NOVEMBER, 1926 PHYSICAL REVIEW VOLUME 28

THE ABSORPTION OF X-RAYS FROM $\lambda = .08$ TO 4.0A

BY S. J. M. ALLEN

ABSTRACT

The mass-absorption cœfficients of a number of elements from C to U for homogeneous rays from λ.709 to 1.933A and for filtered general rays from 1.93 to 4.0A have been measured. A Shearer gas tube, continuously pumped and arranged with a controlled side leak was used. The data from these observations, together with those from the two previous papers by the author at wave-lengths from λ.08 to .709A taken with the same apparatus and on the same samples throughout, form a set of continuous results from .08 to 4A, from which, after a careful analysis, the following conclusions can be stated. Over a wide range of wave-lengths, in which the mass-scattering coefficients σ/ρ can be neglected, the *fluorescent absorptin coefficients* τ/ρ cannot always be represented by the generally accepted formula, $o\tau/\rho = C\lambda^n$ with n and C constant, but each element must be considered by itself, with values of n and C changing rather abruptly in certain regions of wave-length. In general the values of n which best fit the results are a high value 2.92 and a low value 2.6. Nowhere does λ^3 over a long range accurately represent the data. Over the complete range of wave-length, the *absorption of* Al appears to be quite accurately represented by the formula, $\mu/\rho = (13.9 \text{ or } 14.0)\lambda^{2.92} + (.14 \text{ to } .18)$ and is therefore well suited as a standard substance for the determination of effective wave-lengths from absorption coefficients. *The* N^4/A *law*, although holding fairly well in those cases where n is the same for all the elements involved, can at best be only approximate, and it therefore does not seem possible to express the absorption of x-rays by a universal formula of the form $\tau/\rho = C\lambda^3 N^4/A$, in which C is a constant. *The discontinuity magnitudes*, $J_K = C_K/C_L$, extended to Fe, show steadily increasing values with decrease of atomic number N, reaching a maximum value for Fe of about 10. For the L-M series, $J_L(=C_L/C_M)$ shows values following the same law as J_K does, with probably somewhat higher values for the same elements than J_K. For the M-N series, $J_M(=C_M/C_N = 10$ to 12 in the case of U) apparently indicates that J_M with five discontinuities has much higher values than J_L or J_K for the same elements. The effect of impurities in the cases of Al, Cu, Sn and Pb was ascertained, and only the presence of Fe in the Al could make an error as great as 1 percent.

INTRODUCTION

IN TWO recent papers,[1,2] the author gave the results of measurements on the mass-absorption coefficients μ/ρ of a number of elements from carbon to uranium, over a range of homogeneous wave-lengths from λ.08 to λ.709A. The chief conclusions of these papers may be briefly summarized as follows.

(a) As λ decreases, the fluorescent absorption coefficient τ/ρ, becomes less and less important, while the scattering coefficient σ/ρ becomes more

---

[1] Allen, Phys. Rev. **24**, 1 (1924).

[2] Allen, Phys. Rev. **27**, 3 (1926).

and more important, and also in the case of the light elements, when τ/ρ has practically disappeared, σ/ρ decreases very slowly, if at all, with further decrease of λ.

(b) A careful analysis of the data, by trial and error, or by means of graphs, indicates that τ/ρ can be expressed approximately as a function of λ, in the form, $\tau/\rho = C\lambda^n$, if the proper values of σ/ρ are first subtracted from the experimental values of μ/ρ. The author was of the opinion that the value of n which best represented the results was smaller than the generally assumed value 3, being on the average about 2.92. The assumed values of σ/ρ increased with the atomic number and also slowly with λ, reaching in the case of the high atomic weight atoms, values as great as 1.0. The N^4/A law held well for all the elements, except those of high atomic weight.

(c) The magnitudes of the K discontinuities were shown to be smallest for the heavy atoms, and increased with decrease of N, the range being from about 5 for Pb to from 7 to 8 for Ag.

The conclusion (c) has also been confirmed in general by the important work of Richtmyer, who in a recent paper,[3] with a spectrometer of high resolving power investigated the K discontinuities of Pb, Au, W, Sn, Ag, and Mo. His values from Pb to W are somewhat larger than those of mine, as stated in the first paper, but are in better agreement with mine of the second paper, which appeared just after that of Richtmyer. The revised values of mine, which will be discussed later in the present paper, are in even better agreement with those of Richtmyer. In any case there is good agreement as to the general phenomena involved. In obtaining his values of the discontinuity magnitudes Richtmyer draws his graphs, assuming the truth of the λ^3 law. However over the short range necessary it would not make any great difference if $\lambda^{2.92}$ had been used. He states from his data that the following formulas will hold good for Sn and Ag.

$$\mu/\rho_{Sn} = 90\lambda^3 + 1.0; \; \mu/\rho_{Ag} = 70\lambda^3 + 1.0$$

These formulas indicate rather high values of σ/ρ for Sn and Ag, those of the author from the $\lambda^{2.92}$ relation being about half as great. Again Richtmyer gives values of σ/ρ for Pb, Au and W about .85, less than those of Sn and Ag, which would be a somewhat peculiar result.

Any of these formulas, if correct, should predict at λ1.0A the actual experimental value of μ/ρ, since we would have $\mu/\rho = C + \sigma/\rho$ or $\tau/\rho = C$. I shall discuss this question more fully later on.

A number of theoretical formulas of absorption have been deduced on various assumptions, and they all involve as a necessary part, the λ^3 law.

