THIS IS A
PLACE HOLDER
IMAGE

As shown in Fig. 1, the sphere was rotated about a vertical axis. The Rayleigh disc was removed to the inside room below to prevent the disturbance due to air currents and to decrease the absorption due to resonance at the sphere, it having been observed that both of these errors were serious. With the pipe placed as shown in Fig. 1 the intensity produced in it was strictly proportional to the potential energies at the opening. The disc in the room below was attached directly to the pipe. Fortunately, the pipe length was such that there was no tendency for resonance. This statement is verified by the results.

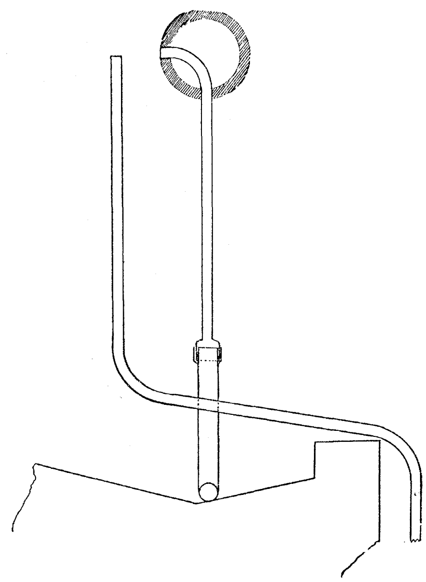

Fig. 1.

The improved apparatus demanded a more sensitive disc and this was obtained by inserting a finer quartz fiber. The period of the disc became 15 seconds.

Theory.—The mean potential energy per unit volume is given by the expression,[1]

$$\tfrac{1}{2}\rho_0(F^2 + G^2)(k/2\pi r\int\int U\,dS)^2.$$

The relative intensities are therefore given by relative values of $F^2 + G^2$. The pipe opening remained at a constant distance r from the sphere center and various values of intensities were obtained by rotating the sphere, this being equivalent to a stationary sphere with the intensities measured at various points in a circumference whose plane includes the diameter of the sphere passing through the source of sound. The relative values of intensities computed as indicated in a previous article[2] for $kr = 2$, are for 0° (the position shown in Fig. 1), 1.00; 30°, 0.560; 60°, 0.187; 90°, 0.065; 120°, 0.034; 150°, 0.033; 180°, 0.033.

Results.—The results obtained for a distance from the center of the sphere equal to twice its radius are shown in Table I. This gives a record of four series of observations. The deviations from the mean can be accounted for by the smallness of the deflection, about 2 cm., and the error due to the setting of the sphere at 30° where the intensity changes

[1] For derivation and meaning of symbols see Stewart, loc. cit.

[2] For numerical values see Stewart and Stiles, loc. cit.

rapidly. In order to compare these results with the theoretical values and to show the great improvement over the earlier experimental arrangement, both the present and former results are plotted in Fig. 2, and the theoretical values are shown by the full line curve. The earlier results are represented by crosses and the later ones by small circles. The

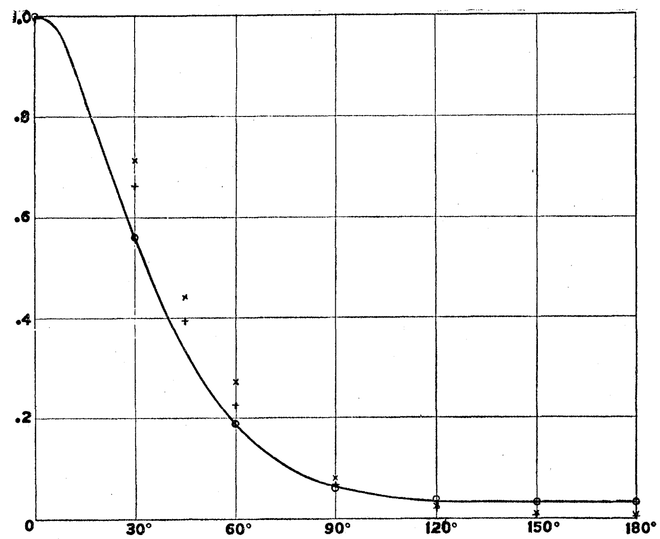

Fig. 2.

TABLE I.

	0°	30°	60°	90°	120°	150°	180°
	1.00	0.580	0.230	0.067	0.035	0.025	0.025
	1.00	0.625	0.175	0.050	0.035	0.035	0.035
	1.00	0.555	0.170	0.050	0.040	0.035	0.030
	1.00	0.475	0.185	0.075	0.045	0.040	0.035
Mean........	1.00	0.558	0.190	0.060	0.039	0.034	0.031

agreement between theory and experiment is better than could be expected. Our belief is that the errors in the earlier results were due to air currents and to the absorption caused by resonance. The distortion produced by the latter would have the effect of "ironing out" the curve, and the former observations show that that is the case.

