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DIRECT MEASUREMENT OF X-RAY MASS SCATTERING COEFFICIENT

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ABSTRACT

The mass scattering coefficients for Li, B, C, H_2O and Na at wave-lengths λ **.32, .43, .54, .66, .79A** were measured directly by collecting the scattered radiation in an ionization chamber surrounding the scatterer, and comparing its intensity with that of a portion of the primary beam going through an auxiliary ionization chamber of equal length along the ray path. The primary beam consisted of the continuous radiation from a Coolidge tungsten tube, filtered through Al and Cu until the spectral width between the half-maximum intensity points was about 25 percent of the mean wave-length. Corrections were made for internal absorption in the scatterer and for the increased absorption of the scattered beam in the ionization chamber due to the Compton effect. The uncertainty in these corrections forms the chief limitation on the accuracy of the measurement. The values of mass scattering coefficient determined are tabulated. The scattering coefficient in all cases starts, for short wave-lengths, below the value computed from Thomson's formula, and slowly increases for longer wave-lengths. For all substances but boron it finally crosses the Thomson value because of gradual entering in of excess scattering.

DIRECT measurements of the mass scattering coefficient of x-rays by measuring the radiation itself were first made by Barkla, and have been repeated with increasing refinement, the more recent determinations being those of Hewlett[1] and Statz.[2] Indirect estimates[3] from the extrapolated intercept (for zero wave-length) of the total mass absorption plotted against the cube (or some power nearly the cube) of the wave-length, have in general given lower values than the direct measurements. Jauncey[4] has recently pointed out that as there are several processes involved in scattering, giving an unmodified beam, a modified beam, and recoil electrons, the scattering coefficient may be separated into three distinct parts. The direct determinations consider only the radiations, while the indirect determinations consider all three parts. The latter should therefore be larger, rather than smaller, than the former.

The present measurements were undertaken to determine the mass scattering coefficient (for the total radiation only) directly by measuring

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[1] Hewlett, Phys. Rev. **20,** 688 (1922).

[2] Statz, Zeits. f. Physik **11,** 304 (1922).

[3] Richtmyer, Phys. Rev. **18,** 13 (1921); Hewlett, Phys. Rev. **17,** 284 (1921); Allen, Phys. Rev. **27,** 271 (1926). Some discussion of this is also given in most articles reporting measurements on the total absorption coefficient.

[4] Jauncey, Proc. Nat. Acad. Sci. **11,** 517 (1925).

simultaneously the radiation scattered at all azimuths, thus obtaining more intensity than by usual previous methods (which also determined the azimuthal distribution), and also reducing the number of readings required. A report of these measurements was presented before the meeting of the American Physical Society last April.[5]

Apparatus and Measurements

The ideal form of ionization chamber for the purpose would be a sphere completely surrounding the scatterer and measuring all the radiation scattered (as in Fig. 1a). On account of the mechanical requirements, however, it is convenient to use only a sector of this sphere, taken symmetrically in such a way that the composition of the radiation measured is a true sample of the radiation that would be measured in the entire sphere. The chamber was mechanically built as shown schematically in Fig. 1b, which also shows the comparison chamber above, for comparing the intensity of the scattered beam with that of the primary beam. The length of each chamber along the ray path was 20 cm. Both chambers were used with air at atmospheric pressure.

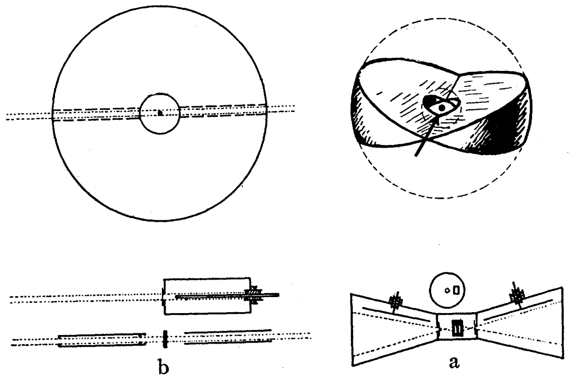

Fig. 1. Ionization chamber.

The electrical connections used are shown in Fig. 2. The electrometer was a Dolezalek type of fairly high sensitivity. The natural leak, which was appreciable because of the large size of the main chamber, was compensated by the small chamber shown here in heavy black, containing uranium oxide and connected in opposition to the main chamber.

The primary beam used consisted of the continuous radiation from a Coolidge tungsten tube, filtered through Al and Cu until the spectral width between the half-maximum intensity points was about 25 percent of the mean wave-length. The estimated spectral distribution of intensity together with the peak voltage and filter, for each of the five beams used,

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[5] Mertz, Phys. Rev. **27**, 795A (1926).

is shown in Fig. 3. The point under each curve shows the effective wavelength as measured from the absorbability of the beam in aluminum, from the data given by Richtmyer.[3] The tube was arranged so that the electron stream was at about 45° to the plane of symmetry of the main ionization chamber, to eliminate corrections which would otherwise be required because of the partial polarization of the primary beam.

