SMALL-SHOT EFFECT AND FLICKER EFFECT

BY W. SCHOTTKY

ABSTRACT

The Flicker Effect.—J. B. Johnson observed, under certain conditions (oxide coated and tungsten filaments, low frequencies, electron currents high but not high enough for space charge effects), voltage fluctuations across connected resonant circuits which were much larger than the theory of the small-shot effect would lead one to expect. Analyzing Johnson's curves, it is found that this effect increases as the square of the electron current i_0 instead of as the first power as in the case of the small-shot effect. This fact supports Johnson's hypothesis that the effect is independent of the small-shot effect and that it should be attributed to fluctuations in the properties of the surface (flickering) resulting in fluctuations in the electron current. The *trend with the natural frequency* of the connected circuit is likewise different from that observed in the small-shot effect. This can hardly be due to a statistical cause, and one must furthermore assume that there is a time element involved in the elementary process underlying the flicker effect. The *elementary atomic process* underlying the flicker effect is the appearance of an individual foreign atom or molecule in the surface of the cathode, changing the ability of the surface to emit electrons so long as the foreign atom remains. The *influence exerted upon the current by foreign atoms* in the surface may be calculated with sufficient approximation from the electrical image theory and Langmuir's doublet theory. The effect is proportional to the current density, and of such magnitude as to indicate that each foreign atom exerts its doublet effect uniformly over the surface. From the assumptions that the elementary atomic processes are independent of one another and that the length of stay of the foreign atoms in the surface conforms to statistical laws, the *frequency distribution* of the flicker effect is derived and compared with Johnson's experiments, whence it is possible to determine the length of stay of the foreign atom in the surface. The total number of foreign atoms in the surface of an oxide coated filament is computed from Johnson's measurements as being about 1/3 of all the atoms present in the surface; while the average length of stay is estimated from the curves to be about .001 second. The number of foreign atoms appearing per unit time on the surface is some 200 times as great as the number of atoms of residual gas striking the surface per unit time, from which it is inferred that the essential cause of the flicker effect with oxide cathodes lies in a continual exchange of positions by the components of the oxide. For tungsten filaments the mean length of stay is greater than 1/20 second; the number of foreign atoms appearing in the surface per unit time is some 20,000 times smaller than with oxide filaments, so that in this case it would be possible to ascribe the flicker effect to residual gas atoms striking the tungsten surface and remaining on it a longer or shorter time. The *variation of the flicker effect with the square of the total current* follows from the assumption that the current change due to one foreign atom is proportional to the total current. From this assumption also follows the relative independence upon *temperature*. A value is given for the *mean square voltage fluctuation* produced by the effect in terms of the impedance of the connected circuit, and formulas are given for circuits of various kinds. Measurements with two circuits, one being resonant and the other a pure resistance should provide a definite check of the calculated length of stay of the foreign atoms.

INTRODUCTION

THE problem of the small-shot effect in thermionic tubes has, since the first discussion by the author[1] in 1918, made several important advances. The subsequent contributions[2] have had to do partly with correcting and simplifying the calculations and with an analysis of the fundamental basis of the problem, and partly with most gratifying additions to C. A. Hartmann's original experimental results,[3] made with improved methods of measurement and over a wider range of conditions. This later work[4] has led to the discovery of new effects and to and analysis of their significance. The authors of these papers have initiated private discussions of the problem by very kindly sending me their experimental results by letter or manuscript, and since their work has appeared in the PHYSICAL REVIEW I wish to present here the results of some new considerations initiated particularly by the research of J. B. Johnson.

The measurements in the radio frequency range, made by A. W. Hull and N. H. Williams with great skill and precision, show a surprisingly accurate agreement with the original theory (numerically corrected), so that the most probable value of the charge on the electron derived from the results comes within 1 percent of the accepted value, with a mean deviation of 2 percent. Under other conditions, however, in their work and in that of Johnson, certain new effects appeared which require us to revise and supplement the meagre hypotheses of the original theory. There was found, for instance, a remarkable and hitherto not entirely explained deviation from the calculated mean square fluctuation when measurements were made while the thermionic current was limited by space charge. Hull and Williams report that in the space charge region, at radio frequencies, the mean square fluctuation falls to 1/5 of the value calculated from the theory. A similar observation is made by Johnson at 1500 periods per second, where the

[1] W. Schottky, Ann. d. Phys. **57**, 541–567 (1918).
[2] J. B. Johnson, Ann. d. Phys. **67**, 154–156 (1922).
W. Schottky, Ann. d. Phys. **68**, 157–176 (1922);
R. Fürth, Phys. Zeits. **23**, 354–362 (1922);
T. C. Fry, J. Frankl. Inst. **199**, 203–220 (1925);
N. Campbell, Phil. Mag. **50**, 81–86 (1925).
[3] C. A. Hartmann, Ann. d. Phys. **65**, 51–81 (1921).
[4] A. W. Hull and N. H. Williams, Phys. Rev. **25**, 147–173 (1925);
J. B. Johnson, Phys. Rev. **26**, 71–85 (1925).

