TERMS ARISING FROM SIMILAR AND DISSIMILAR ELECTRONS

BY R. C. GIBBS, D. T. WILBER AND H. E. WHITE

ABSTRACT

Following the scheme of Hund[1] for similar s, p and d electrons the terms arising from similar f electrons have been worked out and tabulated. Tables have also been prepared for one and two electrons, where in the latter case these electrons are dissimilar i.e. have either different total or different azimuthal quantum numbers, and also for three electrons two of which are similar. These tables along with those for similar s, p and d electrons are found not only to be of frequent use but also to bring out certain rules that may be applied in determining spectral terms arising from any electron configuration.

MODERN spectroscopy depends to such a large extent upon the theoretical considerations of space quantization of the electrons in uncompleted shells of the atom that it seemed desirable to tabulate in compact form the terms arising from some of the more frequent electron configurations. Following the arrangement of tables given by Hund[1] for similar s, p and d electrons it has been possible to work out and tabulate the terms arising from one to fourteen f electrons. According to ideas put forward by Landé,[2] Pauli,[3] and others the terms arising from any electron configuration are obtained from all possible combinations of the magnetic quantum numbers m_a and m_s, but Pauli[3] has shown that for similar electrons certain special configurations must be excluded, i.e. two electrons cannot occupy the same orbit at the same time.

TABLE I

Similar s electrons.

(2)	$s-{}^2S$	
(1)	$s^2-{}^1S$	

Using the notation as proposed by Russell and Saunders[4] and now being widely used, tables have been formulated for some of the more frequently occuring configurations. An electron is denoted by a small letter while

TABLE II

Similar p electrons.

(6)	p^1-	${}^2P$				
(15)	$p^2-{}^1S$		${}^1D$		${}^3P'$	
(20)	p^3-	${}^2P$		${}^2D'$		${}^4S'$
(15)	$p^4-{}^1S$		${}^1D$		${}^3P'$	
(6)	p^5-	${}^2P$				
(1)	$p^6-{}^1S$					

[1] Hund, Zeits. f. Physik **33**, 345 (1925); **34**, 353 (1925).
[2] Landé, Phys. Zeits. **22**, 417 (1921); Zeits. f. Physik **17**, 292 (1923).
[3] Pauli, Zeits. f. Physik **16**, 161 (1923); **31**, 765 (1925).
[4] H. N. Russell and F. A. Saunders, Astrophys. Jour. **61**, 40 (1925).

capital letters are always used for terms. A dot before any small letter signifies that the electron has a different total quantum number than the one preceding it.

TABLE III

Similar d electrons.

(10)	d^1 —	$^2(D)$					
(45)	$d^2-{}^1(SDG)$	$^3(P'F')$					
(120)	d^3 —	$^2(D)$	$^2(P'DF'GH')$	$^4(P'F')$			
(210)	$d^4-{}^1(SDG)$	$^3(P'F')$	$^1(SDF'GI)$	$^3(P'DF'GH')$	$^5(D)$		
(252)	d^5 —	$^2(D)$	$^2(P'DF'GH')$	$^4(P'F')$ $^2(SDF'GI)$		$^4(DG)$	$^6(S)$
(210)	$d^6-{}^1(SDG)$	$^3(P'F')$	$^1(SDF'GI)$	$^3(P'DF'GH')$	$^5(D)$		
(120)	d^7 —	$^2(D)$	$^2(P'DF'GH')$	$^4(P'F')$			
(45)	$d^8-{}^1(SDG)$	$^3(P'F')$					
(10)	d^9 —	$^2(D)$					
(1)	$d^{10}-{}^1(S)$						

The tables given here for similar electrons have all been checked by the combination formula for p things taken q at a time where p is the number of

TABLE IV

Similar f electrons.

