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X-RAY ISOCHROMATS OF COPPER TAKEN IN DIFFERENT DIRECTIONS RELATIVE TO THE CATHODE STREAM

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ABSTRACT

Variation with potential of the intensity of monochromatic x-rays of wavelengths 0.823 to 0.247A for directions making angles of 36°, 90° and 144° with the cathode stream.—X-ray isochromats of copper were taken in three different directions relative to the cathode stream, for a target face making an angle of 25° with the cathode stream. Corrections were made for stray radiation in the neighborhood of the measured beam, for radiation due to secondary hits of cathode electrons which had been reflected backward from the focal spot at large angles, and for absorption in the target. It is established that, within experimental error, an isochromat of frequency ν varies linearly with the potential from potentials about $(5/4)\ H\nu$ to $2H\nu$, where $H\nu$ is the quantum voltage for excitation of frequency ν. The linear portions of the graphs were extrapolated to find the intercept on the intensity axis at $H\nu$. This intercept depended on $H\nu$, and also on θ, the angle between the measured x-rays and the cathode stream. For $\theta=90°$ the intercepts varied between $0.062\ I'$ (for $H\nu=50$ kv.) and $0.085\ I'$ (for $H\nu=15$ kv.) with an average of $0.072\ I'$; for $\theta=36°$ the respective values were $0.076\ I'$, $0.096\ I'$, $0.086\ I'$; for $\theta=144°$, $-0.035\ I'$, $0.010\ I'$, $-0.017\ I'$. I' is the intensity of the isochromat at $2H\nu$. Values of $H\nu$ used were 15, 20, 30, 40, 50 kv. The intercept decreased, in general, for increasing $H\nu$, but for $\theta=36°$ and $\theta=90°$ the change was within the experimental error of $\pm 0.01\ I'$.

Energy distribution in the x-ray continuous spectrum from a thick target.—Kulenkampff's formula for the energy distribution in the x-ray continuous spectrum for $\theta=90°$ does not hold in detail for the higher voltages and different target face inclination used in the present work. Assuming total energy in the continuous spectrum proportional to the square of the voltage on the tube, and assuming suitable modifications of this law, for the forward and backward angles, from Sommerfeld's space distribution of energy as a function of cathode ray velocity, it is shown that the spectrum for the forward angle contains relatively more high frequency rays than the spectrum at 90°, and the spectrum at the backward angle contains relatively more low frequency rays.

Energy distribution in the x-ray continuous spectrum from a thin target.—On similar assumptions as to total energy, the spectra which would have been obtained from a very thin foil of copper are derived from the isochromats by Webster's method. The thin target spectra on a frequency scale, for $\theta=90°$, are horizontal except for a sharp rise in intensity as ν approaches ν_0, the high frequency limit. For $\theta=36°$ the energy is approximately directly proportional to ν except for the region near ν_0, where there is again the sharp increase. For $\theta=144°$ the energy is approximately inversely proportional to ν except near ν_0 where there is a decrease for high voltages.

Theories by Kramers and Wentzel for $\theta=90°$ are in fair agreement with the experiments as to thick target spectra, but if certain assumptions made in this paper are correct, Kramers' predicted thin target spectra are much more nearly in accord with the facts than are Wentzel's.

I. Introduction

PREVIOUS work on x-ray radiation in different directions relative to the cathode stream consists either of measurements of total intensity or of comparisons of the ionization spectrum at forward and backward angles. Inasmuch as a reduction of ionization spectrum measurements to energy spectra is very difficult with our present knowledge, such measurements should be supplemented by others in which these reductions do not occur.

To eliminate these reductions, one may use the isochromat method (measuring ionization at a given wave-length as a function of voltage applied to the tube, for constant current). Since ionization is directly proportional to energy, for a given frequency, and since no errors are involved for which suitable correction cannot be made, the isochromat is of the same form as though energy were directly measured. Although a family of isochromats does not determine the energy distribution in the spectrum without some additional knowledge (such as total energy in the spectrum as a function of voltage), nevertheless the isochromats themselves will often distinguish between a correct and an incorrect theory, and are a very valuable guide in the formulation of a new theory.

II. Apparatus

The x-ray tube was of the Coolidge type, water cooled. The copper target face was inclined at an angle of 25° to the direction of the beam of cathode rays (see Fig. 1). Two parallel iron wires extended from the top of the anode approximately in a plane perpendicular to the cathode stream and in such a position that the cathode stream went between them. This was to prevent deflection of the cathode ray electrons by the electrostatic field near the target.

