THE ABSORPTION OF RADIO WAVES IN THE UPPER ATMOSPHERE*

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ABSTRACT

Recent measurements have shown that radio waves below 150 meters fall off in intensity faster than required by an inverse square law for distances up to 1000 miles. This points to absorption of the wave by the medium, in this case the upper atmosphere. The absorption of the waves variously polarized is calculated on the assumption that it results from collisions between the electrons and molecules of the atmosphere. With reasonable average values of the electronic and molecular densities the amplitude A of the wave λ cms at a distance x cms is $A=\alpha x^{-1}\, exp\,(-11.8\times10^{-16}\lambda^2 x)$, theoretically valid for waves from 16 to 160 meters to distances of 1000 miles. This agrees well enough with the scant range and intensity data, and it is pointed out that an extension of these data may lead to more exact knowledge of the overhead electronic and molecular pressures. From the absorption curves interesting possibilities appear of polarization of waves in the broadcast band 200–600 meters.

IN RECENT papers[1,2] a quantitative theory of the manner in which radio waves pass over the earth has been developed. The waves are shown to reach distant points on the surface of the earth by passage through the outlying regions of the earth's atmosphere being refracted downward by the electrons of those regions. From the simple fact that radio waves, particularly of wave-length below 90 meters, are transmitted successfully with relatively small amounts of power to distances as great as half-way around the earth, it was assumed that the attenuation of the waves in the upper atmospheric strata was slight. As a matter of fact absorption of energy from the wave by the atmosphere was put aside entirely, but the calculations were made in such a way as to be undisturbed by a small absorption which of course exists. In the present paper the influence of absorption is considered with the result that certain facts about the ranges of the waves of the radio spectrum begin to be more clearly understood.

The optical properties of the upper reaches of the atmosphere are assumed to depend upon the molecules, ions and electrons which exist there. The magnetic field of the earth has an important influence which is recognized in the formulas. The electrons contribute largely to the dispersion of the electromagnetic waves, the molecules and ions only a secondary part in so far as they interfere with the electrons. All, however, contribute to the absorption, for we shall assume that the most important cause of the absorption of energy from the wave arises from collisions between the electrons and molecules. The effect of collision is to transfer a portion of the energy which the electron has received from the waves to the molecule and produce the

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[1] Taylor and Hulburt, Phys. Rev. **27**, 189 (1926).

[2] Hulburt, Journal of the Frank. Inst. **201**, 597 (1926).

disorderly motion called heat. The ions may justifiably be treated as molecules, for the effect of their charges on the dispersion and absorption is negligible (see reference 2, page 610). With these assumptions the general formulas for the dispersion and absorption are all available in treatises on magneto-optics, for example Lorentz[3], and therefore for the special formulas developed here we need only record a few of the more essential steps. If an electron of mass m experiences f collisions per second with the molecules, it has been shown[4] that this may be expressed as a frictional force gv on the electron, where v is the velocity of the electron and g is given by

$$g = 2mf. \tag{1}$$

In this formula the collisions are regarded as inelastic; this is a true absorption of energy. Other types of collisions may occur, such as elastic ones which cause scattering of the energy, etc. A more extended treatment may therefore be expected to modify (1). The change will probably not be great and for the present, at any rate, we shall be content to use (1) as it stands.

In writing down the equations of motion of the electron we assume no restoring force due to the medium and no effect of the electrons of each other. Let E_x and ξ be the X components of the electric force and the displacement of the electron, respectively, and η, ζ, E_y and E_z the Y and Z components of the quantities. N is the number of electrons per unit volume, e and m are the electronic charge and mass. The earth's magnetic field H is in the direction of the axis of Z. In c.g.s. electromagnetic units the equations of motion of the electron are

$$\left.\begin{aligned} m\ddot{\xi} &= eE_x - g\dot{\xi} + He\dot{\eta}, \\ m\ddot{\eta} &= eE_y - g\dot{\eta} - He\dot{\xi}, \\ m\ddot{\zeta} &= eE_z - g\dot{\zeta} \quad . \end{aligned}\right\} \tag{2}$$

The exact solutions of (2) are extremely cumbersome. The solutions become much simpler, however, if (following Lorentz) the approximation is adopted that the absorption is small in the space of a wave-length; fortunately this approximation is entirely acceptable in the case of the radio waves.

