FEBRUARY, 1927 PHYSICAL REVIEW VOLUME 29

A DETERMINATION OF THE DIELECTRIC CONSTANT OF AIR BY A DISCHARGE METHOD

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Abstract

Two air condenser systems, one containing the test condenser and the other the balancing condenser, are charged to equal opposite potentials, the opposite charges are mixed and discharged through a galvanometer. The two condenser systems are adjusted until the galvanometer deflection is zero. A special form of rotating commutator was devised for which the contact resistances are small and uniform. This commutator has three pairs of make and break contacts, two for charging and discharging the two condensers, and one pair connected so that a single battery is used to charge both condensers. The capacity of the test condenser is obtained in terms of readings on a condenser which forms part of the balancing condenser system. The ratio of the capacities of the test condenser, with a vacuum and with air for dielectric is then obtained. The calibration for the readings is described. This calibration is made with the apparatus in place, by simple changes of connections. Possible errors from time lag, thermal expansions, and deformations from pressure changes are discussed. The average of thirteen separate measurements gives 1.000594 for the dielectric constant of air at 0°C and 760 mm Hg pressure. The thirteen separate readings agree in the second significant figure of the decimal part of the result.

THE first measurement of the dielectric constant of air was made in 1874 by Boltzmann.[1] He measured the change of potential for constant charge of an air condenser, when the air was exhausted. He obtained the value 1.000590 for the dielectric constant of air at 760 mm pressure and at 0°C. In 1877 Ayrton and Perry[2] determined the dielectric constant of air by comparing, with a quadrant electrometer, the potentials of condensers with and without air. Their value 1.00150 for the dielectric constant of air is generally considered in error. The next determination of the dielectric constant of air was made in 1885 by Klemencic,[3] an assistant working in Boltzmann's laboratory. He obtained the change in capacity of a large air condenser when the air was exhausted, by measuring with a galvanometer the discharges of the condenser. The condenser was charged and discharged sixty-four times per second by a tuning fork commutator. He gives the value 1.000586 as the dielectric constant of air at standard pressure and temperature. Klemencic's individual values vary from 1.000718 to

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[1] L. Boltzmann, Wien. Berichte **69**, Part 2, 795 (1874).

[2] Gordon's Electricity and Magnetism, Vol. 1, p. 130.

[3] I. Klemencic, Wien. Berichte Bd **91**; also Rep. d. Physik, Bd. **21** (1885).

1.000478, and he rejects the last ten of his twenty values on account of irregularities. The average of his twenty values is 1.000599.

The next absolute determination of the dielectric constant of air seems to be that of Fritts,[4] who compared the capacities of air condensers by the beats of a heterodyne circuit,[5] recording the beats by an ingenious photographic method. He obtained the value 1.000540 for the dielectric constant of air. In the same year Zahn,[6] also using a heterodyne method, got the value 1.000572.

In view of these variations in values obtained for the dielectric constant of air, and particularly because the methods using high frequency oscillations have given lower values than Boltzmann's and Klemencic's values, it was thought desirable to make a new determination by one or both of the older methods. This work began in 1923 directly after Fritts' determination in this laboratory. The method used consists in charging two nearly equal air condensers to opposite potentials, and then discharging the two condensers simultaneously through a sensitive galvanometer. The charging and discharging takes place numbers of times per second so that the galvanometer gives a steady deflection for the discharge. The object is to adjust the two opposite discharges until the galvanometer deflection is zero. The

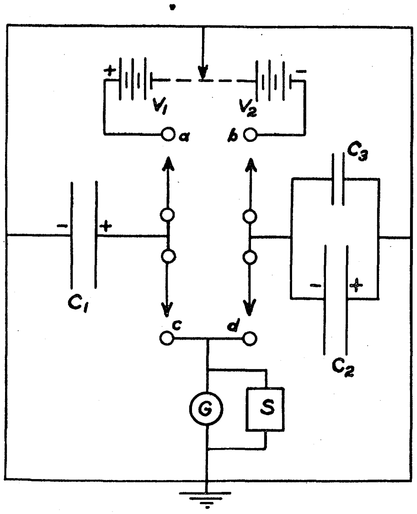

Fig. 1. Diagram of circuit previously used.