The anode was outgassed by cathode ray bombardment during evacuation. Toward the end of the outgassing process, small brilliant spots kept flashing up at the focal spot, and these flashes seemed to be simultaneous with abrupt gas discharges which were occurring at the same time. When the isochromats were being taken, an area of about a square millimeter at the focal spot was extremely bright for high currents. Unfortunately the surface of the target was roughened in the process of outgassing, so that not much value can be attached to the penetration measurements.

The focal spot was about seven millimeters broad, and this made necessary some special precautions about the position of the tube. The tube was provided with a lateral adjustment reading to a tenth of a millimeter, and the total radiation from the focal spot was plotted by moving the tube across narrow slits lined with the crystal table axis. The centroid of this graph was then found, and the tube set so that this centroid was lined with the slits. The position of the centroid shifted slightly with voltage and current, but any error in wave-length due either to the initial setting (re-adjusted for each isochromat) or to shift, or to both, must have been less than one x-unit. For an isochromat, the slit near the tube was widened so that the only slit limiting the beam was the one near the crystal. This is important on account of the variation of the focal spot with voltage and current. Special tests were made to assure that the wires attached to the anode did not limit the beam. To insure definite conditions with regard to polarization, the cathode stream was kept always in the plane of the slits.

The generating plant, the electrostatic voltmeter, and the milliammeter were the same as described by Webster and Hennings,[1] and the calibrations and ionization readings were made in the same way. The spectrometer was of the Bragg type, but with a modification[2] which made possible the reading of wave-lengths directly on a micrometer screw. The Compton electrometer was calibrated by an ionization method, keeping the voltage on the tube constant and varying the current.

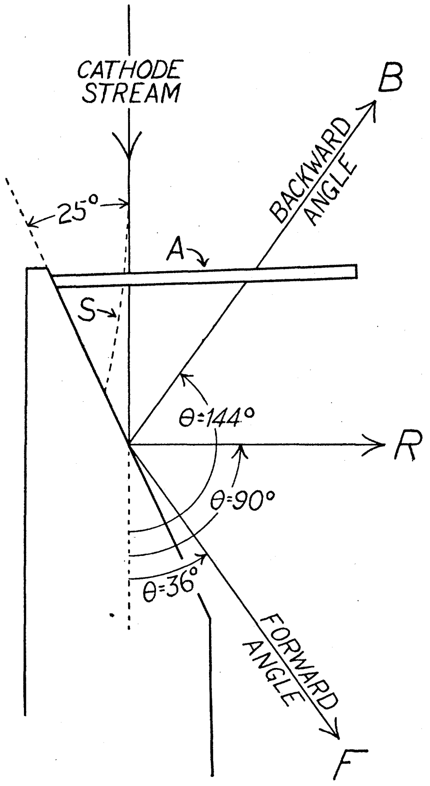

Fig. 1. Schematic diagram of anticathode showing directions of cathode stream and measured x-rays. The cathode stream passed between two wires *A*, which modified the electrostatic field near the anticathode so as to keep the cathode rays from being deflected into some such path as *S*. The continuous x-ray spectra were compared for directions *F*, *R*, and *B*.

III. CORRECTIONS

The isochromats required corrections of three different types, to be discussed in this section. First, the stray radiation in the neighborhood of the

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[1] D. L. Webster and A. E. Hennings, Phys. Rev. **21**, pp. 301–325 (1923).

[2] W. W. Nicholas, J.O.S.A. & R.S.I. **14**, p. 61 (January, 1927).

beam reflected from the crystal was evaluated by plotting it for different positions of the ionization chamber and interpolating to the position for the reflected beam. This radiation was probably due chiefly to diffuse scattering from the crystal.[3] This correction averaged about two percent at the top of the isochromat.