(14)	f^1——$^2(F)$					
(91)	f^2—$^1(SDGI)$	$^3(P'F'H')$				
(364)	f^3——$^2(PD'FG'HI'KL')$	$^4(S'D'FG'I')$				
	2 2 2 2					
(1001)	f^4—$^1(SDF'GH'IK'LN)$	$^3(P'DF'GH'IK'LM')$	$^5(SDF'GI)$			
	2 4 4 2 3 2	3 2 4 3 4 2 2				
(2002)	f^5——$^2(PD'FG'HI'KL'MN'O)$	$^4(S'PD'FG'HI'KL'M)$	$^6(PFH)$			
	4 5 7 6 7 5 5 3 2	2 3 4 4 3 3 2				
(3003)	f^6—$^1(SP'DF'GH'IK'LM'NQ)$	$^3(P'DF'GH'IK'LM'NO')$	$^5(SP'DF'GH'IK'L)$	$^7(F')$		
	4 6 4 8 4 7 3 4 2 2	6 5 9 7 9 6 6 3 3	3 2 3 2 2			
(3432)	f^7——$^2(S'PD'FG'HI'KL'MN'OQ')$	$^4(S'PD'FG'HI'KL'MN')$	$^6(PD'FG'HI')$		$^8(S')$	
	2 5 7 10 10 9 9 7 5 4 2	2 2 6 5 7 5 5 3 3				
(3003)	f^8—$^1(SP'DF'GH'IK'LM'NQ)$	$^3(P'DF'GH'IK'LM'NO')$	$^5(SP'DF'GH'IK'L)$	$^7(F')$		
	4 6 4 8 4 7 3 4 2 2	6 5 9 7 9 6 6 3 3	3 2 3 2 2			
(2002)	f^9——$^2(PD'FG'HI'KL'MN'O)$	$^4(S'PD'FG'HI'KL'M)$	$^6(PFH)$.			
	4 5 7 6 7 5 5 3 2	2 3 4 4 3 3 2				
(1001)	f^{10}—$^1(SDF'GH'IK'LN)$	$^3(P'DF'GH'IK'LM')$	$^5(SDF'GI)$			
	2 4 4 2 3 2	3 2 4 3 4 2 2				
(364)	f^{11}——$^2(PD'FG'HI'KL')$	$^4(S'D'FG'I')$				
	2 2 2 2					
(91)	f^{12}—$^1(SDGI)$	$^3(P'F'H')$				
(14)	f^{13}——$^2(F)$					
(1)	f^{14}—$^1(S)$					

states a given electron may occupy and q the total number of electrons in consideration. This check on the terms shows that in the table given by

TABLE V

One electron systems	Two electron systems	
$s\to{}^2S$ ——	$s.s\to{}^1S$	^3S
$p\to{}^2P$ ——	$p\,s\to{}^1P$	^3P
	$p.p\to{}^1(SP'D)$	$^3(SP'D)$
$d\to{}^2D$ ——	$d\,s\to{}^1(D)$	$^3(D)$
	$d\,p\to{}^1(P\,D'\,F)$	$^3(P\,D'F)$
	$d.d\to{}^1(SP'D\;F'G)$	$^3(SP'D\;F'G)$
$f\to{}^2F$ ——	$f\,s\to{}^1(F)$	$^3(F)$
	$f\,p\to{}^1(D\;F'G)$	$^3(D\;F'G)$
	$f\,d\to{}^1(P\;D'F\,G'H)$	$^3(P\;D'F\,G'H)$
	$f.f\to{}^1(SP'D\;F'G\;H'I)$	$^3(SP'D\;F'G\;H'I)$

TABLE VI

Three electrons. (Two similar).