---

[3] Richtmyer, Phys. Rev. **27**, 1 (1926).

Richtmyer, at considerable length, very clearly analyzes the formulas of Thomson, de Broglie, and Kramers, as to their agreement with experimental facts. The agreement is certainly not very good for any one of them.

Certain European writers of books on x-rays evidently consider that the λ^3 does not always represent the facts. Siegbahn, in his well known book on the spectroscopy of x-rays, gives a table of absorption coefficients, which do not indicate λ^3. Ewald in his book on Roentgen Ray Spectra states that the law of absorption follows more correctly $\lambda^{2.80}$.

Since practically all the data, by various observers using homogeneous rays, upon which the λ^3 law is based, lie in the range from λ.1 to .7A, where in most cases, the unknown values of σ/ρ are an important part of the whole, it would seem difficult to decide the question satisfactorily in this region.

In view of this, and the importance of the question, the author has carried out during the past winter measurements on the mass-absorption coefficient μ/ρ of elements from C to U, for homogeneous rays from λ.709 to λ1.933A, and for specially filtered rays, from λ1.93 to λ4A. With these long wave-lengths σ/ρ becomes negligible (except for C, and in some cases Al), and μ/ρ becomes approximately equal to τ/ρ.

Experimental Arrangement

These experiments were carried out with the same spectrometer, equipped with two Soller collimators, that was used in the previous experiments on shorter wave-lengths. Also the absorbing layers were made from the same materials. They therefore form a set of data from .08 to 4A with the same apparatus, on the same samples, and by the same experimenter, and whatever may be the probable errors of the combination, they should at least be somewhat free from the uncertainties introduced when one attempts to compare the data obtained by different observers, with different samples of substances of unstated purity. The variation of τ/ρ with λ, as ascertained from the data, should be more consistent.

Two chief difficulties were encountered, a slight variation of the tube during observations, and the lack of homogeneity of the excessively thin layers necessary for the long wave-lengths. Many readings and weighted means reduced the former to a minimum, and careful rolling with a good hand roll finally obtained quite satisfactory layers of the elements. With the collimator method a larger area of the layer exposed to the rays can be used, in contrast to the single narrow slit method. In general the area of the layer was just a little greater than the area of the window through which the rays were coming. The average mass per unit area through which the rays penetrated was thus nearly the same as that obtained for the whole layer by weighting. In the few cases where the layers were much larger in area than the window, they were removed around in different positions to obtain an average.

For all cases from λ.709 to λ1.933A several layers of every element were used and complete absorption curves obtained. The average value of μ/ρ from the complete curve was chosen. In the case of Sn from 1.93 to 4A, only one layer was thin enough to be used. The mass per unit area of this layer was obtained by weighing, and also by calibrating it against other thicker layers of the same material, in regions of shorter wave-lengths.

The uranium layer was made in the following manner. Metallic uranium was ground to fine powder, suspended in alcohol with a trace of shellac, flooded into a layer of filter paper, and allowed to dry. A portion of this layer which seemed by examination to be the most homogeneous was so marked that it could in each measurement be placed in the same position in front of the window. Every reading for the absorption at each wave-length was thus taken through the same area. A layer of the same filter paper was also measured at the same wave-length, and from the percentage absorptions of the two layers, the net absorption of the uranium could be readily calculated.

The mass per unit area of the uranium was not found by weighing, but an effective value was obtained, by assuming that the N^4/A law held between uranium and Pb in the M absorption for the same chosen wave-length. Knowing μ/ρ for Pb, the percentage absorption of the uranium layer, and the assumed ratio between uranium and Pb, the mass per unit area of the uranium layer was calculated. Even though the absolute values of μ/ρ for uranium obtained in this manner may be in some error, such an error would be a constant one, and not affect the relative values.

A number of the substances were chemically analyzed for me by Dr. Gray of the Bureau of City tests, of this University, and the effect of impurities will be discussed presently.

The source of radiation for these experiments was a Shearer gas tube, manufactured by Adam Hilger Co., London. This tube is of metal with a glass cylindrical tube separating cathode from anode. Targets of various metals can be easily interchanged. All parts which would become heated are water cooled. The tube during operation is kept continuously pumped, with a side air leak, controlled by a fine capillary tube, rubber tube and pinch cock. For a given current strength, once equilibrium has been obtained between pump and leak, no further attention for hours is necessary except occasionally a touch of the pinch cock. A milliammeter in the tube circuit indicates constancy of current. The source of potential was a small transformer, the tube itself acting as a good rectifier. By means of impedance coils in the primary, any secondary voltage between 3000 and 40,000 was readily obtainable.

The Kα and Kβ radiations of Fe, Ni, Cu, and Zn were used, together with the general radiations for wave-lengths between .709 and 1.293A. The intensities of the characteristic rays were of course much greater than the general radiations, but in all cases the intensities were great enough to be measured to an accuracy of 1 percent. Care was exercised, to be sure that no higher orders were present. The (100) planes of a calcite crystal was used as the reflecting crystal. A large number of readings with different settings of the spectrometer at λ1.0A were made.

For observations between λ1.93 and 4A, an attempt was made to isolate narrow portions of the general spectrum by crystal reflection, but not enough intensity could be obtained for measurement. I have as yet no targets beyond Fe. So, in order to obtain approximate information on the absorption of wave-lengths beyond the Kα of Fe, filtered rays had to be used, the effective wave-lengths estimated from the absorption in Al, which the homogeneous observations had shown to be a constant relation between τ/ρ and λ.