The second correction was for a radiation produced over the whole surface of the target; it is thought to have been due to electrons which had been reflected from the focal spot at large enough backward angles to hit the target face a second time. Curves of the intensity of this radiation at a given wave-length as a function of voltage applied to the tube will be called "secondary isochromats." These secondary isochromats were determined by supporting a piece of lead so as to cut off the radiation from the focal spot and widening the slit nearest the tube in order to measure the radiation from the rest of the target face. They appeared to be linear, from about $(3/2)H\nu$ to $2H\nu$, where $H\nu$[4] is the quantum voltage for production of rays of frequency ν. The extrapolation of the linear portion always gave a negative intensity intercept at $H\nu$. This intercept was always a considerable fraction (1/4 to 2/3) of the intensity of the secondary isochromat at $2H\nu$. At $H\nu$ the secondary isochromats were tangential to the voltage axis. The actual correction was estimated on the assumption that the secondary isochromat radiation was produced uniformly over the target face. The amount of the correction was about three percent at $2H\nu$.

Third is the correction for absorption in the target. The swiftly moving electrons which make up the cathode stream will penetrate into the target, most of them suffering deflections but some of them going in fairly straight lines, and losing energy because of their impacts with the atoms of the target. A few of them will radiate x-rays somewhere along their path, but at any point they cannot radiate a frequency greater than that corresponding to the energy with which they reach the point, according to the Einstein relation. For instance, the maximum frequency of the spectrum must be produced very near the surface of the target, before the electrons have lost appreciable velocity. There will be a maximum depth at which any other frequency may be produced, corresponding to the maximum depth to which a cathode ray can penetrate and still retain enough of its original velocity to be capable of radiating this frequency. But rays of this frequency may be produced at any depth between zero and this maximum, so that in general there will be a depth distribution. The direction in which the absorption of the target for the x-rays is least will be, of course, the direction of the normal to the surface of the target. However, restricting ourselves to directions in the plane perpendicular to the cathode stream, the direction of least absorption will be the direction R (Fig. 1), the absorption increasing as the direction of observation makes greater angles with R, until the angle 90° is reached, when there is infinite absorption.

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[3] For a more complete discussion, see the paper by Webster and Hennings, reference 1.

[4] H is equal to h/e, where h is Planck's constant, and e is the charge on the electron, as follows from Einstein's relation $Ve=h\nu$.

Now it is entirely possible that the fully corrected radiation for the direction R differs from that for directions perpendicular to the cathode stream but not in the plane of the paper, because, although there is symmetry about the cathode stream, there is not symmetry about the normal to the target face. For instance, one may easily imagine that the rays are slightly polarized in a direction normal to the face of the target. But as P. A. Ross[5] has recently shown, polarization in the continuous spectrum is appreciable only near the high frequency limit, and errors in this region will affect the present results very little.

Accordingly, the correction for absorption in the target was made on the assumption that the fully corrected isochromats are dependent only on the angle θ (angle between cathode stream and measured x-rays) and independent of the angle ϕ (specifying angular position of the tube about the cathode stream as axis, and measured from the position of least target absorption, the direction R, Fig. 1). Let $I=f(V)$ represent the fully corrected isochromat of wave-length λ and excitation voltage $H\nu$, for $\theta=90°$. Assume that at a given voltage all the rays of wave-length λ may be considered as having been produced at some definite depth p, measured perpendicular to the face of the target, and let p vary with voltage according to the function $\Pi(V)$ shown in Fig. 2. (The curve of Fig. 2 was obtained by Webster and Hennings[1] for molybdenum at the K absorption limit wave-length and a target face inclination of about 55°. The scale of ordinates is omitted here). For the present target face inclination of 25° the depth measured along the path of the x-rays for $\theta=90°$ and $\phi=0°$ will be $x=p/\cos 25°=1.10p$. If ϕ is increased, this effective depth increases; values of ϕ were actually used corresponding to effective depths x, $2.5x$, and $5x$. The measured isochromats of wave-length λ for effective depth rx should then be represented by

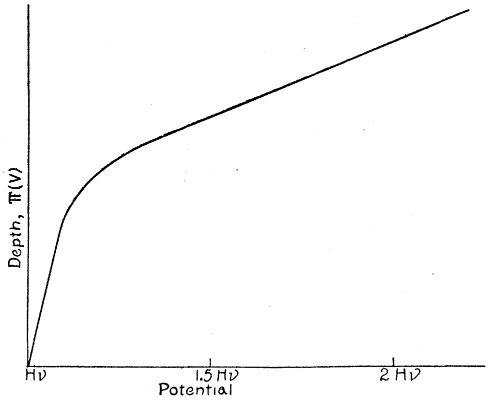

Fig. 2. Relative mean depth of production of rays of frequency ν as a function of voltage on the tube (after Webster and Hennings). $H\nu$ is the excitation potential for frequency ν.