The verification of the theory is very satisfactory. This increases our confidence not only in the theory, but also in the Rayleigh disc. So far as we are aware, there has been no study which has shown that the deflections of the disc are proportional to the potential energy at the opening of the disc tube. It is true that the correctness of the theory of the disc has been experimentally[1] verified so far as the relation between the kinetic energy of the air at the disc and the deflection of the disc are concerned. The writers attempted to show the proportionality of the disc deflection and the potential energy at the disc tube opening by observations of the amplitude of the fork and the deflection of the disc when both were placed in the same room. The dimensions of the room were about 25, 7, and 4 meters. The temperature as measured by wall thermometers was constant to within 1° C. It was found that, if comparisons were made quickly over the range of deflection of the disc, there was a constant proportionality between the square of the fork's amplitude and the deflection of the disc. But the apparent sensibility of the disc measured in this manner changed with time. In other words, the ratio of deflection to the square of the amplitude changed from moment to moment, being quite appreciable in a few minutes. The explanation seemed to be that the maxima and minima intensities shifted their locations with changes in temperature. The changes, however, seemed greater than could be produced by standing waves caused by only one reflection. This might have been anticipated. These tests satisfied our minds of the correctness of the assumption of proportionality of deflection and intensity at the tube opening.

Passage of Sound through Narrow Slits.

Rayleigh has investigated the passage of sound through narrow slits both theoretically[2] and experimentally.[3] The result of the former can be briefly presented. Consider an aperture in a thin plane screen of infinite extent. Let a plane wave be incident from the left. Its velocity potential, omitting the harmonic time factor and considering the modulus unity, is $\varphi = e^{-\iota\kappa x}$. Consider the conditions without aperture and then the supplementary values of the velocity potential representing the changes produced by the aperture. By addition, the velocity potential on each side of the screen is obtained. The resulting velocity potential at a great distance, r, on the right side is,

$$\varphi = e^{-\iota\kappa r}\frac{M}{r}$$

or

$$\varphi = \frac{M}{r}\cos^{\kappa(at-r)},$$

where k is $2\pi \div$ *wave-length* and where M is the "capacity." The term

[1] Zernov, Annal. d. Phys., No. 26, p. 79, 1908.

[2] Rayleigh, Phil. Mag., XLIII., p. 259 (1897); Scien. Paper IV., p. 291.

[3] Rayleigh, Phil. Mag., XIV., p. 153 (1907).

"capacity" is used because M is the total quantity of electricity which can be distributed over the aperture in a manner to produce a uniform potential of unity over the aperture. It is known that for an ellipse,

$$M = \frac{a}{F(e)},$$

where a is the major axis, and small e the eccentricity, and F is the symbol of the complete elliptic function of the first kind. If the ellipse be very elongated,

$$M = \frac{a}{\log_e\left(\frac{4a}{b}\right)}.$$

Inasmuch as the only variable in the expression for the velocity potential at a fixed great distance, r, is M, the same values of M would determine the same sound intensities. The above formula shows that the intensity is much more sensitive to alterations in a than in b, the minor axis, and Rayleigh attempted to verify this by experiment. The difficulties he encountered were great, and in addition he depended upon ear memory for the reproduction of identical intensities. The arrangement of our Rayleigh disc and the possibility of experimentation practically free from reflection, tempted us to test the above formula.

For the infinite plane a rectangular piece of galvanized iron 33 × 38 cm. was utilized, the dimensions and shape being accidental. This plate was fastened on the horizontal end of an elbow placed upon the end of the pipe shown in Fig. 1. In the center of the vertical plate an opening 0.8 × 4.0 cm. was made, and over the opening was constructed the slit. Four small safety razor blades, 1.8 × 4 cm., beveled on both sides, were used. Three were placed in the same plane and to the fourth was attached an edge made out of a copper strip, whose thickness was the desired width of the slit. Changes in the slit width required different copper strips. Changes in the length could be readily obtained by sliding the fourth blade with its attached copper strip. Vaseline was freely used to stop all openings and was found entirely satisfactory.

The source of sound was the electrically operated tuning fork with the open end of the resonator mounted directly in front of the slit and about 150 cm. distant.