For incident plane waves advancing in the direction of H the approximate solution of (2) yields two circularly polarized components of refractive indices μ and absorption coefficients κ given by

$$\mu^2 = 1 - \frac{C\lambda^2(1-\lambda/\lambda_0)}{(1-\lambda/\lambda_0)^2 + G^2\lambda^2}, \tag{3}$$

$$\kappa = \pi CG/\left[(1/\lambda - 1/\lambda_0)^2 + G^2\right], \tag{4}$$

[3] H. A. Lorentz, "The Theory of Electrons," Chap. IV (1916).

[4] Lorentz, Loc. cit., p. 309 or more recently, Houston, Phil. Mag. **2**, 512 (1926).

and

$$\mu^2 = 1 - \frac{C\lambda^2(1+\lambda/\lambda_0)}{(1+\lambda/\lambda_0)^2 + G^2\lambda^2}, \tag{5}$$

$$\kappa = \pi CG/[(1/\lambda + 1/\lambda_0)^2 + G^2], \tag{6}$$

where

$$C = Ne^2/\pi m, \quad \lambda_0 = 2\pi cm/He, \quad G = g/2\pi cm = f/\pi c \ \text{[From (1)]}, \tag{7}$$

c being the velocity of light in vacuum. With the value 0.5 gauss for H, λ_0 from (7) comes out to be 214 meters. The absorption coefficient κ is defined by the relation

$$A = A_0 \epsilon^{-\kappa x} \tag{8}$$

where A_0 is the initial amplitude of the wave and A the amplitude after traversing x cms of the medium.

For incident plane waves advancing normally to the magnetic field the solution of the equations of motion yields two plane polarized components, respectively parallel and perpendicular to H, of refractive indices μ and absorption coefficients κ given by

$$\mu^2 = 1 - \frac{C\lambda^2}{1 + G^2\lambda^2}, \tag{9}$$

$$\kappa = \pi CG/(1/\lambda^2 + G^2), \tag{10}$$

and

$$\mu^2 = \frac{(A^2+B^2)^{1/2} + A}{2Q}, \tag{11}$$

$$\kappa^2 = \frac{2\pi^2}{\lambda^2} \frac{(A^2+B^2)^{1/2} - A}{Q}, \tag{12}$$

where

$$A = pq + rs, \quad B = ps - rq, \quad Q = q^2 + s^2; \quad p = (1+\alpha)^2 - \gamma^2 - \beta^2,$$
$$r = 2\beta(1+\alpha), \quad q = \alpha(1+\alpha) - \gamma^2 - \beta^2, \quad s = \beta(1+2\alpha);$$
$$\alpha = -1/C\lambda^2, \quad \beta = CG/\lambda, \quad \gamma = 1/C\lambda\lambda_0.$$

Expression (10) is an approximation based on small absorption, just as was the case in (3) and (4); it refers to the component with electric vector along H, and therefore to propagation in the absence of the magnetic field. Expression (12), however, which refers to the electric vector normal to H, is exact; the approximation of small absorption did not lead to much simplification. For no absorption, $g = G = \kappa = 0$, and the four formulas for μ, (3), (5), (9) and (11), reduce respectively to the formulas (2), (3), (5) and (6) of the earlier paper.[1]