$$I_{rx}=f(V)e^{-\mu rx}=f(V)e^{-1.10\mu r\Pi(V)}$$

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[5] P. A. Ross, Phys. Rev. **28**, 425 (1926), abstract.

where e is the base of natural logarithms, and μ is the absorption coefficient of copper for wave-length λ. If each ordinate is then divided by the ordinate in the same isochromat at $V=V_1$, the ratio, which will be called I'_{rx}, should be given by

$$I'_{rx}=\frac{f(V)}{f(V_1)}e^{1.10\mu r[\Pi(V_1)-\Pi(V)]}$$

The purpose of this last step is to reduce different isochromats to comparable scales. V_1 was chosen at about two-thirds of the way up the isochromat, where the relative intensity was considered most certain both on account of having a fairly large value and on account of the possibility of comparing it with points on either side.

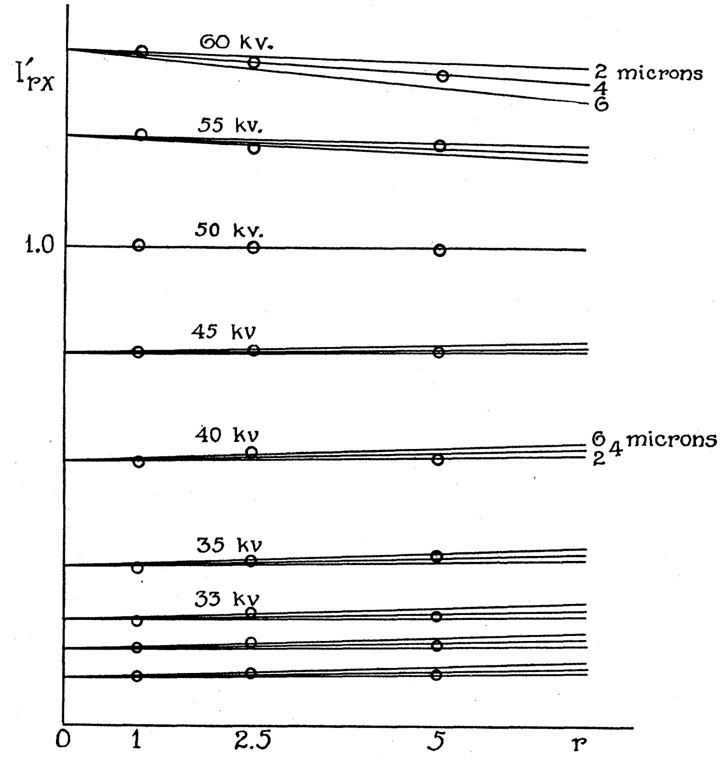

Fig. 3. Illustrating method for determination of the mean depth of production of x-rays in the anticathode.

Experimental values of I'_{rx} can be obtained directly from the data, and their dependence on r may then be compared with this expression as in Fig. 3 (the circles indicate experimental values). According to the formula the points corresponding to a definite voltage ought to lie on an exponential curve. Now the experimental results (compare Fig. 3) show that the slope of the curve must be very small in the vicinity of $r=1$ to 5; this indicates that in reality the coefficient by which r is multiplied in the exponent is very small.

Accordingly, in the region to be considered, the curvature of the exponential line must be inappreciable. This accounts for the fact that the results of Fig. 3 seem to be fairly well represented by straight lines. Therefore the values for $r=0$ (corresponding to no target absorption) were obtained by extrapolating by means of straight graphs. The deviations from the predicted linearity, which were almost always of the kind shown in Fig. 3 (indicating too much absorption for $r=2.5$), are probably due chiefly to the roughness of the target face.

The graphs of I'_{rx} against r for the 30 kv isochromats are shown in Fig. 3. The families of lines correspond to mean depths of production at 60 kv of 2.0, 4.0, and 6.0 microns respectively. Similar families of lines were drawn for each set of isochromats. The depths selected, however, were not those which best fit the curves for that particular isochromat, but rather which lined up best with the curve of depths at $2H\nu$ for all the isochromats. The curve which was finally selected is shown in Fig. 4. In this connection it is worth noting that the values of r used in the corrections were nearly 1 for the backward angle and 5 for the forward angle. For this reason the values of I'_{rx} for $r=2.5$ were given very little weight in the selection of the mean depths for $2H\nu$. From Figs. 2 and 4 all the later absorption corrections could be made on the assumption that p is independent of the direction of observation of the x-rays. Absorption coefficients were calculated from a formula given by Richtmyer.[6]