Two methods of "filtering" were employed, the ordinary "end rays" method, and a method which may be called "filtering from both ends." The first was used in those cases where, in the general radiation, no components were present whose wave-lengths were the same as any of the discontinuity wave-lengths of the substance used as an absorber. Aluminum was used as the filter in those cases.

The second method was used to obtain approximately homogeneous radiation on the long wave-length side of any of the discontinuities. For example, in examining the M absorption of Sn close to the L_1, discontinuity, a filter of Al was used to cut down the long wave-lengths, and a filter of Sn to cut down the short wave-lengths beyond L_1, the small remainder being quite homogeneous. By then varying the tube voltage a range of effective wave-lengths could be obtained. For Ag-M absorption, a filter of Al+Ag was used, and for U-N absorption, a U filter. In all cases a companion measurement in Al was made of such a nature that the total absorption in Al was the same as that for the element in question. In short, the absorption in any substance is being compared with the same total absorption in Al.

It is easy to show, both theoretically and by numerous experimental examples, that under these prescribed conditions, the relative absorption coefficient of any substance to that of the standard substance, is the same as would be obtained if homogeneous rays had been used. To illustrate this point, the results expressed in Table I are given. In the first set the values of μ/ρ for the homogeneous radiation λ1.93A is compared for a number of substances with an Al+Fe filtered beam of the same effective λ. In the second set, the filter is Al+Cu. Within experimental errors the comparative values are the same. In the second set it is to be noticed that Ni is passing through its K discontinuity for this filtered beam.

TABLE I

Comparison between μ/ρ for homogenous, and filtered rays

Radiation	C	Al	Fe	Ni	Cu	Zn	Ag	Sn	Pt	Pb
Homogeneous λ = 1.93	9.2	94	71	90	99	115	410	490	365	420
Filtered λ = 1.93	9.3	94–95	72	92	100	116	415	485	360	425
Homogeneous λ = 1.45	4.5	41.5	294	335	43.5	52	197	241	176	207
Filtered λ=1.45	4.4	42	295	Kγ	44	51	200	240	178	205

For wave-lengths up to λ2A, the thin aluminum window furnished with the tube was sufficiently thin to transmit the rays, but for longer wave-lengths recourse had to be had to a less absorbing substance. Considerable difficulty was experienced, but finally thin windows of celluloid and gelatine were found satisfactory.

EXPERIMENTAL RESULTS AND DISCUSSION

The mass-absorption coefficients μ/ρ with homogeneous rays for the substances, paraffine wax, carbon, iron, nickel, copper, zinc, silver, tin, tungsten, platinum, gold, and lead, are expressed in Table II. The range of wave-lengths is from the Kα of Mo to the Kα of Fe. The results with

TABLE II

Values of μ/ρ for homogeneous rays reflected from calcite (100)

	K			L						M			
λ	Paraffine	C	Al	Fe	Ni	Cu	Zn	Ag	Sn	W	Pt	Au	Pb
1.933A	8.0	9.2	94	71	90	99	115	410	490		365		420
1.752	5.9	7.3	73.2	56	69	76	90	325	385		280		335
1.655	5.1	6.4	61.5	430	60	64	76	280	275		243	265	290
1.537	4.1	4.9	48.5	330	47.5	50	59	225	325		199	210	230
1.432	3.5	4.3	40.3	290	323	42.0	50.5	192	235	130	171	179	202
1.389	3.1	3.8	36.0			38.0	45.0	173	205		155		180
1.293	2.6	3.2	29.8	213	237	266	38.0	146	176	105	131	137	154
1.170	2.1	2.4	22.5	160	175	206	226	113	132		100	104	120
1.090	1.7	2.0	18.5	128	148	165	186	95	110		87	88	98
1.000	1.25	1.50	14.2	102	118	133	152	73	87		165	110	77
.910		1.25	10.8	75	88	102	115	57	66		170	165	150
.748	.73	.76	6.20	45.0	55	61	69	33.0	39.0		134	141	145
.709	.60	.63	5.29	38.4	47.0	52.0	59	26.8	32.5		117	122	140

filtered rays are shown in Table III, the range of wave-lengths being 1.08 to 3.93A. In discussing the results with homogeneous rays it will be advantageous to place them in certain groups.

TABLE III

Values of μ/ρ *for strongly filtered "end rays" from general radiation.*
λ *calculated from* Al, $\tau/\rho = 14\lambda^{2.92}$

λ	K absorption			L absorption				M		N
effective	Paraffine	C	Al	Fe	Ni	Cu	Zn	Ag	Sn	U
3.93A			760					540	680	490
3.74			660	510		666		570	610	424
3.66			620	490		624			560	410
3.44			516	394		530		660	460	564
3.37			485	374	460	496	590	700	440	580
3.22			426	338	402	430	534	1000	390	600
3.18			410	322	388	420	500	1100	420	600
3.04			360	284	340	364	430	1100	600	600
2.97			338	270	320	350	420	1150	660	600
2.60	17.0	21.5	225	176	210	230	276	870	980	570
2.49			200	156	190	206	246	806	910	560
2.35			170	132	160	176	210	720	780	540
2.24	11.5	14.7	148	113	142	156	182	640	715	525
2.18			136	105	128	144	172	585	660	516
2.06			116	90	110	120	144	510	560	495
1.97			101	79		112		490	505	485
1.93			94	72	92	100	116	415	485	470
1.80			80							442
1.54			48						250	360
1.45		4.4	42	29.5	Kγ	44	51	200	240	285
1.24			26.5						150	190
1.08			18.0							150

Group (1). Those light substances whose absorption lies wholly in the K series. In this group, aluminum is the most useful element to discuss. The chemical analysis of the aluminum used shows Al 99.4 percent, Fe 0.4 percent, Si 0.15 percent. The only impurity present which will have any appreciable effect upon μ/ρ for Al is the Fe, and its effect for all values of λ less than the Kγ of Fe has been calculated as about 1 percent, about equal to or a little greater than the experimental error. For values of λ greater than Kγ the effect would be much less than the experimental error. The effect of the silicon would also be negligible.