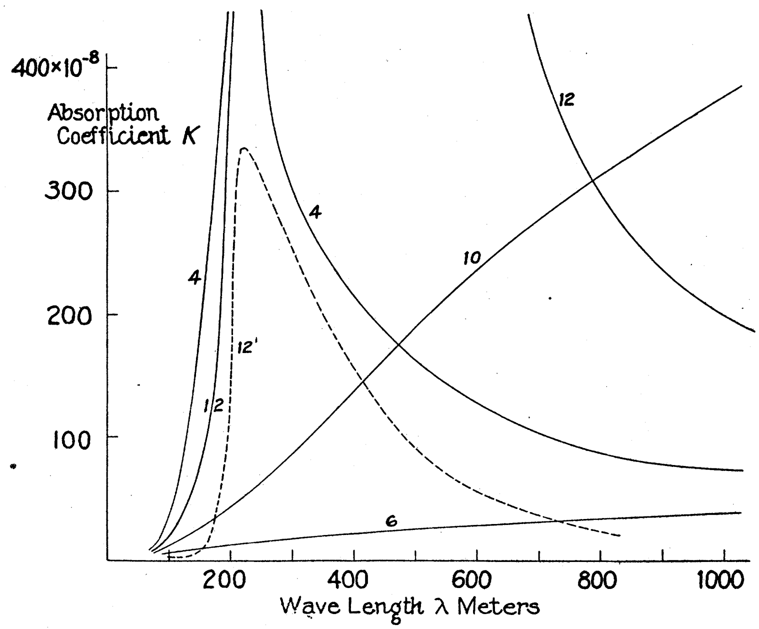

Fig. 1. Absorption coefficients of the upper atmosphere as a function of the wave-length for rays in and perpendicular to the earth's magnetic field, from equations (4), (6), (10) and (12). Curve 12′ is curve 12 with ordinates reduced ten times.

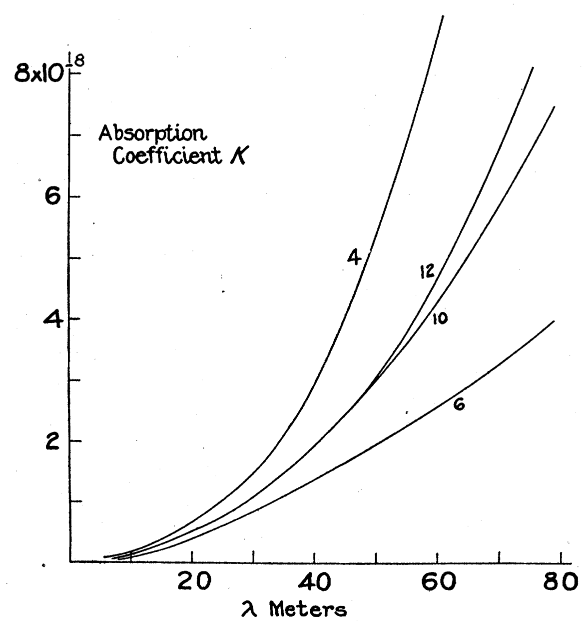

Fig. 2. The short wave portion of Figure 1 on a larger scale.

To bring out the general character of the absorption, κ from formulas (4), (6), (10) and (12) is plotted in curves 4, 6, 10, and 12, Fig. 1, respectively with $\lambda_0=214$ meters, $C=2.5\times10^{-11}$ or $N=280$ electrons per cc, and with $G=1.5\times10^{-5}$. Curve 12 rises to such high values as to be off the page, so that it has been drawn again in curve 12′, Fig 1, on the same abscissas, but with ordinates ten times reduced Since we shall be interested in the curves in the region of shorter waves below 100 meters, the respective formulas for κ have been plotted on a larger scale in the curves 4, 6, 10, and 12 of Fig. 2.

Experimental Data

A program of measurements of the electric field A of the received wave at various distances from the transmitter for the shorter waves, below 200 meters, has recently been entered upon by Heising, Schelleng and Southworth.[5] We choose a portion of their data for wave-lengths 44 and 66 meters under full daylight conditions; these are plotted respectively in curves 1 and 2, Fig. 3, in which the ordinates are the field strengths A in arbitrary

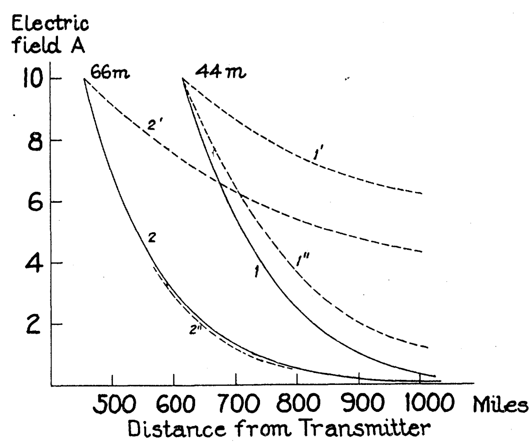

Fig. 3. The electric field as a function of the distance from the transmitter for a 44 and a 66 meter wave; curves 1 and 2 observed, curves 1′ and 2′ calculated by the inverse distance law, curves 1″ and 2″ calculated from (15).