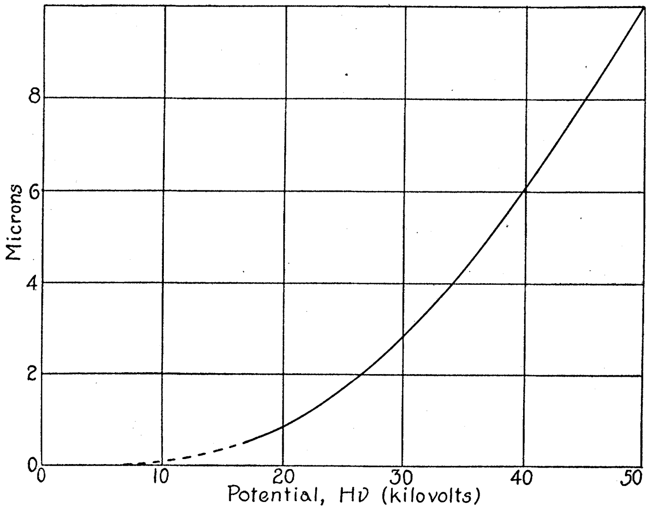

Fig. 4. Mean depth of production of rays of frequency ν at $2H\nu$.

IV. Results

In Figs. 5, 6, and 7 are shown the fully corrected isochromats for $\theta=90°$, $\theta=36°$, and $\theta=144°$, reduced to the same intensity at $2H\nu$. The forward angle, 36°, and the backward angle, 144°, were selected supplementary so as

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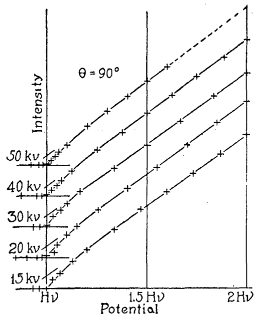
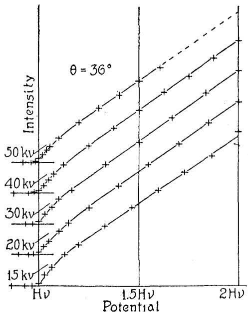
[6] F. K. Richtmyer, Phys. Rev. **27**, 1 (1926).

to have polarization the same. For a copper target, the face of which is inclined at 25° to the cathode stream, and for the voltage range here investigated, it seems to be established that:

(1) An isochromat of frequency ν is linear, within experimental error, from about $(5/4)H\nu$ to $2H\nu$, where $H\nu$ is the quantum voltage for excitation of frequency ν.

(2) The linear portions of the graphs were extrapolated to find the intercepts on the intensity axis at $H\nu$. These intercepts, for $\theta=90°$, are within the limits of $+0.073I'$ $\pm 0.02I'$, where I' is the intensity of the isochromat at twice the excitation voltage. The variation of the intercepts with $H\nu$ for $\theta=90°$ is within experimental error.





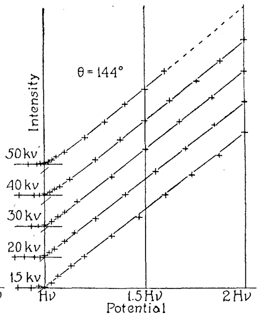


Fig. 5. Isochromats for $\theta=90°$ reduced to equal intensities at $2H\nu$. θ is the angle between cathode stream and measured x-rays.

Fig. 6. Isochromats for $\theta=36°$ reduced to equal intensities at $2H\nu$.

Fig. 7. Isochromats for $\theta=144°$ reduced to equal intensities at $2H\nu$.

(3) The intercepts for $\theta=36°$ were greater than those at 90° by about $0.01I'$, but this amount is not outside the limits of error. These intercepts were likewise independent of $H\nu$.

(4) The intercepts for $\theta=144°$ show a variation with voltage which is outside the experimental error, the intercept decreasing for increasing $H\nu$. The difference between these intercepts and the ones for $\theta=90°$ and $\theta=36°$ is well outside the limits of error, especially at high voltages, where the intercepts for $\theta=144°$ have a *negative* value.

A table of values for the intercept, obtained from the graphs, is shown below. The experimental error is about $\pm 0.01I'$. If anything, the results for $\theta=36°$ are less reliable than the others on account of the larger correction necessary for absorption in the target.