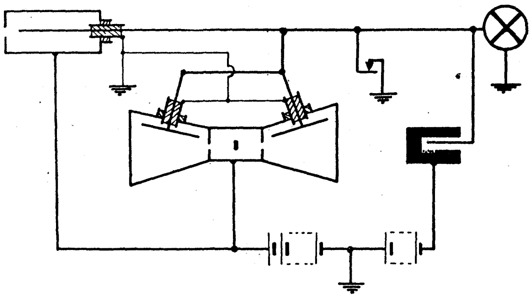

Fig. 2. Electrical connections.

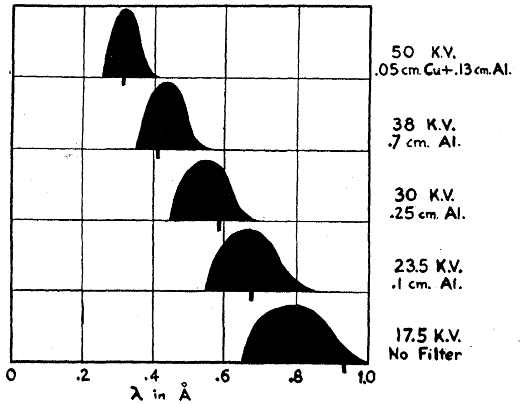

Fig. 3. Spectral distribution of intensity in the primary beam.

The scatterers were in the same form as that used by Hewlett,[1] i.e., a cylinder with axis perpendicular to the primary beam and to the plane of symmetry of the main chamber. The lithium and boron were obtained from Eimer & Amend. The carbon was cut from an arc light carbon containing a total of about 0.05 percent of impurity. The sodium was obtained from the department of chemistry of this university. Those substances which required it were held in capsules similar to those used by Hewlett, but made of thin (.002 cm) paraffined paper. The various samples and capsules varied somewhat in size, but were all about 1/3 cm in diameter, with a length of about 1 cm exposed to the beam.

The readings were taken by measuring the rate of drift of the electrometer needle under the following conditions:

(a) Scattering beam stopped (by suitable piece of lead)

1. Comparison primary beam stopped
2. Comparison primary beam measured

(b) Comparison primary beam stopped

3. Scatterer out of scattering beam (or empty capsule in scattering beam, if a capsule was used to hold the scatterer)
4. Scatterer in scattering beam

The differences between the rates of drift for cases 1 and 2, and for cases 3 and 4, were proportional to the radiation collected by the auxiliary and main ionization chambers, respectively.

COMPUTATIONS

The scattering coefficient was computed from the following considerations:

The total scattered radiation (J_s) collected by the main chamber when a mass m of scattering substance is irradiated by a primary beam of intensity I, is

$$J_s = Im\frac{\sigma}{\rho}\frac{\phi}{\pi}F$$

where φ is the dihedral angle (in radians) between the planes bounding the sector composing the main chamber, and F is a correcting factor, the ratio between the actual scattered energy getting out of the scatterer and the amount which would get out if there were no internal absorption in the sample.

The radiation (J_p) collected by the auxiliary chamber is $J_p = IA$, where A is the cross-sectional area of the beam entering this chamber. Hence

$$\frac{\sigma}{\rho} = \frac{J_s}{J_p}\,\frac{A}{m\phi F}$$

The correcting factor F was estimated according to the method used by Hewlett.* It was computed as

$$F = \tfrac{1}{2}(F' + F'')$$

where

$$F' = 1 - \frac{8}{3\pi}b + \frac{3}{8}b^2 - \frac{16}{45\pi}b^3 \cdots;$$

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* Hewlett, loc. cit.,[1] §VI, Appendix, pp. 706–08 (1922). Several typographical errors in the appendix to his paper may be noted. The algebraic signs of $y \cos \theta$ and of $y \sin \theta$ should both be $-$ instead of $+$ in both his second and fourth equations. The *numerical* coefficient of the a^3 term in his third equation should be $22/45\pi$ instead of $24/45\pi$.

$$F''=1-\frac{4}{3\pi}\frac{b^2-a^2}{b-a}+\frac{1}{8}\frac{b^3-a^3}{b-a}-\frac{4}{45\pi}\frac{b^4-a^4}{b-a}\cdots;$$

$$a=\tau D;\quad b=\mu D=(\tau+\sigma)D;\quad D=\text{diameter of specimen.}$$

The correction varied from less than 2 percent for Li at $\lambda=.32$A, to 31 percent for Na at $\lambda=.54$A.