measurements give 1/3 the expected value. These estimates refer to tubes containing grids, the grid being held at a small negative potential. In the case of two-electrode tubes the suppression of the fluctuations is of quite a different magnitude. Johnson finds in one instance a reduction of the small-shot effect by a factor of fifty.[5] This remarkable behavior has not yet been satisfactorily accounted for, although the most direct hint has been given by Norman Campbell[2] when he suggests that in the space charge region "electrons tend to follow each other at regular intervals." In this connection a fact concerning macroscopic flow of electrons should be emphasized. It is well known that an increase or decrease in the cathode temperature, and therefore in the primary emission, causes but little variation in the space-charge-limited current. Perhaps the tendency to chaotic increase and decrease in number of electrons which causes the small-shot effect may be regarded as simply the equivalent of fluctuations in the emission, so that the space-charge-limited current reproduces these fluctuations in the same reduced measure as those caused by variation in the cathode temperature.

It is not the purpose of this article to deal further with this interesting question. Neither have I much to contribute to the neat experiments of Hull and Williams which seek to reveal the character of secondary emission through measurements on the small-shot effect.[6] I am rather concerned with the explanation of a third fundamental effect, discovered by J. B. Johnson in the region of low frequencies and large temperature-limited electron currents. This effect manifests itself as a hundred fold to a thousand fold increase in the value of the small-shot effect measurements when the circuit associated with the tube is of low natural frequency. Johnson has found this effect both with tungsten filaments and with oxide coated filaments. He has investigated its relation to the frequency and to the emission current, and has proved that for a given tube the effect[7] depends upon these two variables alone. He shows that the divergence of this effect from the small-shot effect is the greater the greater the specific emission from the filament and the lower the frequency. The independent character of this effect leads him to think that it is not caused by the atomic structure of electricity,

[5] This reduction is not the result of the lowering of resistance of the tube in the space charge region, a resistance which makes for high damping of the resonant circuit. This effect is taken into account in the computation. We may assume also that the authors considered sufficiently the variation of amplification with reference in the cases of high damping where contribution to the total effect are made from a considerable frequency range.

[6] See also N. Campbell, l.c.[2]

[7] Or rather on the ratio of the observed to the calculated effect.

but rather by fluctuations in the properties of the cathode surface, resulting in greater irregularities than those associated with the small-shot effect. If we had to do with emission of light instead of electrons, we would speak of a chaotic variation of light intensity taking place over the surface of the cathode, a phenomenon which we should describe by the word "flicker." If, as I shall endeavor to further confirm, Johnson's explanation of the phenomenon is the correct one, then we may use the analogy and call the new effect the "flicker effect."

Variation of Flicker Effect with Current

Let us attempt, with the aid of Johnson's results, to differentiate more clearly between the two effects. Fig. 1 reproduces the observa-

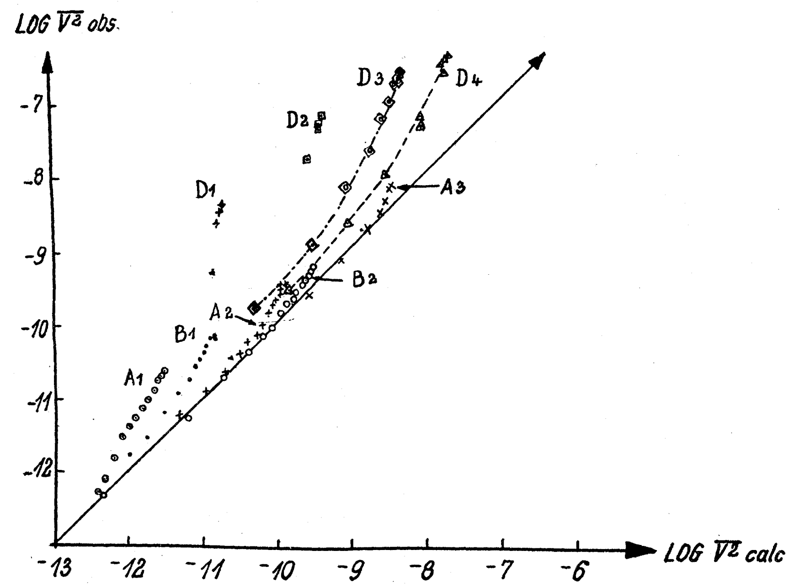

Fig. 1. Reproduction of observations by Johnson

tions collected by Johnson in his Fig. 10.[4] The letters $A-D$ signify different series of measurements with different inductances in the measuring circuit. The natural frequency of the circuit was lowest for the series D, and highest for A_3, the frequency range being from 160 to 33,000 radians per second. The current emission from the oxide coated filament ranged from .1 to 5 milli-amperes. The abcissa are the mean square voltage fluctuations at the terminals of the resonant circuit, as calculated from the current and the circuit constants by the formula for the small-shot effect; the ordinates are the corresponding observed values.