In analyzing the data for Al, the mass-scattering coefficient σ/ρ has been taken from the previous papers as .14 to perhaps .18. Except for the shorter wave-lengths its effect will not greatly affect the discussion.

The analysis of Al by the "trial and error" method is shown in Table IV where one vertical column gives the values of C in the formula, $\tau/\rho = C\lambda^n$, for $n = 3$, and the other column, those for $n = 2.92$.

Very few data are available to establish the value of μ/ρ for Al at 1.0A, but what there are show values generally between 14 and 15. Hewlett's[4]

---

[4] Hewlett, Phys. Rev. **17**, 284 (1921).

results would indicate at λ1.0A a value about 14 or a little less. The weighted average of the author's results at λ1.0A lies between 14.1 and 14.2, which, subtracting the value of μ/ρ, would give a value of $\tau/\rho = 13.95 - 14.00$. The effect of the iron impurity would be to make this value for pure Al 1/2 to 1 percent smaller, considering our average experimental error as less than 1 percent. It is very probable therefore that the value of C for pure Al is close to 14.0, perhaps slightly less. Any formula then should predict at λ1.0A a value of C close to 14.0.

Contrasting in Table IV the values of C for $n=3$ and $n=2.92$, we see that $n=3$ gives at short wave-lengths, values of C close to 15, and at long wave-lengths values close to 13, a steady systematic decrease; while $n=2.92$ gives values, which though fluctuating + and −, are on the average close to 14.0. It is very evident, to the author at least, that the λ^3 law does not express the results over this range of long wave-lengths. Whether $\lambda^{2.92}$ expresses them completely over this range, or whether there may be a slight change in the function of τ/ρ with λ, would take precision work with absolutely pure Al to decide. Certainly aluminum comes the nearest to showing a constant function of τ/ρ with λ of any of the substances studied. It therefore is well chosen as a standard for estimating effective wave-lengths, over a very wide range.

TABLE IV

Values of C in equation, $\tau/\rho = C^{\lambda}$ *for various values of n which best fit the data.*

λ	Al		Cu			Sn			Pb		
	$n=3$	$n=2.92$	$n=3$	$n=2.92$	$n=2.7$	$n=3$	$n=2.92$	$n=2.6$	$n=3$	$n=2.92$	$n=2.6$
.430	15.0	14.0	155	144	120	94	87	68	482	450	343
.500						92	87				
.525						92	87				
.560	14.3	13.7	150	143		90	85				
.600						90	87				
.631	14.4	14.0	149	143	130	91	88		400	380	
.650						90	87				
.675						91	88				
.710	14.4	14.0	145	141	131	88	86	78	400	380	350
.750	14.4	14.1	145	142	130	89	87				
.850	14.4	14.1	138	137		86	85				
.910	14.0	13.9	134	133	131	87	86				
1.000	14.0	14.0	132	132	132	87	87	87	77	77	77
1.109	14.2	14.3	128	129	131	85	86	90	76	76	78
1.170	13.9	14.1	129	130	134	83	84	90	75	76	80
1.293	13.6	14.0	119	123	130	81	83	90	71	72	78
1.389	13.3	13.7	14.1	14.5	15.6	76	80	89	67	68	77
1.432	13.6	14.0	14.4	14.6	15.8	79	82	92	69	70	79
1.537	13.3	13.8	13.7	14.2	15.7	76	78	90	63	66	75
1.655	13.7	14.1	14.3	14.6	16.4	73	75	88	64	67	78
1.752	13.5	14.2	14.0	14.7	16.7	71	74	90	62	65	77
1.933	13.1	13.7	13.7	14.4	16.8	68	72	89	58	61	76

A considerable amount of work has been done by other observers at the Kα of Mo, λ.709 to .710A. Most of these give values of C from 5.27 to 5.30. Taking 5.28 as an average, and solving for C we obtain, for $n=3$, a value 14.4, and for $n=2.92$, a value 14.0.

At the characteristic radiations of Fe, λ1.752 and λ1.933A, we get for $n=3$, a value for C of 13.3, and for $n=2.92$, a value for C of 14.0, a difference between the two of about 7 to 8 percent.

The graphs for Al, for $n=3$, and $n=2.92$ are shown in Fig. 1.

Carbon and paraffine wax are apparently following the same law as aluminum, the C of carbon being on the average a little more than 1/10 the value for Al. Paraffine wax, which the author in the previous paper on very short wave-lengths showed had a value of μ/ρ greater than that of carbon, in this present range of wave-lengths has values less than those of carbon. The effect of the richness of hydrogen in paraffine upon the scattering coefficient has been discussed there.