TABLE I

Quantum voltage of isochromat (kilovolts)	Intercepts		
	$\theta=36°$	$\theta=90°$	$\theta=144°$
15	+0.096 I′	+0.085 I′	+0.010 I′
20	+0.093 "	+0.075 "	0 "
30	+0.077 "	+0.072 "	−0.018 "
40	+0.086 "	+0.065 "	−0.040 "
50	+0.076 "	+0.062 "	−0.035 "

V. DISCUSSION

i. Empirical formulas for spectra at 90°. Kulenkampff[7] has proposed the formula[8]

$$I_\nu = C\{Z(\nu_0 - \nu) + Z^2 b\} \tag{1}$$

to express the results of his work on the continuous spectrum at voltages from 7 to 12 kilovolts for an angle of 90° between cathode stream and measured x-rays. I_ν is the intensity on a frequency scale at frequency ν, C is a constant, Z is the atomic number of the metal of the anticathode, ν_0 is the high frequency limit of the spectrum, dependent, of course, on the applied voltage, and b is approximately equal to 0.0025 when ν is measured in units 10^{18} sec^{-1}. This formula would give, for copper, isochromats whose intercepts (see above) are very approximately 0.01 ZI'/V_0, where V_0 is the excitation voltage of the isochromat. The present work, and also the work of Webster and Hennings on molybdenum,[1] indicates very definitely that this formula for the intercepts cannot be correct at the higher voltages. On the other hand, the condition that the isochromat intercept shall be nearly constant (as indicated by the isochromat work) is approximated by the formula

$$J_\nu = C\{Z(\nu_0 - \nu) + b' Z^2 \nu_0\} \tag{2}$$

Putting $b' = 0.0019$ to give the isochromat intercepts for copper as low a value as allowable ($0.05I'$) from the present experiments, the spectrum intercepts coincide with those of formula (1) for 5400 volts, but are 2.2 too big at 12,000 volts.

It may be that some more general formula is required which will approximate (1) at low voltages and (2) at high voltages, but it is perhaps more likely that several dissimilarities in experimental conditions are responsible for the disagreement. For instance, in Kulenkampff's experiments the cathode stream was perpendicular to the crystal table axis, while in the present work the two were parallel. The former arrangement would cause a loss of intensity due to the reflection of polarized rays from the crystal, but since the spectrum is strongly polarized only near the high frequency limit,[5] the factor should be negligible. A more promising source of explanation is the following extension of an idea of Kramers[9] to explain the discrepancy between his theory and Kulenkampff's experiments. Kramers' theory predicted that the second term on the right hand side of formula (1) should be zero, and Kramers offered the suggestion that since many of the electrons of the cathode stream suffer large deflections on entering the target, some will be deflected out of the target with fairly high speed and in such a manner that they do not return to the focal spot and therefore are lost, in so far as concerns the radiation

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[7] H. Kulenkampff, Ann. der Physik, **69**, 548 (1922).

[8] A formula based on Webster's formula for isochromats (Phys. Rev. **9**, p. 220, 1917) would represent much more closely the actual shape of the spectra near the high frequency limit, but for the present discussion of the magnitude of the intercepts, Kulenkampff's simpler formula is adequate.

[9] H. A. Kramers, Phil. Mag. **46**, 869 (1923).

actually measured. The form of the secondary isochromats of the present work suggests that the explanation may be well founded. These deflections vary with the atomic number of the metal of the anticathode, and it would be surprizing if the number and speeds of the electrons lost did not depend on the angle between target face and cathode stream (90° in Kulenkampff's experiments; 25° here).[10] More detailed discussion of the point must, however, await further experiment.

ii. Thick target spectra in different directions. As noted above, a family of isochromats does not determine the energy distribution in the spectrum without some additional knowledge, such as total energy in the spectrum as a function of voltage. It is very likely that this total energy will not be the same function of voltage for forward angles as it is for backward angles, and still different for 90°. As a basis for an approximation to these laws, the usually accepted law that the total energy is proportional to the square of the voltage[11] was assumed for 90°, and modifications for forward and backward angles were estimated from Sommerfeld's curves[12] of spatial distribution of energy as a function of cathode ray velocity. The estimated modifications of the V^2 law were: Total energy $=aV^3$ for $\theta=36°$, and total energy $=bV^{1.3}$ for $\theta=144°$, where θ is the angle between cathode stream and measured x-rays, and a and b are constants.[13]