We see that the lower part of the curve has a tendency to follow the theoretical line $\log \overline{V^2}_{obs} = \log \overline{V^2}_{calc}$. We conclude, so far as the observations go, that at every frequency of the resonant circuit there is a region of sufficiently low current in which the small-shot effect contributes by far the most to the fluctuations. This result, even though further confirmation is desired, seems to me to indicate positively that the theory of the small-shot effect holds also in the range of low frequencies provided the rate of emission is low enough.

Let us examine the deviation from this simple relation which sets in at higher values of the emission current. As $\overline{V^2}_{calc}$ increases,

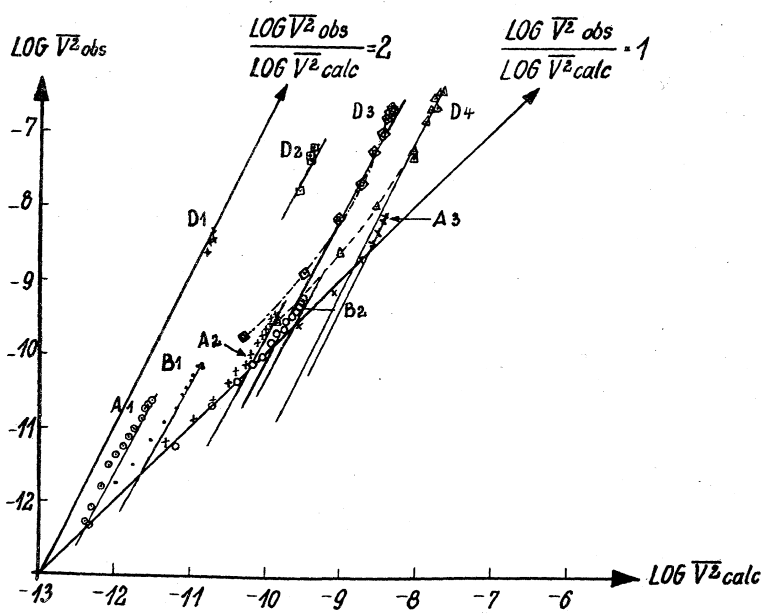

Fig. 2.

that is, as the current becomes larger, all of the observational curves depart upwards from the theoretical curve based on the theory for the small-shot effect. The nature of this departure I have tried to show somewhat roughly in Fig. 2. It seems not to conflict with Johnson's measurements to suppose that as i_0 increases the curves again approach the slope of straight lines with twice their initial slope. This means, however, since the scales are logarithmic, that at the higher values of $\overline{V^2}_{calc}$ the observed quantity $\overline{V^2}_{obs}$ becomes proportional to the square of $\overline{V^2}_{calc}$, and that therefore in this region where the small-shot effect contributes little the *observed mean square voltage fluctuations become proportional to the square of the emission current.*

If this law is found to hold, it will give strong support to Johnson's belief that we here have a new effect which is independent of the small-shot effect. It may readily be supposed that the curves of Fig. 1 result from the superposition of two effects, one proportional to the first power and the other to the square of the space current. If this is so, the magnitude of the flicker effect may be obtained either by subtracting $\overline{V^2}_{calc}$ from the observed curves, or by working in the region where the contribution of the small-shot effect to the fluctuations is negligible and where the proportionality with i_0^2 holds.

Interpretation of the Flicker Effect: Frequency Variation

Is the proportionality of the flicker effect to i_0^2 to be expected if the origin of the effect is what Johnson assumes? If, in accordance with this conception, the fluctuations are caused by spontaneous changes in the cathode surface which alter the emission, then clearly the proportionality with i_0^2 must hold. We have only to suppose that as i_0