units and the abscissas are the distances from the transmitter in miles. The curves do not claim great accuracy, their smoothness in the drawing really gives a wrong impression of the precision. They represent averages of field strengths which fluctuated over wide limits from day to day. Qualitative observations of the strengths of signals with waves below 100 meters have been recorded by Prescott,[6] these are, however, unsuited to quantitative calculations. We shall have use for Taylor's[7] ranges of transmission. These ranges in miles, under full daylight, averaged throughout a year for uniform

[5] Heising, Schelleng and Southworth, Proc. Inst. Rad. Eng. **14,** 613 (1926).
[6] Prescott, "QST" **10,** 9 (1926).
[7] Taylor, Proc. Inst. Rad. Eng. **13,** 677 (1925).

transmitting conditions, i.e. five kilowatts in a normal transmitting antenna, are plotted in curve 1, Fig. 4 as ordinates against the wave-lengths as abscis-

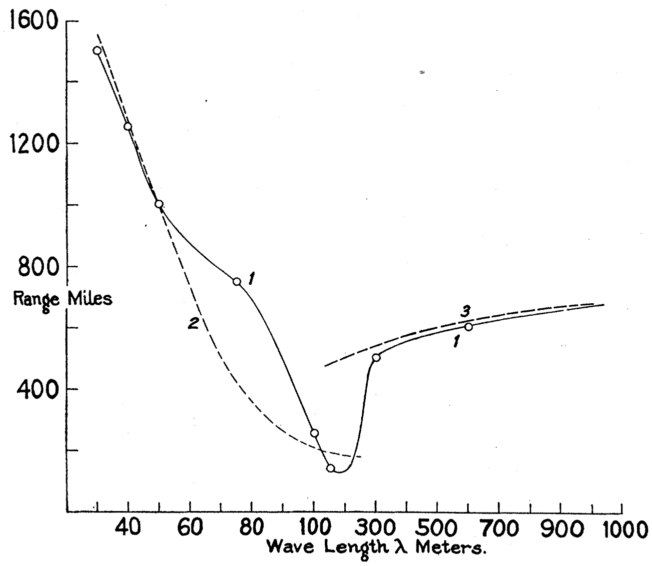

Fig. 4. The average daylight ranges, for uniform transmitting conditions, as a function of the wave-length; curve 1 observed, curves 2 and 3 calculated from (15) and (14) respectively.

sas. For the sake of clearness the scale of the abscissas for waves below 100 meters is ten times that for waves above 100 meters.

Application of the Formulas

Before entering upon a discussion of the experimental data in the light of the theoretical formulas, it is well to fix ideas by referring to Fig. 5, which shows the ray path *ace* of a 66 meter wave, and *afe* of a 44 meter wave, from

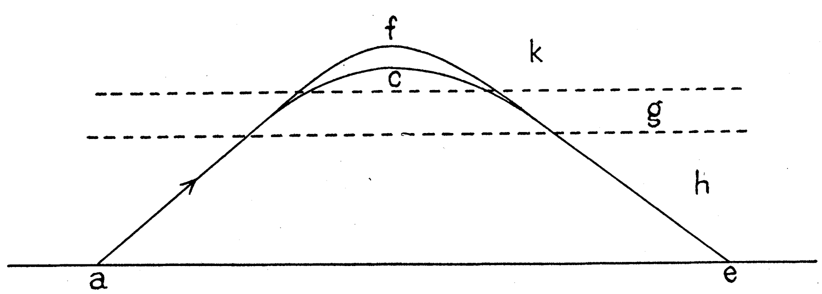

Fig. 5. Refraction of rays in the electronic regions *g* and *κ*.