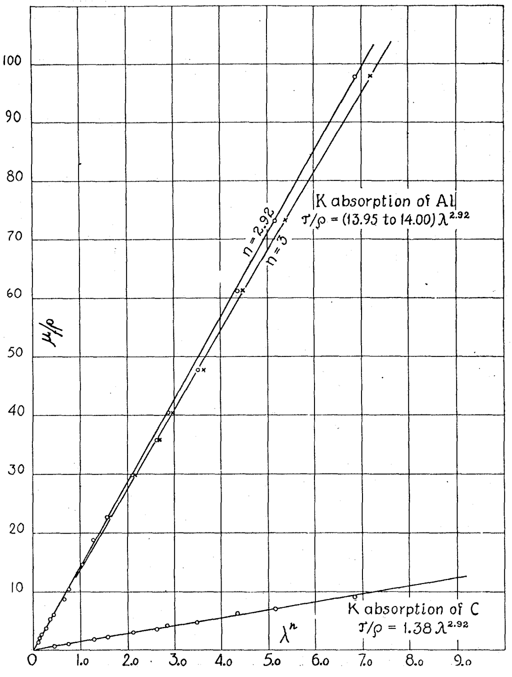

Fig. 1. μ/ρ against λ^n.

Group (2). Elements, whose absorptions lie partly in the K and partly in the L series. This group comprises Fe, Ni, Cu and Zn. Of these, Cu was chemically analyzed as Cu 99.83 percent (no Sn, Zn, or Pb present). This is very pure Cu, and any traces of impurities can not effect results. The analyses of these four elements shows in each case the same characteristics, viz., $n=3$ does not fit the data over the complete range, nor does $n=2.92$, but the function of τ/ρ with λ in the K absorption seems to be made up of two distinct functions, a higher value of n for short wave-lengths, and lower values for long wave-lengths near Kγ.

The results for Cu are chosen for illustration, and are shown in Table IV. It is seen that $n=3$ gives values of C steadily decreasing, both in the

K and L absorptions. $N=2.92$ fits the results well up to a certain value of λ, after which a smaller value of n seems best.

The data of the past, notably that of Richtmyer,[5] and Stoner and Martin,[6] on the λ^3 law would predict a value of C at $\lambda 1.0$ in the neighborhood of 150 for Cu. The actual experimental value by the author at 1.0A is about 132 or 133, a much smaller value. Data of the past on Zn would predict for C at 1.0A values from 160 to 170, while the experimental value is about 152. This apparent rather sharp break in the function in these four elements occurs at a value of λ less than 1.0A which accounts for the predicted values always being greater than the experimental.

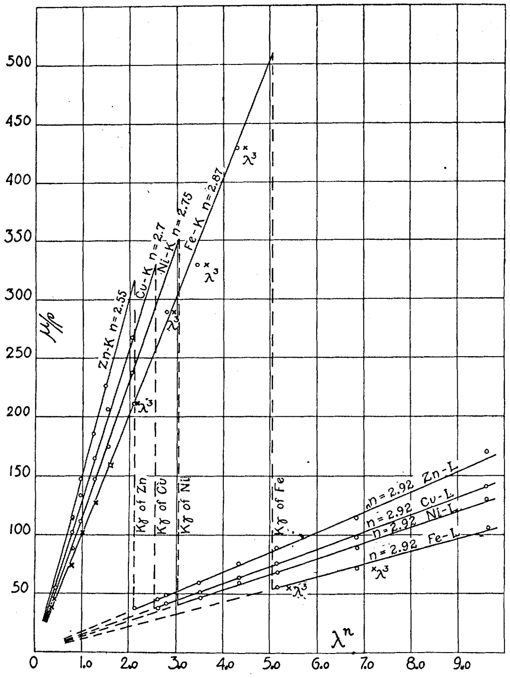

Fig. 2. μ/ρ against λ^n.

Very few data are at hand by other observers above 1.0A for these elements, but what there are show on analysis to be of the same general character. All I can find in the literature are a few scattered results of Bragg and Pierce,[7] a few results of Hewlett on Fe, and the original classic results of Barkla. It is noteworthy that in Hewlett's graph of Fe, is shown this "break" and he calls attention to it.

The author's data for the L absorption of these elements, shows that $n=2.92$ holds extremely well, and also that if these graphs are produced backward they pass very close, if not actually through, the origin. In fact $\lambda^{2.92}$ seems in all cases to be the only graph that will do this. Barkla's data if translated into effective wave-lengths equal to the mean of $K\alpha$ and $K\beta$ are in very good agreement with those of the author. In Fig. 2 will be found the graphs, showing the relation of τ/ρ to λ, for the four elements.

---

[5] Richtmyer, Phys. Rev. **18,** 13 (1921).

[6] Stoner and Martin, Proc. Roy. Soc. A **107,** 312 (1925).

[7] Bragg and Pierce, Phil. Mag. **8,** 608 (1914).