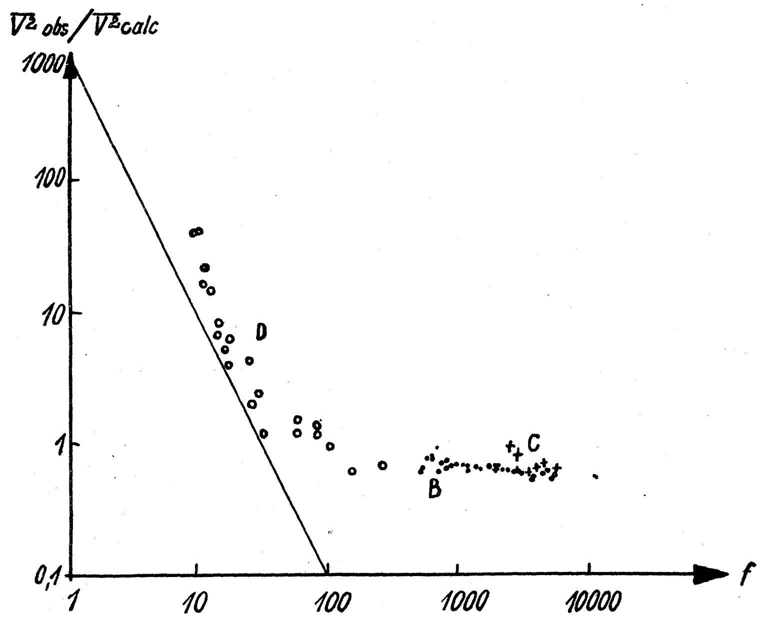

Fig. 3.

varies, the character of the surface fluctuations does not change; and that the absolute value of the corresponding fluctuations in the emission current remains proportional to i_0, or, in other words, that alteration in the surface condition is always accompanied by the same relative change in emission. The second assumption agrees completely with all that we know about variation of electron emission. It is substantially correct for all the possible sources of variation named by Johnson, such as evaporation, diffusion, chemical action, structural changes, bombardment by gas ions, etc. Less certain is the first assumption, that regarding the invariant character of the surface changes as i_0 varies. I shall return to this point later in the discussion of the theory.

It was the dependence of the observed effect upon frequency which suggested to Johnson more definite conclusions regarding the origin of the spontaneous perturbation in the emission from the surface. This frequency variation is shown, to the same logarithmic scale as

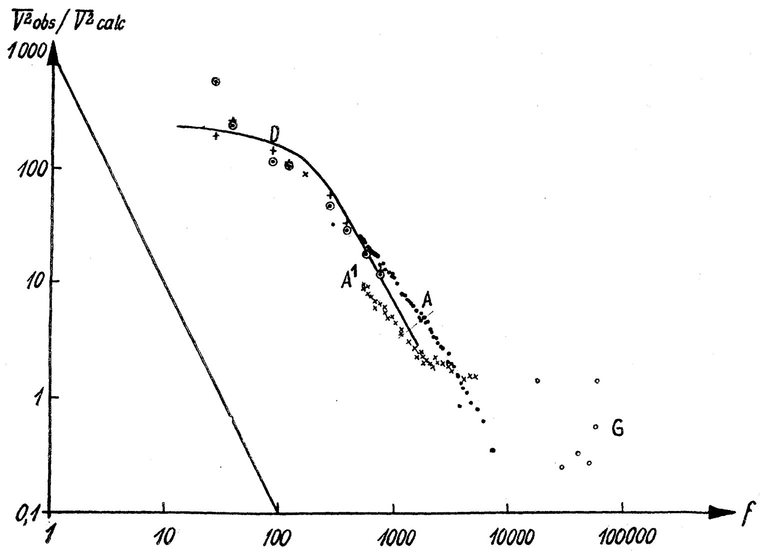

Fig. 4.

before, in Figs. 3 and 4, taken from Johnson's paper. Fig. 3 corresponds to a tungsten filament, Fig. 4 to an oxide coated one, and all observations were made at an emission of 5 milli-amperes. (The meaning of the solid line drawn in Fig. 4 will be explained later). It is to be observed that in the region where the flicker effect predominates, where $\overline{V^2}_{obs}/\overline{V^2}_{calc} >> 1$, the curves rise rapidly with decreasing frequency. The rate of rise seems to be given approximately by the straight lines in the figures, drawn with the slope $-1/v^2$ where v is the frequency. This rapid rise of the flicker effect with decreasing frequency and increasing period of the circuit, as well as the high absolute value of the observed fluctuation, leads Johnson to suppose that we are dealing with a phenomenon which is not a result of the emission of individual electrons, but which is rather of a more intensive and extensive nature, affecting at once the emission of a great number of electrons.[8] Whence, however, the rapid rise of the effect with the period of the circuit? Johnson sees the reason simply in the fact that fluctuations having greater intensity or area also demand a longer duration than feeble or less extensive ones. Illuminating as this argument is, it requires a little closer scrutiny. We may ask, for instance, why this argument cannot similarly be used for the small-shot effect, in which also there is greater opportunity for large deviations from the average during long time intervals than in a very short time. Apparently some explanation other than the above must be found for the frequency variation of the flicker effect. Indeed, there is a difference between the two phenomena deducible directly from Johnson's interpretation. In the small-shot effect the number of transferred individual charges obeys the laws of probability. In the flicker effect, on the other hand, the primary variations assumed by Johnson depend not upon the number of charges but upon *the condition of a small surface region during the time in which it emits a certain increased or decreased current.* The number of charges transferred in consequence of this condition depends furthermore upon the duration of the condition. In general, any characteristic of the phenomenon having the dimension of time, and being therefore in some way relatable to the natural period of the resonant circuit, will give rise to a frequency variation, in contrast with the small-shot effect which involves no such time constant.