For an approximation to the spectrum energy distribution from a thick target, it will be assumed that the isochromats are linear, with zero intensity intercept at V_0, i.e. that they can be represented by the formula

$$I(V,\ \nu)=k(\nu)(V-V_0)$$

where $I(V,\ \nu)$ is the energy at frequency ν in a range $d\nu$, for applied voltage V, and V_0 is the quantum voltage for frequency ν. If $k(\nu)$ can be represented by $c\nu^n$, where c is a constant, the exponent n can be readily found by use of the total energy law

$$\int_{\nu=0}^{\nu=\nu_0} I(V,\ \nu)d\nu=AV^{2+\epsilon}$$

where A is a constant. It turns out, following Webster's analysis,[14] that $\epsilon=n$, and the equation becomes

$$I(V,\ \nu)=c\nu^{\epsilon}(V-V_0)$$

Thus when $\epsilon=0$ (spectrum at 90°, see above) the spectra on a frequency scale are represented by straight lines passing through the high frequency limit, and the lines for different voltages are parallel. When ϵ is equal to 1.0,

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[10] It is partly for this reason that the correction equivalent to the present secondary isochromat correction cannot be readily estimated for Kulenkampff's work.

[11] See Siegbahn, Spektroskopie der Rontgenstrahlen, p. 201.

[12] See Sommerfeld, Atombau und Spektrallinien, 4th Ed. p. 37.

[13] It should be emphasized that there is no direct experimental evidence for these assumptions in their present form. But the general form of Sommerfeld's space distribution of energy for steady voltages has been confirmed by Loebe (Ann. d. Phys. **44**, 1033, 1914) and others.

[14] D. L. Webster, Proc. Nat. Acad. **5**, 163 (1919).

corresponding to the forward angle of the present work, the spectra are curved concave downward, and have relatively more energy in the high frequencies than the straight line distribution has. For ϵ equal to -0.7 the spectra are curved concave upward, and have relatively more energy in the low frequencies.[15] A consideration of the fact that actually the isochromat intercepts are greater, algebraically, for forward than for backward angles, leads to even greater asymmetry in the spectra.

iii. Thin target spectra in different directions. The difference in the isochromats for the various directions is perhaps most strikingly brought out by a consideration of the spectra that would have been obtained from an exceedingly thin foil of copper. A method for obtaining thin target spectra, involving the Thomson-Whiddington penetration law, and the total energy law, together with isochromat data has been given by Webster and Hennings.[1]

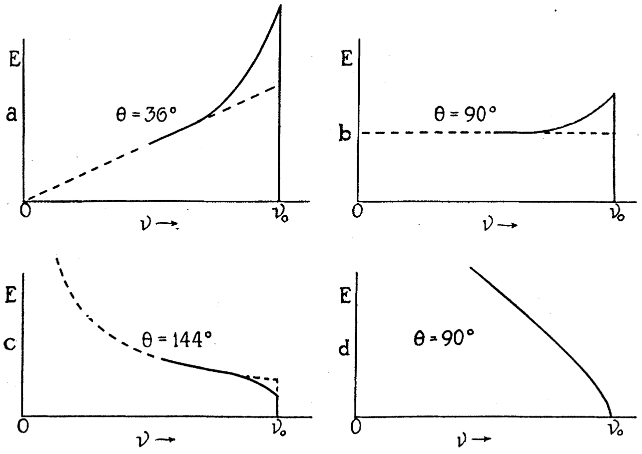

Fig. 8. Energy distribution in thin target spectra. E is the energy on arbitrary scales, θ is the angle between the cathode stream and measured x-rays, ν_0 is the high frequency limit. Curves a, b, c were derived from the isochromats and an assumed total intensity law for the various directions. Curve d is predicted by Wentzel's theory. Curve b, dotted portion, is predicted by Kramers' theory.