[4] E. C. Fritts, MS Doctor's thesis, Univ. of Ill. Library, Feb. 1923; Physical Review **23**, 345 (1924).

[5] Hyslop and Carman, Phys. Rev. **15**, 243 (1920).

[6] C. T. Zahn, Phys. Rev. **23**, 781; **24**, 401.

arrangement for this method as used by A. P. Carman and K. O. Smith[7] in 1923, is shown in Fig. 1.

At first the well known commutator of Fleming and Clinton[8] was used for the charging and discharging of the condensers, but the galvanometer deflections were so unsteady because of irregular contacts of the brushes of the commutator, that finally an entirely new commutator was devised and used with satisfaction in all this investigation. In the new commutator, contact is made by bringing a platinum-tipped rod end-on against a plate. The rod is connected with insulation bushings to a brass block which fits on an eccentric part of the commutator shaft. The rods are kept horizontal by suitable bearings, and are in pairs, one on the right side and the other on the left side of the eccentric block. The eccentricity in this particular instrument is 3 mm, so that the rods move backwards and forwards through 3 mm in a harmonic motion. The platinum plate is carried through a flat cushioning spring on an adjustable support, so that the end of the rod makes contact with the plate at the rod's maximum outward position. Thus when the contact on the right side is made, the contact on the left side is broken, and vice versa. In the final form of this commutator, as used in this investigation, there are four pairs of rods, so that four connections can be made (or broken) at the same time. It will be seen that in our final circuit, the third pair proved important in allowing the use of a single undivided battery. Various other improvements are found in the final form of this commutator not found in the form described in 1924.[9]

In the equation for the above arrangement of apparatus, it is assumed that the commutator speed and the voltages V_1 and V_2 are absolutely constant for the five or ten minutes of an experiment. Also, the galvanometer was found to vary more or less, unless the shunt was made too low for extreme sensitivity. These difficulties led to the development of a new circuit in which there is a single battery for the charging, and there are better conditions for the balance. The new arrangement has come gradually after many experiments from the first circuit. For the work on this development with the long careful tests and observations involved, credit is due to Mr. Hubbard who took up the investigation with the senior author for the last two years.

The essential parts of the final circuit are shown in Fig. 2. The charging battery of about 50 "B" storage cells is shown at V. The

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[7] MS of Master's thesis by K. O. Smith, Library of Univ. of Ill., June, 1923.

[8] Fleming and Clinton, Proc. Phys. Soc. London, **18**, 389 (1903).

[9] A. P. Carman, J.O.S.A., **9**, 175 (1924).

test-condenser C_B is built rigidly and is under a bell jar so that the gaseous dielectric can be exhausted. Its capacity is 0.04 mf. The balancing condenser C_N has approximately the same capacity as C_B. The variable condensers C_D and C_M are in parallel with C_B and C_N respectively and serve for final balancing. The commutator with three pairs of contact rods is indicated by the arrows. The commutator speed was about 1500 r.p.m. The key K_1 is at the start in position Q, so that contacts 6 and 7 are at the same potential. While contacts 5, 6, and 7 are closed, contacts 1, 2 and 3 are open, and vice versa. To insure mixing of the condenser charges, contacts 1 and 2 close just an instant before 3 closes. If the capacity of the test condenser is equal to that of the balancing condenser, there is zero charge remaining after the mixing and consequently no current through the galvanometer. The possible sensitivity can be calculated as follows. Suppose the test and balancing condensers are equal so that there is zero galvanometer current. Now change the capacity of C_B+C_D by C, and the resulting current is I_g. Then $I_g=CVn$, where n is the commutator speed in r.p.s. The galvanometer deflection is $d=I_g/F=CVn/F$ where F is the figure of merit of the galvanometer. The sensitivity of the system to change of capacity is then $d/C=Vn/F$. The figure of merit of the galvanometer is about 10^{-10} amperes per mm deflection. We thus calculate the sensitivity, $d/C=25$ mm galvanometer deflections for

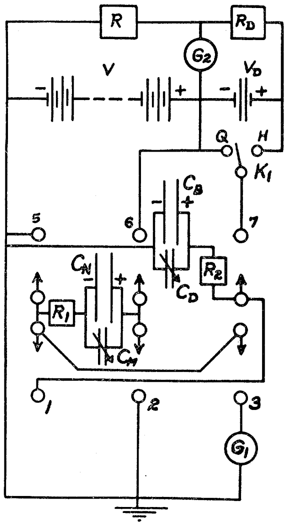

Fig. 2. The new circuit (simplified).