Group (3). Those elements whose absorptions lie wholly in the L series. These include Pd, Ag, and Sn. The results for Pd are not shown, because they indicated below λ1.1A a sudden increase to values somewhat above those of Ag, which is undoubtedly caused by the presence of platinum as an impurity. Above λ1.1 the data for Pd are of value since the M absorption of Pt is nearly the same as the L absorption of Pd. The Ag consisted of both commercial and very pure samples, and no difference was detected in the values of μ/ρ over the entire range. The chemical analysis of Sn, gave Sn 99.81 percent (no Sb, Cu or Pb), which indicates high purity. The analysis of the results on Sn is shown in Table IV for $n=3$, $n=2.92$, and $n=2.6$. From λ.43 to λ.71A the data used for calculation include all the data from the papers of Richtmyer, Stoner and Martin, and Bragg and Pierce, the average value being chosen. It is seen again that $n=3$ does not hold, but $n=2.92$ holds very well up to about λ1.2A, when a value of $n=2.6$ fits most satisfactorily. We have again this "break," but it now occurs above 1.0A, which results in the predicted value of C from short wave-length data being less than the actual value at λ1.0, just the reverse of what occurs for Cu. We can now test Richtmyer's formulas for Ag and Sn, quoted in the introduction to this paper. In the case of Sn the formula would predict at λ1.0A a value of C about 89 or 90, which is in fair agreement with the actual value, but at λ1.933 it would predict a value of 649, which is about 32 percent greater than the experimental value. The formula for Ag would predict values with just as great a discrepancy. The graphs for Ag and Sn are shown in Fig. 3, and Fig. 4, for $n=3$, 2.92, and 2.6.

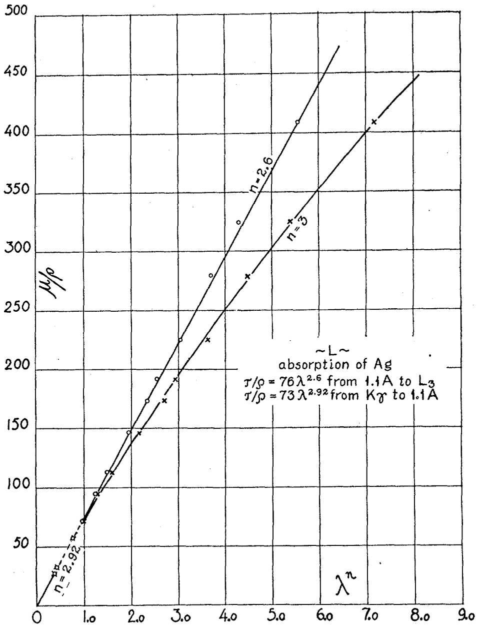

Fig. 3. μ/ρ against λ^n.

Group (4). Those elements whose absorptions lie wholly in the M series. This includes W, Pt, An, and Pb. The analysis of the results for Pb is shown in Table IV. The chemical composition of the lead samples was Pb 99.83 percent (no Sb or Sn) which is very pure. It is seen that neither $n=3$ nor $n=2.92$ fit the results, except possibly near λ1.1A, but that $n=2.6$ fits very well. If the 2.6 graph is produced backward it will pass above the origin indicating that probably a "break" to the 2.92 graph occurs about 1.1A. Any evidence of this "break" at shorter wave-lengths is hidden by the L discontinuities. However, all these elements in their L absorption (which includes the M) show better agreement with $n=2.92$ than a lower value of n. (See second paper.) In Fig. 5 and Fig. 6 are shown the graphs for Pt and Pb.

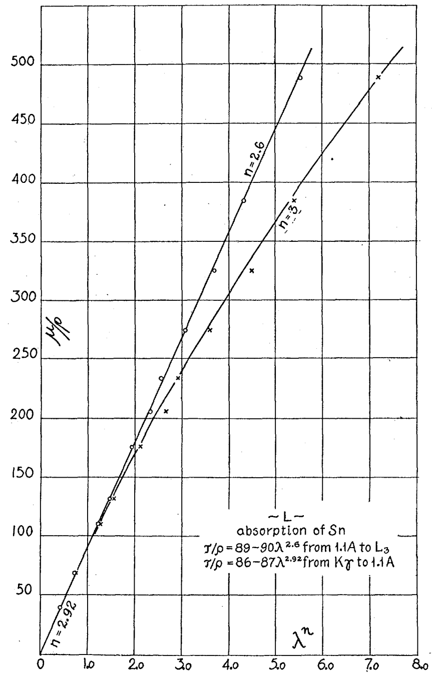

Fig. 4. μ/ρ against λ^n.

The data for the filtered rays from Table III have been analyzed in the same manner as already discussed for the homogeneous radiation. The graphs are shown in Figs. 7 and 8. Aluminum being chosen as the standard substance from which to calculate the effective wave-lengths, cannot be directly analyzed. The only data available to compare it with are two values of Barkla (the Kα+Kβ of Cr, and the Kα+Kβ of Ca), and the data of Miller,[8] who used filtered general rays and calculated his wave-lengths from the minimum quantum wave-lengths, which would of course be too low. Taking the effective wave-lengths of Barkla's rays as 2.18A and 3.22A we obtain for $n=3$ values of $C=14.2$ and 14.4, in very good agreement with the author's for homogeneous rays. If Miller's effective wave-lengths are chosen as about 9 or 10 percent greater than the minimum quantum wave-lengths we obtain an average value