$s^2.s-{}^2(S)$									
$s^2p-{}^2(P)$									
$s^2d-{}^2(D)$									
$s^2f-{}^2(F)$									
$p^2s-{}^2(S)$	$^2(D)$			$^2(P')$			$^4(P')$		
$p^2.p-{}^2(P)$	$^2(PD'F)$			$^2(S'PD')$			$^4(S'PD')$		
$p^2d-{}^2(D)$	$^2(SP'DF'G)$			$^2(P'DF')$			$^4(P'DF')$		
$p^2f-{}^2(F)$	$^2(PD'FG'H)$			$^2(D'FG')$			$^4(D'FG')$		
$d^2s-{}^2(S)$	$^2(D)$	$^2(G)$		$^2(P')$	$^2(F')$		$^4(P')$	$^4(F')$	
$d^2p-{}^2(P)$	$^2(PD'F)$	$^2(FG'H)$		$^2(S'PD')$	$^2(D'FG')$		$^4(S'PD')$	$^4(D'FG')$	
$d^2.d-{}^2(D)$	$^2(SP'DF'G)$	$^2(DF'GH'I)$		$^2(P'DF')$	$^2(P'DF'GH')$		$^4(P'DF')$	$^4(P'DF'GH')$	
$d^2f-{}^2(F)$	$^2(PD'FG'H)$	$^2(PD'FG'HI'K)$		$^2(D'FG')$	$^2(S'PD'FG'HI')$		$^4(D'FG')$	$^4(S'PD'FG'HI')$	
$f^2s-{}^2(S)$	$^2(D)$	$^2(G)$	$^2(I)$	$^2(P')$	$^2(F')$	$^2(H')$	$^4(P')$	$^4(F')$	$^4(H')$
$f^2p-{}^2(P)$	$^2(PD'F)$	$^2(FG'H)$	$^2(HI'K)$	$^2(S'PD')$	$^2(D'FG')$	$^2(G'HI')$	$^4(S'PD')$	$^4(D'FG')$	$^4(G'HI')$
$f^2d-{}^2(D)$	$^2(SP'DF'G)$	$^2(DF'GH'I)$	$^2(GH'IK'L)$	$^2(P'DF')$	$^2(P'DF'GH')$	$^2(F'GH'IK')$	$^4(P'DF')$	$^4(P'DF'GH')$	$^4(F'GH'IK')$
$f^2.f-{}^2(F)$	$^2(PD'FG'H)$	$^2(PD'FG'HI'K)$	$^2(FG'HI'KL'M)$	$^2(D'FG')$	$^2(S'PD'FG'HI')$	$^2(D'FG'HI'KL')$	$^4(D'FG')$	$^4(S'PD'FG'HI')$	$^4(D'FG'HI'KL')$

Hund[1] for similar "d" electrons two terms are missing, namely a ^3G term arising from d^4 electrons and a ^2S term from d^5 electrons. In the tables for similar electrons we have indicated, in parentheses, before each electron configuration the total number of possible combinations of p(= number of possible values of $m_a \times$ number of possible values of m_s) things taken q(=total number of electrons in consideration) at a time. The terms for similar f electrons from one to fourteen have been worked out and checked in this same way. These have been arranged as given in Table IV. We originally arranged the terms from f electrons according to the scheme used by Hund[1] for similar p and d electrons and found a high degree of symmetry throughout the table, but owing to the length of such a table a more compact form has been given here. The multiplicity of all terms included in the parentheses is indicated by the superscript in front of the parenthesis. Where several identical terms appear, for any given electron configuration, we have indicated the total number of such terms by small arabic numerals placed directly below the term, for example $^4\underset{3}{D}$ means that there are three ^4D terms. Where there is but one such term to be represented this numeral has been omitted.

From the terms arising from dissimilar electrons, Table V, the simple rules for determining all the terms arising from the addition of another electron to any one given term may be inferred. From these rules the terms corresponding to any electron configuration can be elaborated. This we have done in Table VI for the case of three electrons, two similar and one different. The electron configurations given in the above tables include nearly all of the cases that are of primary importance. It may here be pointed out that there is a striking similarity between the terms as given in Table VI and the table of inner quantum numbers as given by Russell and Saunders.[4]

If our present idea is correct for the rare earth group of elements a *d* electron must be added to the configurations in Table IV to obtain the ground terms of the arc spectra of the corresponding rare earths since in lanthanum a $5d$ valence electron has been added. Furthermore when one of the rare earth atoms is excited the most probable electron to be excited will be an *s* electron from the already completed $6s$ shell. This means that to the terms in Table IV we must add not only a *d* electron but also two dissimilar *s* electrons. It may be seen that the rare earth spectra are likely to be extremely rich in raes-ultimes.

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Note added with proof, April 26, 1927. After this paper was submitted for publication we learned from Dr. H. N. Russell that he had independently worked out the terms arising from similar *f* electrons and that his results check with ours exactly. By mutual arrangement he has given in the preceding paper a detailed explanation of the principles and rules underlying the formation of these tables. The simple rule used by us to distinguish between primed and unprimed terms is given by him on page 785. The ^3G and ^2S terms which we have referred to above as missing in Hund's original paper are included in his book, "Linien Spectren und Periodisches System der Elemente" a copy of which we have just seen.