A small correction (1.5 percent at most) was made for the absorption of air between the scatterer and the window of the main ionization chamber, as the scattered ray traverses this path in addition to the path traversed by the primary beam.

Since, because of the Compton effect, the scattered radiation is softer than the primary, the ionization chamber will absorb the first more readily than the second, and the main chamber will be more efficient than the auxiliary chamber. It is a complicated matter to compute this correction exactly, as the change in wave-length, intensity ratio between modified and unmodified beams, and total intensity all vary with scattering angle. No data are at present available on the last two quantities for most of the substances and wave-lengths here used. A first approximation to the correction was obtained by assuming all of the scattered beam at all angles to be shifted in wave-length by .0242A. The resulting correction varied from -6.7 to -11.8 percent. The error involved in making the approximations above was estimated in the case of carbon at $\lambda=.712$A, for which Hewlett[1] has studied the azimuthal intensity distribution of the scattered beam, and for which Woo[6] has studied the ratio between the intensities of the modified and unmodified beams. The accurately computed correction is -5.8 percent, as compared with a rough computation of -7.3 percent. In view of the nature of the approximations made, this is probably as close agreement as may be expected. On all of the measurements except those taken at $\lambda=.79$A, and on water and sodium, the accurate correction would be expected to be proportionately larger than in the above example, as the proportion of modified to unmodified scattered radiation increases with decreasing wave-length and with decreasing atomic number. It is therefore probable that for most of the data the discrepancy between the actual and the roughly estimated corrections is not much larger than in the example just considered.

The comparatively large value and uncertainty of the corrections for internal absorption and for the Compton effect form the chief limitations on the accuracy of this method for the measurement of the mass scattering coefficient. The correction for internal absorption limits the use of scatterers to elements not much heavier than sodium. For elements of

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[6] Woo, Phys. Rev. **27**, 119 (1926).

higher atomic number the size of the sample must be so small, if the correction is to be kept reasonably small, that the scattered radiation is too weak to measure.

RESULTS

The values of mass scattering coefficient as determined are given in Table I:

TABLE I

Mass scattering coefficient (σ/ρ, in cm^2/gram)

Scatterer $\lambda =$	.32	.43	.54	.66	.79A
Li	.133	.165	.157	.169	.200
B	.154	.162	.169	.165	.179
C	.166	.182	.194	.214	.234
H_2O	.198	.206	.210	.216	.228
Na	.173	.191	.248		

These have been plotted in Fig. 4, together with the data obtained by Hewlett[1] and Statz.[2] There is also included in the plot the value of the scattering coefficient for each substance as computed from Thomson's formula.[7] The scattering coefficient in all cases starts, for short wave-lengths, below the Thomson value, and slowly increases for longer wave-lengths. For all substances but boron it finally crosses the Thomson value because of the gradual entering in of excess scattering. The diminished scattering at short wave-lengths explains why indirect determinations of the scattering coefficient from the extrapolated intercept for zero wave-length of the total absorption curve gives consistently low

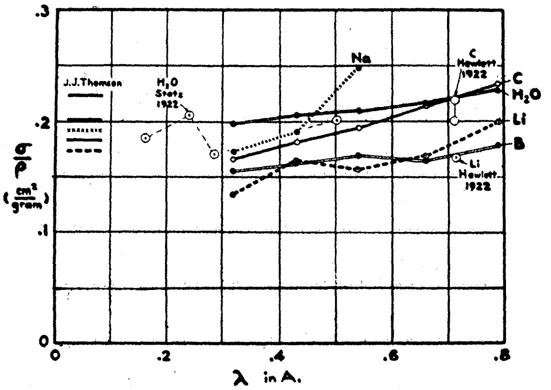

Fig. 4. Mass scattering coefficient.

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[7] J. J. Thomson, Conduction of Electricity Through Gases, p. 325 (1906).

results. For the extrapolated value will be less than the value corresponding to any actual point in the data.

It will be noted that if "true" absorption rigorously proportional to the cube of the wave-length is added to a small amount of scattering of which the increase with wave-length is slower than as the cube, the total absorption plotted against λ^3 will give a curve which is not far from a straight line, but is somewhat concave to the λ^3 axis. This slight concavity has uniformly been observed on all measurements of total mass absorption coefficient. From the present discussion it is probably due to a variation of the mass scattering coefficient with wave-length which is of a lower power than the cube (it is almost linear for the present data). This would seem more likely than that the "true" absorption should vary with a power of the wave-length less than the cube. This is all, of course, entirely aside from the more marked variation which is usually observed near the γ ray region.

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