1 mmf change. Let C_1 be the capacity of the test condenser C_B, and K_1 the dielectric constant for a low pressure, and C_2 and K_2 the corresponding capacity and dielectric constant of C_B at normal air pressure. If C_B is the capacity with perfect vacuum as the dielectric then $C_1 - C_2 = C_B(K_1 - K_2)$. Putting this in the above equation for sensitivity, we get $d/(K_1 - K_2) = C_B Vn/F$. Using approximate values, we get $d/(K_1 - K_2)$ equal to 10^6 mm galvanometer deflections per unit change in K. That is, applied to the dielectric constant of air, a change from 1.000590 to 1.000591 should make a difference of one millimeter in the galvanometer deflection. The final results show that we approximate this calculated sensitivity in having variations only in the sixth decimal place.

It is seen directly that the circuit is sensitive to change of voltage. This makes it important to guard against stray induced e.m.f.'s. Thus it was found that for certain commutator speeds, the galvanometer was unsteady. This was traced to voltages induced from the field of the 60 cycle power circuit, these producing considerable galvanometer deflections if the commutator speed was $\frac{60}{2}$, $\frac{60}{3}$, $\frac{60}{4}$, or etc. r.p.s. These stray e.m.f.'s disappeared completely at suitable commutator speeds. The procedure for the calibration of the apparatus and the calculation of the dielectric constant is as follows:

I. With the key at Q, the condenser C_M is adjusted to give zero galvanometer deflection. The pressure of the gas in C_B is P_1 and its dielectric constant is K_1. Then

$$C_B K_1 V + C_1 K_D V + C_s K_s V = 0 \tag{1}$$

where C_B and C_1 are the capacities of the condensers C_B and C_D for vacuum, and C_s is the algebraic sum of all other capacities in the circuit, and K_1, K_D and K_s are dielectric constants of the respective dielectrics.

II. Change pressure of gas in C_B to P_2 with dielectric constant K_2, and bring galvanometer deflection to zero by changing capacity of C_D to C_2. Then

$$C_B K_2 V + C_2 K_D V + C_s K_s V = 0. \tag{2}$$

From Eqs. (1) and (2), we get

$$K_1 - K_2 = (C_2 - C_1) K_D / C_B \tag{3}$$

The calibration to determine $(C_2 - C_1)/C_B$ is carried out by adding the resistances R and R_D, the galvanometer G_2, and the two opposing cells giving the resultant e.m.f. V_D.

(1) The key, is on position Q and the condensers are adjusted to zero galvanometer deflection

$$C_B K_B V + C_3 K_D V + C_L K_L V + C_{s1} K_{s1} V = 0 \tag{4}$$

where C_L is a part of the capacity (C_N+C_M) (see step 3 below), and C_{s1} is the algebraic sum of all remaining capacities in the circuit. K_B, K_D, K_L and K_{s1} are dielectric constants.

(2) The key is turned to position H, and the galvanometer deflection is brought to zero by changing condenser C_D. Then

$$C_BK_BV'+C_4K_DV'+C_LK_LV+C_{s2}K_{s2}V+C_{s3}K_{s3}V'+C_{s4}K_{s4}V_D=0 \tag{5}$$

Here $V'=V+V_D$ and the meanings of the other terms are apparent.