the transmitter *a* to the receiver *e*. In the region *h* of the atmosphere there are no electrons and the rays are straight. In the region *g*, marked off from *h* by a dotted line in Fig. 5 there are a considerable number of electrons and air molecules; it is here that the absorption may be supposed to occur. Above this there is a region k of greater electronic density, but of lower molecular density, so that the absorption is again slight. The electronic density increases with the height reaching a value of the order of 10^5 at about 100 miles above the earth (an average for daylight over a year[1]). In the regions g and k the rays bend over and return to the earth, and as seen from the refraction formulas (3), (5), (9) and (11) the 66 meter path is below the 44 meter ray. There is evidence[8] that the electron density sets in rapidly above h so that the curved portions of the ray paths are relatively short compared to the whole length of the path, and therefore the paths of a 44 and a 66 meter wave, for example, will not differ greatly.

We may now turn to the data of curves 1 and 2, Fig. 3, which have been chosen for numerical discussion for particular reasons. These curves give the electric fields of the 44 and 66 meter waves at distances from 600 to 1000 miles from the transmitter. At these distances the direct ground wave was absent so that the field strengths were due to rays refracted downwards from the upper atmosphere. Furthermore, multiple reflections and focussing of the rays by the electronic regions are probably of small influence here, so that for a given wave-length and state of polarization a single ray from the transmitter to a receiver may be assumed. The transmitter is considered to deliver its rays impartially in all directions to perhaps 30° above the horizontal. We shall further assume that the length of the ray path may be roughly calculated as if the ray travelled in a straight line to a height of 100 miles and there experienced sharp reflection. Thus, the ray path distance x is related to d, the great circle distance to the transmitter, by the equation, $x=(d^2+4h^2)^{1/2}$, h being 100 miles, and for $d>400$ miles, $x=d$, approximately. If the electric field of the wave diminished inversely with the distance (i.e. the energy inversely with the square of the distance) the field strength would be shown in curves 1′ and 2′ of Fig. 3. In short, the observed field strengths fall off much more rapidly with the distance than the inverse first power; this too for the case, purposely selected, in which one might expect the inverse first power law to hold full sway. Evidently an additional cause of energy degradation is at work, and we take this to be the absorption of energy from the wave by the atmosphere.

We then write

$$A=A_0(x_0/x)\epsilon^{-\kappa(x-x_0)}, \qquad (13)$$

where A and A_0 are the field strengths at ray path distances x and x_0, respectively, and κ is the absorption coefficient defined in (8).

For the moment we must leave the ray picture of Fig. 5, to return to it later, and assume that the optical properties of the atmosphere are constant along the entire path of the ray. The absorption coefficient κ may then be calculated for λ66 meters by substituting in (13) the coordinates of the ends of curve 2, Fig. 3, and comes out to be 5×10^{-8}. With this value of κ the curve 2″ was calculated corresponding to the observed curve 2; it is seen that the two curves are in close agreement. We might now determine κ for λ44

[8] Breit and Tuve, Phys. Rev. **28**, 554 (1926).

meters from curve 1, Fig. 3, in a similar manner, and then by substituting the respective values of κ for $\lambda 44$ and $\lambda 66$ meters in, say, formula (10) obtain two equations in the two unknowns C and G, and thereby determine C and G. Unfortunately such a procedure yields meaningless values for C and G, merely because the data of curves 1 and 3, Fig. 3, are not sufficiently accurate. It seems therefore best to proceed by making a frank assumption, namely, that throughout the course of the ray the average number of molecules per cc is 5×10^{14}; this pressure corresponds to a height above the earth of about 50 miles.[9] With a value $-50°$C for the temperature the number of collisions f which an electron makes per second with the molecules is calculated from classical kinetic theory to be 1.4×10^6. From (1) and (7) this gives $G = 1.5 \times 10^{-5}$. Referring only to the state of polarization represented by (10) we introduce the value of G into (10) together with $\kappa = 5 \times 10^{-8}$ for $\lambda 66$ meters, and find $C = 2.5 \times 10^{-11}$ and 280 for the electronic density N. With the constants thus determined κ is calculated from (10) to be 2.29×10^{-8} for $\lambda 44$ meters, and the intensity degredation curve for this wave-length corresponding to the observed curve 1, Fig. 3, is calculated. This is given in curve 1″, Fig. 3, and is seen to agree with curve 1 within the error of observation.