This general inquiry could perhaps be pursued even a little further. If the above characteristic time is large compared to the period of the

[8] This hypothesis, by which I attempted to explain Hartmann's results, I had abandoned even before the appearance of Johnson's article. Numerical calculations based on the heat conduction constant for tungsten convinced me that any such cooling is much too evanescent to influence the electron emission. In a letter to Dr. Hull on this subject in May, 1925, I also took exception to the statements made by Hull and Williams regarding the apparent capacity effect associated with the electron current, discovered by Mr. Hartmann. We concluded that the effect was caused by a lag in the establishment of temperature equilibrium as the space current was altered. Hull and Williams cite their own results in refutation of the existence of such an effect, the capacity C_i being supposed by them to represent constant capacity shunted across the terminals. This supposition, suggested no doubt by Hartmann's notation, is not in agreement with our explanation. The thermal lag theory demands that the effect should vanish at high frequencies in agreement with Hull's observations. Dr. Johnson, also, has sent me some results of measurements upon the tubes he used (oxide coated filaments). These tend to show a capacity effect at low frequencies decreasing toward higher frequencies. They also show remarkable hysteresis phenomena. These effects, easily to be derived from the data of emission and heat conduction, may be worthy of further experiments.

circuit then the disposition of the surface to emit a positive or negative current increment is very ineffectually transmitted to the resonant circuit, since only the more or less abrupt beginning or end of the current increment contributes much to the induction of oscillating current in the circuit. The action has an analogy in the interruption of an alternating current at time intervals that are great compared with the period of the circuit, the residual direct current being very small. The effective part of the current increments may easily be shown to be proportional to the period τ of the resonant circuit when τ is small compared with the duration of the current increment. Consequently, the mean square voltage amplitude at the terminals of the circuit will show a variation with τ^2, in contrast with such phenomena as the small-shot effect which do not have their origin in a characteristic time constant that is large compared with the period of the circuit. This seems to furnish a plausible explanation for the frequency variation observed by Johnson.

The same arguments lead to yet another conclusion which is not so directly evident from the data in Figs. 3 and 4. When the period τ approaches and eventually exceeds the assumed characteristic time constant, the cause of the increase of the observed effect with rising period τ gradually disappears, and the variations in the state of the surface result merely in increments to the electron current. This condition seems to be quite analogous to that producing the small-shot effect, except that the quantity of charge emitted as the result of a single surface fluctuation may be considerably greater than that of one electron. We may therefore at sufficiently low frequencies expect the frequency curve again to bend over and approach a horizontal course toward zero frequency. Perhaps it is bold to surmise evidence for such an inflection toward the horizontal in the series of points D of Fig. 4. If further experiments should confirm this surmise, such curves would give directly a measure of the charge associated with the individual fluctuations. From Fig. 4 we should expect that groups of 100 to 1000 electrons are involved, corresponding to the 100 to 1000 fold increase in the observed effect over the small-shot effect. Furthermore, if the shape of the solid curve should turn out to be correct, then the value of the period τ at which the curve bends toward the horizontal would give directly the value of the assumed time constant. Judging from Fig. 4 one estimates this time constant to be $1/2\pi \times 1/100$ sec.[9]

[9] We evidently must consider not the whole period but only such a fraction as $\tau/2\pi$ over which the phase is roughly constant.

Fundamentals of the Flicker Effect

After what has already been said about the small-shot effect it is hardly possible to suppose that the time constant which is characteristic of this effect can have any connection with the spatial or secular *statistics* of the process; in other words, it can not be something which concerns simultaneously a large number of independent invididual events. Seemingly, we must search for a *time constant which is defined by a single event or process, independent of other elementary fluctuation events.* If this is so, what type of elementary independent events might compose the surface perturbation which manifests itself as flicker effect?