From this analysis the energy distribution in the thin target spectrum is given by

$$(b/2V_0)D_V I(V,\ \nu)$$

where the only quantity not previously defined is b, the coefficient of the Thomson-Whiddington law in the form $V_s^2 = V^2 - bs$. For the simplified case of linear isochromats, this may be written

$$(b/2V_0)k(\nu)D_V(V-V_0) = (c/2V_0)k(\nu)$$

where c is a constant. Taking the formerly determined values for $k(\nu)$, it is obvious that the thin target spectra on arbitrary intensity scales are of the forms indicated by the dotted lines in Fig. 8. If, now, account is taken of the

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[15] Compare Wagner's spectra in different directions J. d. Rad. Elek. **16**, 212 (Dec. 1919).

marked deviation from linearity of the isochromats in the region near the quantum voltage, the thin target spectra are modified as shown approximately by the full lines of Fig. 8, a, b, c.

iv. Comparison with theories for spectra at 90°. A theory of the x-ray continuous spectrum proposed by Wentzel[16] predicted that for $\theta=90°$ the spectrum from a thin target should resemble approximately the graph (d) of Fig. 8. Wentzel assumed that the quantum spectrum can be derived from the classical spectrum (emitted by an electron describing a hyperbolic orbit about the nucleus) by a continuous compression of the classical spectrum energy toward lower frequencies, an infinite frequency in the classical spectrum corresponding to the high frequency limit of the quantum spectrum. This is analogous to the method of deriving the quantum line spectrum (e.g. Balmer's series) from the classical spectrum from an electron in an elliptical orbit (fundamental frequency with harmonics). A theory proposed by Kramers[9] proceeded differently, Kramers' method of treatment being simply to chop off the classical spectrum at the high frequency limit given by the Einstein relation, and assume that the frequencies higher than this limit correspond to transitions to stationary states (which practically do not occur). His thin target spectra resembled the dotted portion of Fig. 8*b*. Both theories agree fairly well with experimental results on thick target spectra in spite of the striking disagreement for thin target spectra. This is due to wholly different methods of treating the loss of velocity of the cathode rays on penetrating the target. Kramers assumed that practically all the electrons are slowed up gradually within the target according to the Thomson-Whiddington law. (The efficiency of production of x-rays is so low that comparatively very few electrons will suffer any sudden losses of energy by radiation). Wentzel, on the other hand, followed Lenard in assuming that a large part of the electrons suffer absorption before they have penetrated far enough to have lost much velocity according to the Thomson-Whiddington law.

It is of very great theoretical importance to determine which of these two fundamentally different forms of the Correspondence Principle is more nearly correct. Now the present thin target spectra were derived from the isochromats on the assumption that practically all of the electrons are slowed up gradually within the target (i.e. that Lenard's absorptions are inappreciable). But on the other hand, if Lenard's absorptions had been assumed to be of chief importance, the derived thin target spectra at 90° would have resembled Wentzel's. Evidently then, one cannot, on the basis of the present work, discriminate between the two theories without a knowledge of what actually happens to the cathode rays on entering the target.

Kulenhampff[17] has recently discussed these absorptions for the passage through air of electrons having an initial velocity corresponding to voltages in the neighborhood of 10 kv. This was in connection with experimental work on the total ionization in air produced by photoelectrons which had been

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[16] Gregor Wentzel, Zeits. f. Physik, **27**, 257 (1924).

[17] H. Kulenkampff, Ann. der Physik, **80**. 3. p. 261, June 2, 1926.

ejected by x-rays of known energy. It appears that for these conditions Lenard's absorptions are extremely rare. If it could be concluded from this that the absorptions are also negligible in the present work, then the continuous compression theory of Wentzel could be said quite definitely to be wrong. It is scarcely permissible, however, to carry over the results for low voltages and gaseous media to high voltages and solid media.[18] Nevertheless, it is difficult to understand what becomes of the energy of the electron in these atomic absorptions, considering the conservation of energy and the conservation of momentum together with the fact that the probability of production of an x-ray is extremely small.

v. Comparison with theories for spectra in different directions. As to the isochromats in the different directions relative to the cathode stream, neither Kramers' nor Wentzel's theory is developed to such an extent as to allow ready comparison of experiment with theory. It may be pointed out simply that the present results are in qualitative agreement with what would be expected if these theories were modified in some such way as that developed by Sommerfeld (see above) to account, by means of classical electrodynamics, for the early work in this field. It should be strongly emphasized that although considerations such as are elaborated in the first part of this discussion make it difficult to interpret theoretically the absolute magnitude of the intercepts of the present work, the difference between the intercepts for various angles between cathode stream and measured x-rays is not to be explained in this manner.

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Cornell University,
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[18] See also two papers by P. Lenard, Ann. d. Physik, **80**, pp. 1–32 (May 18, 1926).