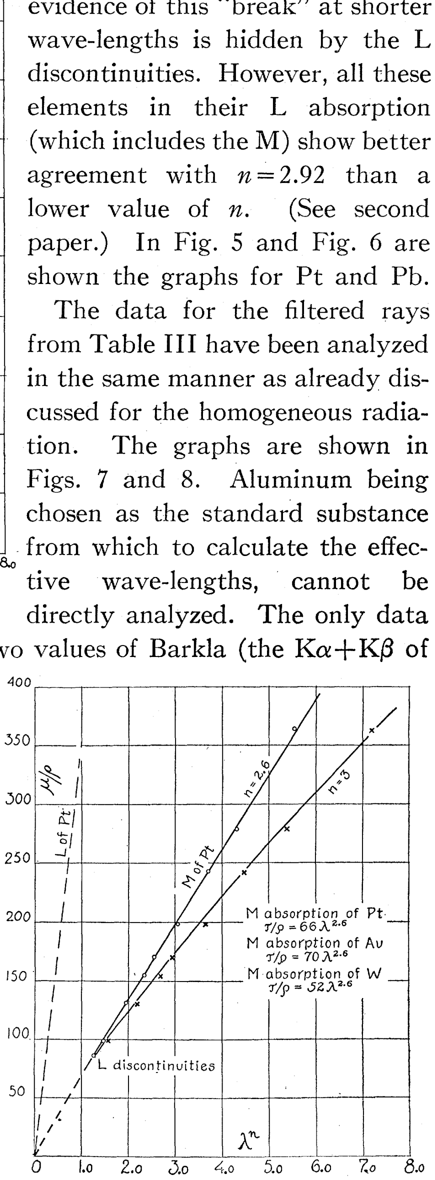

Fig. 5. μ/ρ against λ^n.

---

[8] Miller, Phys. Rev. 8, 329 (1926).

of C about 13.9. On the whole there is very good evidence that Al is continuing to function as it did for shorter wave-lengths.

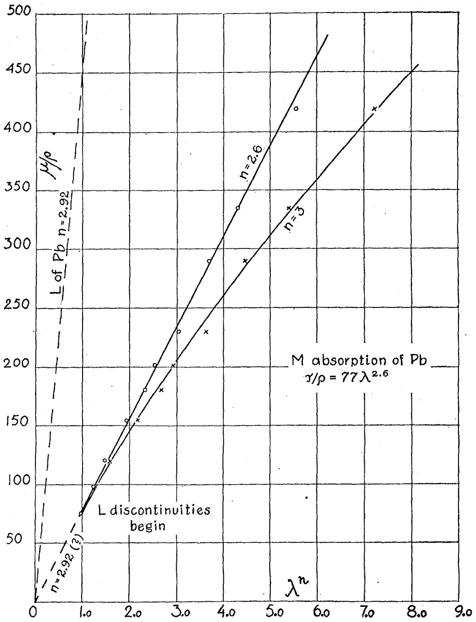

Fig. 6. μ/ρ against λ^n.

The results for Fe, Ni, Cu and Zn form a continuation of the data for homogeneous rays, being absorbed according to the 2.92 law, and having the same values of C. Ag and Sn continue to be absorbed approximately according to $n=2.6$ up to a point where the nearness of the L discontinuities affect the data. In the case of Sn, the M absorption apparently follows once more the 2.92 law, being in this respect similar to what occurred in Cu when it passed through its Kγ. Only one value in the M absorption of Ag was obtained, so its variation in the M absorption cannot be stated, but is assumed to be the same as that in the case of Sn. The data for uranium would indicate that it is being absorbed in the M series according to the $\lambda^{2,6}$ law, and in the N absorption according to the 2.92 law (data rather meager). It is interesting to note that at a wave-length of 4A, uranium is mass for mass more transparent than aluminum (U being in the N, and Al in the K series).

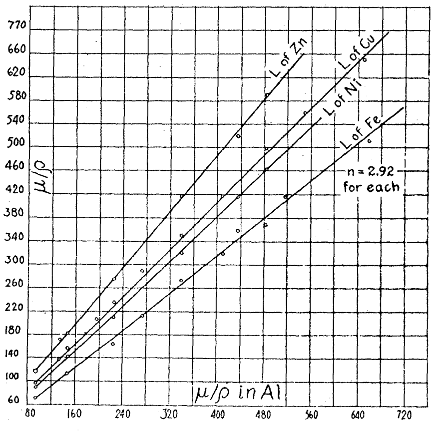

Fig. 7. μ/ρ against μ/ρ_{Al} for filtered rays.

The complete data of this paper and the two previous ones are summarized in Table V, which gives a complete experimental history of the absorption of x-rays from $\lambda .08$ to $\lambda 4.0$A. The range is the approximate region within which the chosen values of N and

C hold good. The values of the discontinuity magnitudes "J" are calculated from the values of $C(J_K=C_K/C_L,\ J_L=C_L/C_M,\ J_M=C_M/C_N)$. Since there are several values of C corresponding to the different series, I have given the greatest and the least.

TABLE V

Summary of analyses of data.
$\mu/\rho=\tau/\rho+\sigma/\rho$, $\tau/\rho=C\lambda^n$, *J is the discontinuity magnitude at the K, L, or M absorption edge,* $J=C_K/C_L$ *or* C_L/C_M *or* C_M/C_N