Almost all the phenomena named by Johnson involve the appearance of foreign atoms on the surface.[10] The far-reaching effects of foreign atoms in the surface of an electron emitter have been worked out quantitatively, particularly by Langmuir. We know that in certain circumstances it is possible for a single layer of atoms to raise or lower the emission a thousand fold. It seems plausible to suppose, therefore, that the origin of the flicker effect discovered by Johnson lies in *fluctuations in the surface layer of foreign atoms.* These atoms reach the surface by diffusion from within, or by deposit from the remanent gas of the tube. With compound materials such as barium oxide the surface may be changed also by partial evaporation of one component or by spontaneous rearrangements of both.[11] These processes probably consist of the advent or departure at the surface of a single atom or a small group of atoms in the molecular condition. The presence of a few foreign atoms thus furnishes a certainly permissible and for a first calculation sufficient basis, in the assumption that the *coming or going of separate foreign atoms or molecules may be regarded as elementary* events which are independent of one another.

We have thus, as in the case of the small-shot effect, arrived at the *atomic nature of matter* as the ultimate cause of the observed fluctuation. It is a short step to give a definite interpretation to the time constant characteristic of the elementary processes. Presumably we have to do with the duration τ' of the existence of an individual atom or molecule in the surface, for the change in emissivity caused by a single

[10] The direct contribution to the current from ion bombardment and from secondary electrons is clearly of little moment, and the same is true of the temperature effect. It is therefore only the capture of gas ions on the surface that matters. The effect of recrystallation I think is negligible.

[11] On the other hand, the evaporation of an atom from a pure substance, such as a tungsten atom from a tungsten filament, can have little influence on the emissivity at that particular place.

foreign particle would probably be co-extensive with the presence of the particle in the surface. The particle would give rise to a steady change γ in the average current, and this process is identical with the one with which we attempted to explain the variation of the flicker effect with the frequency.

For the determination of γ we shall temporarily assume only the general relation

$$\gamma = F i_0 \ ,$$

where i_0 is the space current and F is a very small numerical factor, of the order of magnitude 10^{-16}. The determination of F with the aid of Langmuir's extension to the electrical image theory will be considered after the discussion of the fluctuation processes. Suffice it to say that F is the same for all foreign atoms of the same type, and is therefore constant for definite values of τ' and i_0.

If different types of foreign atoms are present in the surface, then the typical elementary currents and factors must be considered separately:

$$\gamma_1 = F_1 i_0 \ , \ \gamma_2 = F_2 i_0 \ , \ \text{etc} \ .$$

This generalization will be omitted, however, and the presence of only one type of foreign atoms will be postulated.

Mathematical Treatment of the Fluctuation

The procedure proposed in the following is closely related to that originally used in calculating the small-shot effect. We make a Fourier analysis or spectral resolution of the current fluctuations and we ascertain the action upon a connected current circuit by summing the actions of their various harmonic components upon it. This procedure, though it is perhaps not mathematically the simplest and perhaps will later be replaced by a more elegant one (as in the case of the small-shot effect), has the advantage of greater generality and in addition makes it possible to carry out an entirely separate investigation of the fluctuation process itself and of its action upon the circuits. For the engineer this method is convenient in that the only properties of a connected circuit which enter into the calculation are its directly measurable impedance values (within certain frequency ranges). Finally as regards T. C. Fry's objection[2] that unordered processes of this type admit of no Fourier resolution and possess no definite spectrum, I may say that we are not concerned with the individual Fourier coefficients but with their root mean square values within small frequency ranges; and these mean values have a definite physical meaning as I emphasized in speaking of the small-shot effect[1] and as we shall see similarly in dealing with the flicker effect.[12]

Fig. 5 represents the electrical connections of the experimental tube and circuit, all batteries being omitted. Let R represent the impedance of the tube and Z' the impedance of the connected circuit (both of these impedances will in general be complex) and represent

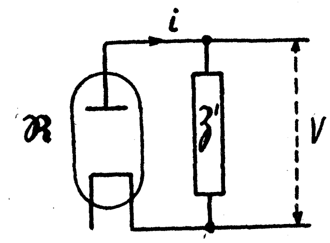

Fig. 5.

by D_k the amplitude of the current flowing in Z'; and by V' the voltage amplitude across the ends of Z' (which in general has a phase displacement relative to D_k). Then we have

$$V'=D_k\, Z'. \tag{2}$$

Now we introduce the primary current in the tube of amplitude A_k, whose phase is generally shifted with respect to D_k. This is by definition the current which would flow in the tube under the action of any forces producing alternating current if the tube were short-circuited so that there were no reactions upon the current flowing through it. This amplitude we can take as real, referring all phases to that of the primary current. We then evidently have

$$A_k=D_k+V'/R\ ; \tag{3}$$

from (2) and (3), we have

$$\begin{aligned} A_k &= V'(1/R+1/Z') \\ V'^2 &= A_k^2\frac{1}{(1/R+1/Z')^2} \quad \text{or} \\ V'^2 &= A_k^2 Z^2, \end{aligned} \tag{4}$$

in which Z represents the combination resistance of the circuit composed of the tube plus the connected circuit, each measured from the two electrodes of the tube. This procedure eliminates the problem

[12] Certain changes in the previously published argument indicated by the substitution of the symbols a and c instead of A_k and C_k are due to detailed explanations and criticisms in letters from Dr. Fry, for which I am deeply grateful.

as to whether the tube capacity should be considered as belonging to the tube or to the external circuit.