It is of interest to see what can be done with the range curve 1 of Fig. 4, which gives the ranges reached by the various waves. We take this to mean the distance at which the wave amplitude is a constant small value, i.e. the value just detectable. (It will be appreciated that this is exactly the case of the intensity distribution curves of spectrum lines or bands derived from neutral wedge spectra.) Thus A in (13) is a constant, and (13) can therefore be used to calculate the ranges x or d, (x being the ray path distance and d the great circle distance) for values of λ by using the κ corresponding to each λ determined from (10) with the constants $C = 2.5 \times 10^{-11}$ and $G = 1.5 \times 10^{-5}$ given in the preceding paragraph. The quantity x_0 being entirely arbitrary is taken to be small with respect to x so that it disappears from the exponent in (13). The calculated range curve, made to pass through one observed point, i.e. 1000 miles for $\lambda 50$ meters, is given in the dotted curve 2 of Fig. 4 and agrees with the observed curve within the error of experiment for the waves from 16 to 150 meters. For waves longer than 200 meters the calculated ranges are below the observed values, as indeed is to be expected, since for these wave-lengths the ground wave is predominant at the shorter distances and the overhead ray is relatively feeble.

At first sight the fair agreement which has been reached between theory and experiment might appear meaningless. So many simplifying assumptions have been made that the theoretical picture might seem to have but little resemblance to reality. The considerations which follow, however, indicate the agreement to be genuine and to offer strong support of the theoretical ideas. The foregoing calculations are valid, within limits, for the general case of random polarization, in spite of the fact that they have been based on the absorption formula (10) which refers to a ray polarized in a particular

[9] Humphreys, "Physics of the Air," 1920.

way. For, an exactly similar calculation with each of the other types of polarization represented by (4), (6), and (12) yields agreement with the experimental curves of Fig. 3 and 4 similar to that obtained with (10). This comes about because the values of κ from one of the curves of Fig. 2 are roughly proportional to those from any of the other curves. We have assumed that the rays pass through a homogeneous medium composed of, for example, 5×10^{14} molecules and 280 electrons in each cc, these numbers giving the desired values for the absorption. However, because of the approximately linear relation between κ, N and the molecular density we may keep the absorption values undisturbed and at the same time suppose that the absorption exists only in a certain fractional part of the ray path (the electrons having a density greater than 280, of course). Thus we return to the more exact conception of the ray paths sketched in Fig. 5, in which the absorption occurs only in the region g. The numbers 5×10^{14} and 280 are therefore to be regarded as properly chosen equivalent average values over the ray path.

The values of the molecular and of the electronic density are of course in no wise fixed by the foregoing calculations. The one has been a proper guess and the other has been calculated from it; when regarded as average values over the ray path the numbers are perhaps reasonable. With the experimental radio data at present this is about all that can be done. As these data improve and become more numerous we may hope that they may be fitted into a general scheme which in the end will depict the electronic and molecular pressures at all heights above the earth to which we may direct a radio ray so that it will return. We may take this occasion to mention that throughout this and the earlier papers[1,2] we have been at pains as far as possible to draw conclusions as to the electrons, etc., of the upper atmosphere from the radio wave phenomena only. With the idea, wise or unwise, that these inferences may be entirely independent of, and therefore worthy of comparison with, similar inferences from other fields of experiment[10,11]. Meanwhile others[12,13] have been attacking the converse problem of calculating the ionization by means of the absorption of radiation, or in other ways, and from this to derive the facts of radio. However, we prefer to regard the radio waves as a means, and a fruitful one, of plumbing the outer depths just as the earthquake waves of seismology serve to search the earth beneath.

From the present viewpoint two programs, rather prosaic perhaps, of experimental measurements suggest themselves, namely, measurements of average field intensities of waves below 200 meters and observations of the polarizations of waves, say, 100 to 1000 meters, throughout distances to 2000 miles or more from the transmitter. The first is an extension of the curves of Fig. 3 and the work of Austin,[14] the second is touched upon,

[10] Chapman, Roy. Meteor. Soc., Quarterly Journal **52**, 225 (1926).