Element	Range	Series	C	n	J Max	J Min
C	.08 to 2.6A	K	1.38	2.92	Not known	
Al	.08 to 3.93A	K	13.95 to 14.00	2.92	Not known	
Fe	.08 to .6A	K	104 to 108	2.92	10	9.5
	.6 to Kγ	K	102	2.87		
	Kγ to 3.74A	L	10.7	2.92		
Ni	.08 to .8A	K	128 to 132	2.92	9.8	8.8
	.8A to Kγ	K	116 to 118	2.75		
	Kγ to 3.37A	L	13.3	2.92		
Cu	.08 to .85A	K	142 to 144	2.92	9.8	9.1
	.85A to Kγ	K	132	2.70		
	Kγ to 3.74A	L	14.5	2.92		
Zn	.08 to .9A	K	162 to 165	2.92	9.5	8.8
	.9A to Kγ	K	152	2.6 ?		
	Kγ to 3.24A	L	17.25	2.92		
Ag	.08A to Kγ	K	540 to 580	2.92	7.7	7.3
	Kγ to 1.10A	K	73	2.92		
	1.10 to 2.50A	L	76 to 78	2.6	8.0	7.5
		M	10.0	2.92		
Sn	.08A to Kγ	K	650 to 670	2.92	7.6	6.9
	Kγ to 1.20A	L	86 to 87	2.92		
	1.20 to 2.50A	L	89 to 90	2.6	7.5	7.0
	L_1 to ——	M	12 to 13	2.92		
W	.08A to Kγ	K	1840	2.92	6.4 or less	
	Kγ to L	L	285	2.92		
	L to about 1.0A	M	52	2.92	5.5	5.5
	about 1A to M	M	52	2.6		
Pt	.08A to Kγ	K	2000 to 2300	2.92	6.0	
	Kγ to L	L	345	2.92		
	L to about 1.1A	M	65	2.92	5.3	5.2
	about 1.1A to M	M	66	2.6		
Au	.08A to Kγ	K	2000 to 2300	2.92	5.8	
	Kγ to L_3	L	360	2.92		
	L_1 to about 1.1A	M	69	2.92	5.2	5.1
	about 1.1A to M	M	70	2.6		
Pb	.08A to Kγ	K	2000 to 2400	2.92	5.0	
	Kγ to L_3	L	440	2.92		
	L_1 to 1.15A	M	77	2.92	5.7	5.6
	1.15A to M	M	76 to 78	2.6		
U	1.0A to M_5	M	110 to 120	2.6	10 to 12	
	M_1 to ——	N	9 to 10	2.92		

It has already been shown that in the K-L series, J_K is least for atoms of high atomic weight and increases as N decreases. Formerly J_K was only known to Mo, but we have now the values for Zn, Cu, Ni and Fe, which as seen show a still further increase to a maximum value near 10. For the L-M series we have values of J_L for Pb, Au, Pt, W, Sn and Ag, which indicate the same kind of phenomena as has been observed for J_K. The values of J_L seem on the whole to be somewhat greater than the corresponding values of J_K. The one value for J_M, that of U, would indicate that U in passing from the N absorption to the M through its five discontinuities "jumps" 10 or 12 times.

If J_M behaves like J_K and J_L the N-M "jumps" for the lighter elements must be very large. J_K for U is about 4–5, and J_L about 5. It must be considered, however, that this estimated J_M for U is derived from data which may not be free from objection, and that homogeneous rays, and a homogeneous metallic layer of U might give different results. It is merely given here as an interesting possibility from the only data at hand at present.

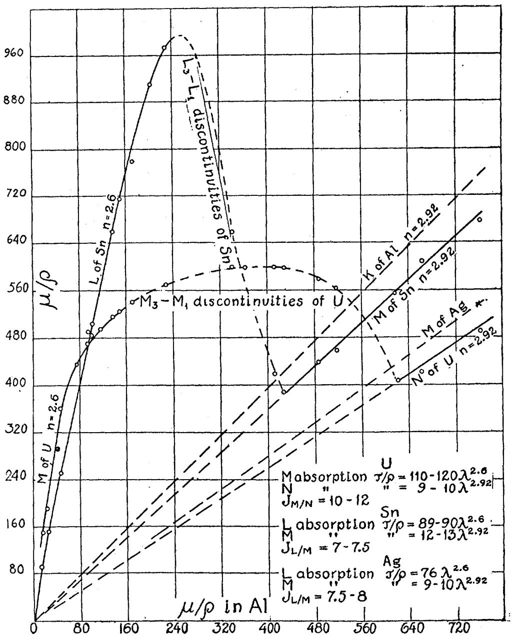

Fig. 8. μ/ρ against μ/ρ_{Al} for filtered rays.

Conclusion

The author is convinced from a study of the data of this paper and that of the two previous ones, that the process of absorption of x-rays over a wide range of λ and N is not simple, and not capable of being always represented accurately by a simple formula of the type $\tau/\rho = C\lambda^n$ in which n and C remain constant, but that each element is a law by itself, changing at certain regions rather abruptly its functionality. This change may occur in the K, L, M, etc., series depending on the atomic number of the substance.

Since both J and n change with the elements, the N^4/A law can at best be only approximate.

The theoretical formulas of absorption, whose derivation necessitates the λ^3 law, would seem to have great difficulty in fitting the present experimental facts. It is too early yet to attempt to give theoretical explanations of the phenomena herein described, but the empirical relations given in the summary may be very useful in predicting the absorption coefficients of any elements not included in the tables.

It can be shown in several cases that the absorption of a low series of an element of high atomic weight is the same as that of the next higher series of some element of lower atomic weight, which again is the same as that of a still higher series of a still lower element. For example, the N absorption of U, the M absorption of Ag or Pd, the L absorption of Mn, and the K absorption of Na or Mg are about the same, and all probably obey the $\lambda^{2.92}$ relation, while the M absorption of U, the L absorption of I, and the K absorption of Ni, are about the same, and follow the $\lambda^{2.6}$ relation.

The author is at present engaged in perfecting means of obtaining more homogeneous rays beyond 2A, and is also awaiting the arrival from Kahlbaum of samples of chemically pure elements in thin sheets, so that the present results can be checked at a few properly chosen wave-lengths by methods of greater precision.

UNIVERSITY OF CINCINNATI,
June, 1926.