The mean value over time of the square of the voltage is $\frac{1}{2}V'^2 = \frac{1}{2}A_k^2 \times Z^2$, in which the symbols V' and Z now signify the *absolute values* of the quantities previously designated. Suppose now that there is a very great or infinite number of partial current amplitudes A_k and B_k which correspond to various independent frequencies as in a Fourier analysis: then at the ends of the tube there will be a resultant fluctuation of voltage of which the root mean square value is obtained by simple addition of the root mean square values of the components. Representing by V the momentary value of the voltage fluctuation at the terminals of the tube (apart from constant voltage differences) we have

$$\overline{V^2} = \frac{1}{2}\sum_1^\infty A_k^2 Z^2 + \frac{1}{2}\sum_1^\infty B_k^2 Z^2. \tag{5}$$

In this equation A_k, B_k and Z are in general functions of frequency which must be known in order to determine $\overline{V^2}$. We suppose this function already known for Z by calculation or measurement. A_k and B_k remain to be determined.

It follows at once that if the readings of the measuring apparatus are not proportional to B'^2 for every frequency, we have to deal with a quantity

$$\overline{V_e^2} = \frac{1}{2} \sum \beta^2 (A_k^2 + B_k^2) Z^2 \tag{6}$$

in which β stands for the coefficient by which V' must be multiplied to give the observed reading and may depend on the electrical connections and on the properties of the measuring instrument and must be known as a function of frequency.[13]

Spectrum of the Flicker Effect

For calculating the equations (5) and (6) we select a frequency interval so small that Z and β can be regarded constant for this range. This interval, however, must not be regarded as infinitely small but as containing many Fourier components, which is possible because we take the fundamental period T of the Fourier analysis as very large

[13] This fact, which I have called attention to before (Ann. d. Physik **68,** 169 (1922)), was used by Hull and Williams in reducing their measurements. This reference contains also the generalization of the small-shot effect formula which was later introduced by Fry and used by Johnson.

and continue to employ the representation by sums merely for the sake of simplicity. Now we identify the partial amplitudes A_k and B_k with those designated by the same symbols in the Fourier series

$$j = \sum_{k=1}^{\infty} A_k \cos \omega_k t + \sum_{k=1}^{\infty} B_k \sin \omega_k t$$

in which j stands for the primary fluctuation current of the flicker effect, that is to say, the primary current minus its constant component. Then the quantity $\sum_{\omega}^{\omega+\Delta\omega}(A_k^2+B_k^2)$ is the sum to be taken for any number of terms and can be written $(\overline{A_k^2}+\overline{B_k^2})\Delta k$, in which the overlined symbols are the mean values of A_k^2 and B_k^2. Consequently we are concerned, as we should be, only with the mean square values of the amplitudes of enormously many components. Considering the unordered character of the process, we naturally have $A_k^2 = B_k^2$ and therefore we may now write Eq. (5) or (6):

$$\overline{V^2} = \sum_{k=1}^{\infty} \overline{A_k^2}\, Z^2\, \Delta k \tag{7}$$

in which

$$\overline{A_k^2} = \frac{4}{T^2} \int_0^T (j \cos \omega_k t dt)^2 \,,$$

T being the duration of the chosen Fourier period. In calculating the small-shot effect it was shown that the mean square value of the above integral in that case has the value

$$\tfrac{1}{2} T\, e i_0$$

so that

$$\overline{A_k^2} = \frac{2}{T} e i_0 \text{ and } \overline{A_k^2}\ \Delta k = T \cdot \Delta v \cdot \frac{2}{T} e i_0 = 2 e i_0 \Delta v$$

Hence, in going over to Fourier integrals it is permissible to represent the spectrum by an expression $a^2 dv$, wherein $a^2 = 2ei_1$ represents the sum of the mean square values of the current amplitudes belonging to the cosine vibrations per unit of frequency change, and an equally large value is obtained for the sine vibrations.