(3) The key is placed at Q, and condenser C_B is disconnected, and capacity C_L of (C_N+C_M) being also removed, the galvanometer deflection is zero. Then

$$C_5K_DV + C_{s1}K_{s1}V = 0. \tag{6}$$

(4) The key is placed at H, and zero galvanometer deflection is produced by changing C_D. Then

$$C_6K_DV' + C_{s2}K_{s2}V + C_{s3}K_{s3}V' + C_{s4}K_{s4}V_D = 0. \tag{7}$$

From Eqs. (4), (5), (6), (7), we get

$$C_BK_B(V' - V) + K_D[V'(C_4 - C_6) - V(C_3 - C_5)] = 0. \tag{8}$$

Substituting $V_D = V' - V$, we get

$$C_B = \frac{K_D[V'(C_4 - C_6) - V(C_3 - C_5)]}{- K_BV_D}. \tag{9}$$

In the above calibration, V and V' are assumed constant, and there were actually no variations of sufficient magnitude to cause appreciable error. If the resistances R and R_D are adjusted to give zero deflection of galvanometer G_2, then R', R and R_D, are proportional to V', V and V_D, where $R'=R+R_D$. Introducing these proportional quantities into Eq. (8), we get

$$C_B = \frac{K_D}{K_B}\,\frac{R'(C_4 - C_6) - R(C_3 - C_5)}{- R_D} = \frac{K_D}{K_B}A\,; \tag{10}$$

where A is a constant determined by the calibration. Eq. (10) thus gives the capacity C_B in terms of capacity differences as observed on condenser C_D. By combining Eqs. (3) and (10) we get

$$(K_1 - K_2) = (C_2 - C_1)K_B/A. \tag{11}$$

If we use the same gas at the same pressure and temperature we have $K_B=K_1$, then

$$(K_1 - K_2)/K_1 = (C_2 - C_1)/A$$

or

$$K_2/K_1 = (C_1 - C_2 + A)/A. \tag{12}$$

The reduction to standard temperature and pressure is made on the assumption that the dielectric constant is proportional to the density of the gas, and hence directly proportional to the absolute temperature and inversely proportional to the pressure. We get the equation in the form

$$(K_0 - 1) = \frac{760T(C_2 - C_1)}{273(P_1 - P_2)[A - (C_2 - C_1)]} \tag{13}$$

where

$$A = \frac{R'(C_4 - C_6) - R(C_3 - C_5)}{- R_D}$$

as indicated in Eq. (10). We thus get an absolute determination of the dielectric constant of air by the above procedure. The calibration is made with all parts in place, by simply throwing a switch, and is indeed a part of the procedure each time.

In the above method, it is of course assumed that the insulation is "perfect" and this was found by repeated tests to be the case except on some days of very high humidity and consequent surface condensation. No measurements were made on such days. It is assumed that the charging and the discharging are complete in the time of the commutator contact. The inductance and resistance of the circuit were such that no error was caused by time lag. This was verified experimentally.

The importance of the test condenser is such that a brief description of it is desirable. This condenser is constructed of iron plates, each plate being five inches square and one-sixteenth of an inch thick. We will call the two armatures of the condenser, *A* and *B*. Armature *A* consists of fifty plates with forty-nine spaces, each space being three thirty-seconds of an inch wide. The *A* plates are held together by four quarter-inch steel rods or bolts which pass through holes in the corners of the plates. Iron washers of uniform thickness were carefully turned, and used on the rods between the plates, thus securing uniform spacing. The rods are threaded at the ends, and strong nuts are used to clamp the plates rigidly together. Armature *B* consists of forty-nine plates held together by rods, with spaces and nuts similar to those for *A*. The plates of *B* occupy the spaces between the plates of *A*, with small but safe air clearance. A common arrangement in an air condenser of this kind is to have the diagonals of the plates of *A* at right angles to those of *B*. To get more effective use of the surfaces, we placed half of the *B* plates with their diagonals making an angle of 35° to the right of the diagonals of *A*, and the other half of the *B* plates making the same angle to the left. These *B* plates are clamped together on the rods by strong nuts in groups of two or three for rigidity and for uniform spacing. All rods are clamped to a thick plate of Pyrex glass, holes being bored through the glass and the rods are held firmly in place by nuts and washers on each side of the plate. This glass plate is at the top, so that the *B* plates are in fact suspended from the glass plate. The insulation between *A* and *B* was found "perfect" for our purpose. The *A* armature is joined to earth. The condenser and the glass plate are completely shielded by grounded sheet iron disks and cylinders. The only dielectric in the field, except the air, is the glass between the widely separated rods; and for a condenser of this size the correction for this dielectric is negligible.