Our first experiments were made with sharpened brass edges but these were found unsatisfactory when the slit became narrow. Experiments with these edges with a width of 1 mm. (b = 0.5 mm.), showed that the variation of intensity with length was practically linear. This fact simplified our experiments with the razor blades. We obtained a deflection with a 17.5 mm. and b 0.1 mm., then a deflection with a 17.5 mm. and b 0.5 mm. and finally a deflection with a 10.0 mm., and b 0.5 mm. The ratio of intensities of the first and second slits were found to be approximately 0.52, the separate values being 0.51, 0.49, 0.55. The ratio of intensities of the third and second was approximately 0.49, the separate values being 0.47, 0.50, and 0.49. By assuming that the linear relation above cited holds, it is readily seen that the value of a giving the ratio of 0.52 with b 0.5 mm., would be 10.5 mm. We would then have the same value of M for the two slits, 35. × 0.2 mm. and 22. × 1.0 mm. Substituting these experimental values, we have

$$M = \frac{a}{\log_e\left(\frac{4a}{b}\right)} = \frac{17.5}{\log_e\left(4\frac{17.5}{0.1}\right)} = 2.66$$

$$= \frac{10.5}{\log_e\left(4\frac{10.5}{0.5}\right)} = 2.37.$$

Thus the formula for M is verified as nearly as could be expected with the lack of conformity with theoretical conditions. For it is to be noted that the presence of the pipe behind the slit destroyed the plane of infinite area, and further, that there is resonance in the pipe leading to the disc. This resonance would of course be modified by the size of the aperture. Other experiments cited below would indicate that the error due to resonance is of the same order as the variation between the two values of the M given above.

Passage Through Circular Apertures.

Tests were made by varying the area and by varying the number of circular apertures of equal area. The value of M for a circle is $2a \div \pi$ where a is the radius. If the velocity potential at a great distance is proportional to M, then the intensity at a given distance, proportional to $\dot{\varphi}^2$, is proportional to M^2, or to a^2. An experimental test of this relation did not prove satisfactory, for as M changed the resonance in the pipe was altered and the conditions of the experiment were thus seriously modified. The values actually obtained are shown in the accompanying table, Table II. The deflections were reduced to the same scale, the aperture F giving a deflection of unity. The ratio in the last column is the relation of experimental and theoretical values of M. The ratio varies not more than 10 per cent. from the mean for a range of radii from 0.42 cm. to 1.28 cm. These variations, as well as those for the smaller apertures are accounted for by variations in resonance. The indications are, however, that if resonance could be avoided a close agreement between experiment and theory would be found.

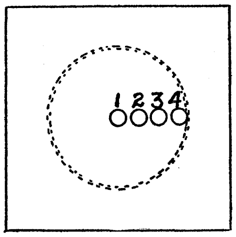

Fig. 3.

The value of M for N circles is $N \times M$, and the corresponding intensity is proportional to N^2M^2. If the M's are not alike, the intensity is proportional to the square of their sum. A test was made as follows. Four circular apertures, each 0.30 cm. in radius were made in a screen, see Fig. 3, and intensities obtained by opening them separately and simultaneously as indicated in the accompanying Table III. The agreement between theory and experiment is indicated by the ratio in the last column. It should be observed that the variation is not great and can readily be

TABLE II.

Screen.	Deflection.	$\sqrt{\text{Defl.}} \propto M$.	$2a$.	Ratio.
L	5.45	2.34	2.56	0.91
K	3.67	1.91	1.88	1.02
J	2.00	1.41	1.27	1.11
F	1.00	1.00	1.00	1.00
D	0.56	0.75	0.83	0.90
C	0.20	0.45	0.60	0.75
B	0.06	0.25	0.40	0.62

TABLE III.

Apertures Used.	Relative Observed Values of Intensity.	Relative Values of M $\sqrt{\text{Intensity}} \propto M$.	Relative Computed Values of M $M = M_1 + M_2 + M_3 + M_4$.	Ratio.
1	1.00	1.00		
2	1.00	1.00		
3	0.80	0.90		
4	0.58	0.76		
1 + 2	3.73	1.93	2.00	0.96
1 + 2 + 3	6.23	2.50	2.90	0.86
1 + 2 + 3 + 4	9.66	3.10	3.66	0.83

interpreted as due to change of resonance in the pipe. The variations of M for the separate apertures is doubtless due to the proximity of the apertures to the pipe. The arrangement is faulty also in that the apertures are close together. The lack of agreement between the experimental and theoretical results is explained by the lack of conformity to theoretical conditions and the variation in resonance in the pipe. The latter seems to produce the greater error. Indeed, all the experiments with apertures indicate that the errors due to changes in resonance are of the same order as the differences between experimental and theoretical results.

The results with both slits and apertures show the difficulty of securing satisfactory experimental conditions. Experiments of this character should be continued with improvements in apparatus that will avoid the errors due to resonance.

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