[11] Bendorf, Phys. Zeitschrift **27**, 686 (1926).

[12] Rice and Baker, Journ. Amer. Inst. Elect. Eng. **45**, 535 (1926).

[13] Lassen, Zeits. f. Hochfrequentztechnik, **28**, 109 and 139 (1926); Elias, ibid., **27**, 66 (1926).

[14] Austin, Bull. Bur. Stand. **7**, 317 (1911).

although slightly, in the experiments of Picard,[15] of Smith-Rose and Barfield,[16] and of Appleton.[17] One must however, refrain from being too sanguine of the success of such programs. An experimental analysis of the received wave to discover its state of polarization is not simple. The ground causes complicating effects and the state undoubtedly varies more or less rapidly with the time. Particularly in the broadcast range of wave-lengths from 200 to 600 meters a glance at the absorption curves of Fig. 1 and the refraction curves (readily plotted from (3), (5), (9), and (11).) indicates a rather bewildering array of possible states of polarization. Apparently one might simplify things by dealing with east-west or north-south propagation, particularly at the equator where the earth's magnetic field is horizontal, or at the magnetic poles where it is vertical.

A discussion of the amplitudes of radio waves as a function of the distance from the transmitter is hardly complete without reference to the experimental measurements of Austin[14] on waves longer than 300 meters. These, for daylight conditions, are expressed in condensed form by the Austin-Cohen formula, which may be written

$$A=(aA_0/d)\epsilon^{-bd/\lambda^s}, \tag{14}$$

where a, b and s are constants and d is the great circle distance to the transmitter. If the curves of Fig. 3 are calculated from (14) values much too low are found, as indeed is to be expected since (14) is not valid for the shorter waves. Using (14), with Austin's values for b and s, to calculate the range curve of Fig. 4 we obtain the dotted curve 3, which agrees very well with the observations at the longer wave-lengths. This merely means that Taylor's independent measurements were consistent with those of Austin. Formulas (13) and (14), although somewhat similar, should not be confused; they supplement each other. (13) gives the theoretical received amplitude due to a single overhead ray, for waves below 150 meters, whereas (14) gives the actual received amplitude, which in the general case may be due to several overhead rays and a ground ray, for waves longer than 300 meters. The complete theoretical amplitude degradation formula embracing all wave-lengths will perhaps be similar in form to these. Even now for practical purposes formula (13) with the value of κ from (10) may be used to calculate the relative amplitudes of the shorter waves at various distances from the transmitter under full daylight conditions. We obtain to a sufficiently close approximation,

$$A=(\alpha/x)\epsilon^{-\pi CG\lambda^2 x}=(\alpha/x)\epsilon^{-11.8\times 10^{-16}\lambda^2 x}, \tag{15}$$

where α is a constant and λ and x are in cms, x being the ray path distance. (15) is valid for wave-lengths from 16 to 150 meters to distances of 1000 miles from the transmitter, and therefore covers as great a range of frequencies and distances as the Austin-Cohen formula. Because of the direct ground wave

[15] Picard, Proc. Inst. Rad. Eng. **14,** 205 (1926).

[16] Smith-Rose and Barfield, Proc. Roy. Soc. **110A,** 580 (1926).

[17] Appleton, Proc. Roy. Soc. **109A,** 621 (1926).

the formula does not hold too near the transmitter, for 75 meters waves within, say, 100 miles, and for longer waves within longer distances. The ground wave, however, can be calculated from the Sommerfeld formula.[18] Expression (15) of course is not true at great distances. For instance, in the case of a 30 meter wave it gives at 5000 miles from the transmitter 10^{-7}, and at 3000 miles 10^{-4}, of the energy at 1000 miles, and qualitative experience indicates that the signal strengths are usually greater than these numbers. At great distances the showering down of energy from the upper atmosphere, or, in other words, the contribution of many possible ray paths, is sufficient to overshadow an inverse square law of energy attenuation further weakened by exponential absorption.

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January 7, 1927.

[18] Sommerfeld, Ann. d. Phys. **28**, 665 (1909).