The constancy of the capacities of "test" and the "balancing" condenser system is, of course, a fundamental requirement, and this was tested with great care. One annoying variation was finally eliminated by putting the comparison, as well as the test condenser, and the rotating commutator on separated supports, from which it was inferred that the mechanical vibrations introduced a small capacity change in the comparison condenser. The temperature effects on the capacities of the condensers proved troublesome until understood. The condensers are made of several materials, the thermal expansions and contractions, due to even a very moderate temperature change, introduce deformations which persist for a considerable time. The resulting change in capacity is shown by the creeping of the galvanometer deflections. This creep was fairly constant and so could be allowed for, but it was possible to find times and conditions when the creep was very small. A more fundamental difficulty was a capacity change which we ascribed to the displacement of the parts of the condenser by the pressure of the gaseous dielectric. Our test condenser as described above is very strongly built and all parts are of iron except a pyrex glass insulating plate. It was found that, if the gas in the condenser had been at normal pressure for several hours, and it was then evacuated, and allowed to return to normal pressure, there was a galvanometer deflection showing a small capacity change. After the first evacuation and return to normal pressure, the change in capacity was not observed until the condenser had again rested. This effect, though small, is difficult to evaluate. It is an effect equally important in all methods of obtaining the absolute dielectric constant of a gas, and may exist in small condensers in the same ratio as in large condensers. The important adjusting condenser C_D is of the rotating wing type, and is very carefully built with heavy brass plates turned from cast brass. It has a circular scale and vernier, but the readings for the calibrations were made by means of a telescope and circular scale, a mirror being mounted on top of the rotor shaft of the condenser. The readings were kept in that part of the calibration curve which is a straight line. The bell jar over the test condenser was exhausted by a Hy-vac pump, and the admitted air was carefully dried by passing slowly through tubes containing phosphorus pentoxide. The temperatures were measured by means of sensitive thermocouples, two couples being inside of the bell jar, and one couple on the outside.

This method, as indicated, has been developed slowly after many tests and changes. The special form of commutator devised in the course of the work is satisfactory in making and breaking contact regularly in time and with small and constant resistance. The speed of the commutator can be varied easily through a wide range. Theoretically, changes in speed of the commutator are eliminated in the equation, but these changes in speed are so very small that no appreciable error enters from any secondary effect due to speed change. Speeds of about 1500 were used in the final measurements. The very difficult requirement of constancy in the ratio of potentials of two charging sources has been met by using a single battery for charging both condenser systems, so that variations of the voltage of the charging battery do not come in. The arrangement for calibrating the apparatus in place, by only connecting and disconnecting keys, also reduces possible errors.

Results: After many preliminary trials, the following final measurements were made when the temperature conditions were fairly satisfactory. Early morning hours were used in three of four runs to secure even temperatures. All measurements are included.

	Dielectric constant	Pressure change (mm Hg)
Morning, June 18, 1926 2–6 A.M.	1.000598	732.4
	1.000597	730.9
	1.000593	732.9
	1.000595	734.4

Afternoon, June 18	1.000597	738.1
2–4 P.M.	1.000598	737.8
	1.000590	543.2
Morning, June 19	1.000594	731.9
2–4 A.M.	1.000592	730.1
	1.000591	544.3
	1.000592	548.7
Morning, June 27	1.000592	723.2
2–6 A.M.	1.000592	723.5

It will be noted that all of the thirteen separate measurements agree in the second significant figure of the decimal. The final average is 1.000594. Boltzmann's value of 1.000590 is in closer agreement with this value of 1.000594 than the more generally quoted value of 1.000586 of Klemencic. These values by static charge methods are larger than the values by oscillation methods obtained recently by Fritts and by Zahn.

Physics Laboratory,
University of Illinois,
October, 1926.