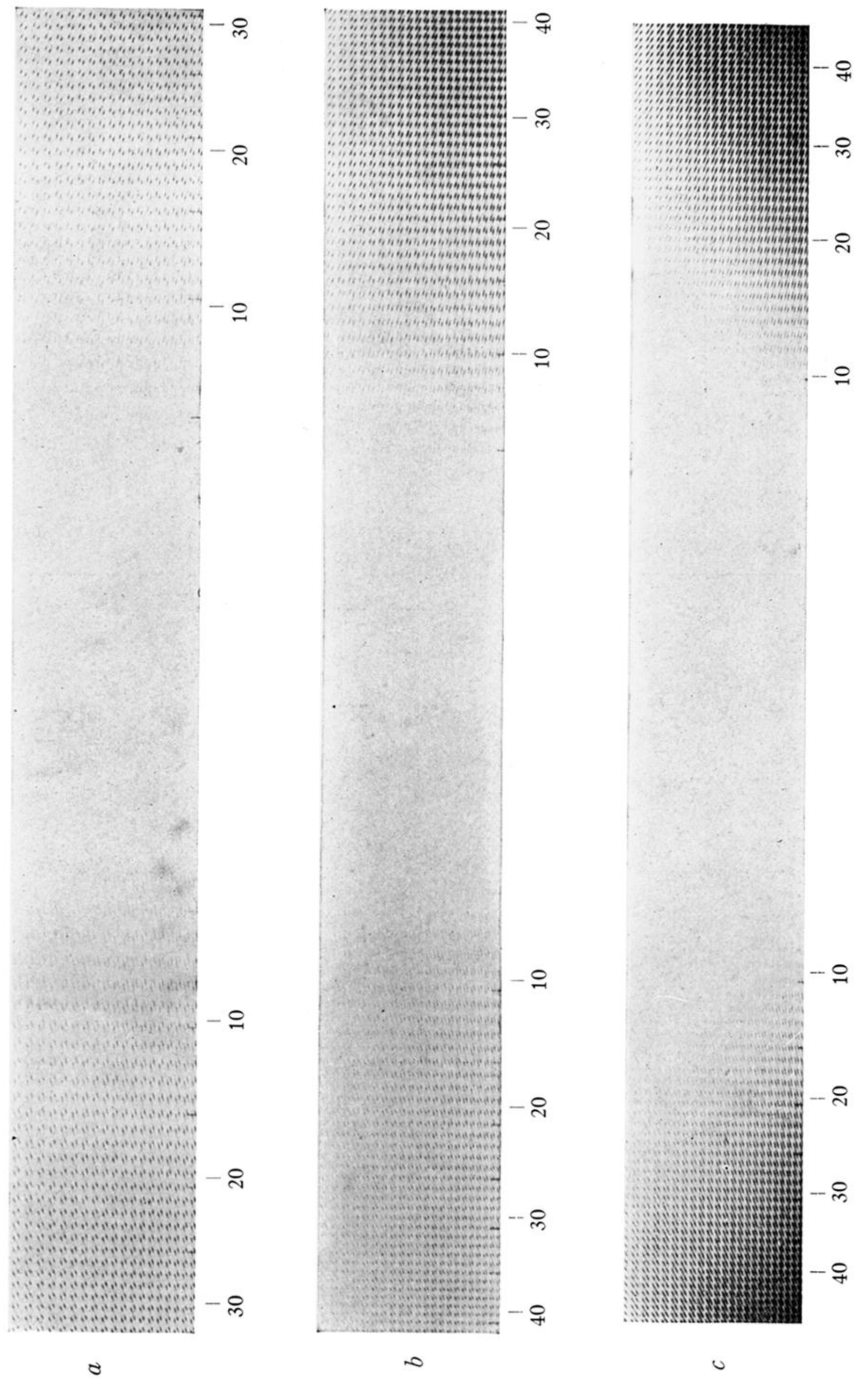


Plate I. Pattern *a* is Plate 4 for Hα; *b* is plate 10 for Hβ; *c* is plate 25 for Hγ. The magnification is 4.2 fold. See Fig. 3 and Plate II as aids in studying the pattern. The numbers refer to rows.

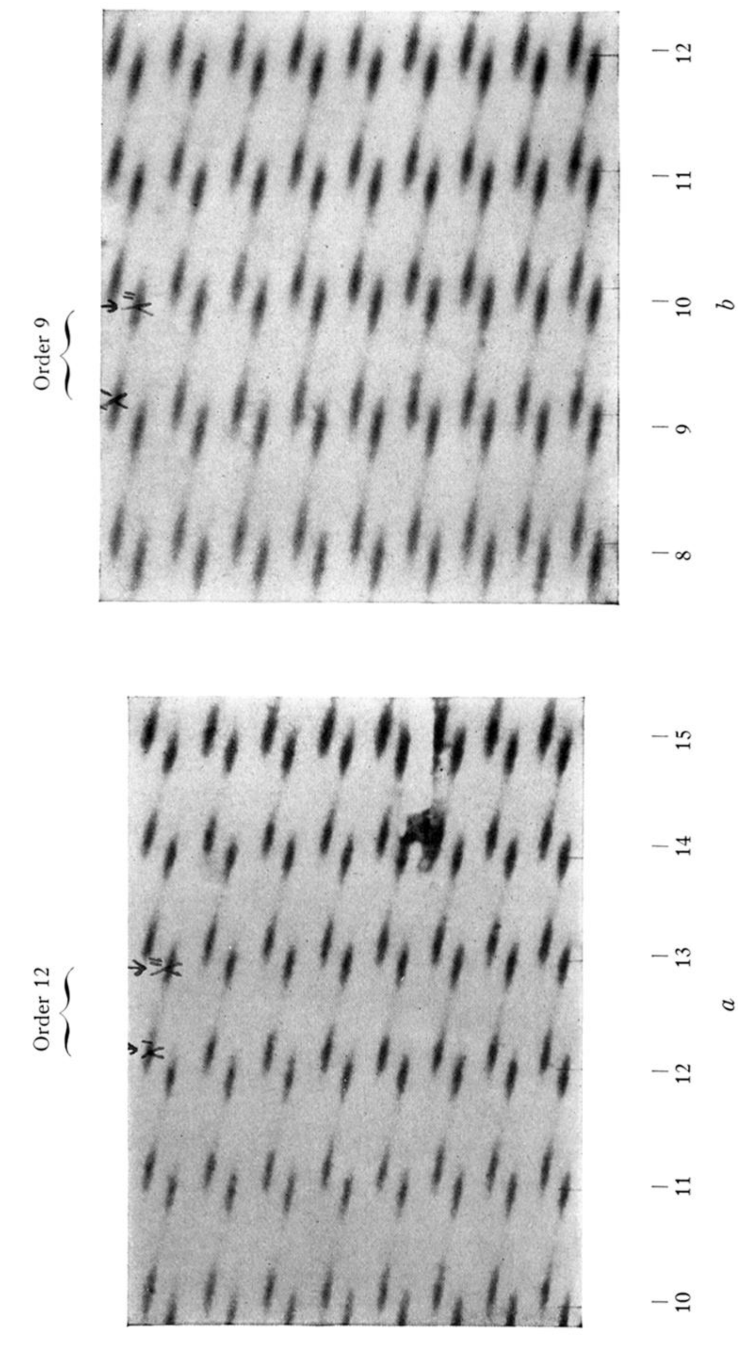


Plate II. Pattern *a* is plate *L*2 for Hα, magnification 32 fold; pattern *b* is plate *L*7 for Hα, magnification 28 fold. Observe the persistent assymetry toward the right in λ'.