Consequently $c^2(=4ei_0)$ is in the case of the small-shot effect the square of the average effective amplitude of the small-shot current per unit of frequency. This quantity (which corresponds to the coefficient introduced by Planck into the radiation formula) is independent of frequency, that is to say, the spectrum of the small-shot effect shows, for all periods which are not smaller than the time of transit of an electron, a perfectly uniform distribution over all frequencies affected.[14] We now have the task of determining the corresponding spectrum distribution $\overline{A_k^2}$, a^2 or c^2 for the flicker effect and we know already from the *measurements* that this distribution cannot be independent of frequency.

First of all we calculate the value of A_k^2 for a single value of k,

$$A_k^2 = \frac{4}{T^2}\left(\int_0^T j \cos \omega_k t dt\right)^2 .$$

In calculating the squared integral, which can be written

$$\int_0^T \int_0^T j\, dt\, j'\, dt' \cos \omega_k t \cos \omega_k t' ,$$

we first select a time element dt corresponding to a time t'. The contribution of this time element is

$$\cos \omega_k t_1\, dt \int_0^T j(t_1) \cdot j(t') \cos \omega_k t'\, dt' .$$

In accordance with the previous discussion, j may be expressed by

$$j = n\gamma = (N - N_0)\gamma .^{15} \tag{9}$$

In this equation N represents the number of foreign atoms or molecules on the surface of the cathode at the time in question; N_0 represents the time average of N, and n represents the momentary deviation from the time average. Eq. (9) will according to our assumptions always be valid when the fluctuations in number of foreign molecules on a spot of the surface are on the average so small that the partial value of the emission current flowing out of this spot fluctuates relatively little. Later we shall be able to prove this more convincingly.

[14] This result seems to contradict the solution given in my first paper, according to which the mean square of the fluctuation current flowing during period τ should depend upon τ according to $\overline{j_\tau^2} = ei_0/\tau$. However, c^2 is a quantity measured per unit change of frequency. If we form the equation

$$c^2 \Delta v = (c^2/\tau)(\Delta\tau/\tau)$$

we recognize that $\overline{j_\tau^2}$ is proportional to the integral for the squares of the amplitudes for a constant fraction $\Delta\tau/\tau$ of the period in question. Closer agreement cannot be demanded under the circumstances.

[15] The assumption contained in this substitution, that the current due to a foreign atom during the sojourn of the atom in the surface is constant in time, naturally does not include the small-shot effect. However, it will be possible to calculate the two effects independently.

Using Eq. (7), the integral which must be computed assumes the form

$$\gamma^2 \int_0^T n(t_1)n(t') \cos \omega_k t' \, dt' \; . \tag{10}$$

In the second integration the variable t runs through all values from 0 to t for a given time t_1. Since however, many cases will occur in which n is numerically equal to $n(t_1)$ we need not calculate any individual value but only the mean value of the integrand for a given difference between the times t_1 and t'. First therefore, we are concerned with the mean value $\overline{n_1 n'}$ for a given value of the quantity $t'-t_1$. If this quantity is 0 we have $n'=n_1$. As the difference between t_1 and t' becomes greater however, the less becomes the coherence of n_2 and n' and for somewhat larger values of time n' can quite as well have the same as the opposite sign to n_1; consequently, we need take account only of residual effects.

Now in order to make an exact calculation of the quantity $\overline{n_1 n'}$ for any arbitrary value of $t'-t_1$, at many repetitions of the "instants t_1" (so we shall designate the times at which $n=n(t_1)$) we must introduce the probable time of sojourn of the foreign molecules in the surface. If we represent by $1/\alpha=\tau'$ the average time of sojourn of the molecules in the surface, it follows from the definition that out of N_1 molecules present at a given time, the number present at a time Δt later will be $N_1 e^{-\alpha\Delta t}$. The molecules which are not present at both the beginning and the end of the interval have no particular relation to one another. They can influence the numbers n_1 and n' either in the same sense or in opposite senses. Consequently, so far as I can see, we need only consider the foreign molecules which are present at both the beginning and the end of the interval, when computing $\overline{n_1 n'}$. We obtain

$$n_1 n' = (N_1 - N_0)\,(N' - N_1) \; .$$

If we take the average over many "instants t_1," holding n_1 and Δt constant we have

$$\overline{n_1 n'} = (N_1 - N_0)\,(N' - N_0) = (N_1 - N_0)\,(N_1 \epsilon^{-\alpha\Delta t} - N_0) \; .$$

Introducing the relation $N_1 = N_0 + n_1$ we get

$$\overline{n_1 n'} = n_1^2 \epsilon^{-\alpha\Delta t} + (1 - \epsilon^{-\alpha\Delta t}) n_1 N_1 \; , \tag{11}$$

since $\bar{n}$ is equal to 0 by definition. Inserting this result into Eq. (10) and making the substitution

$$\cos \omega_k t' = \cos \omega_k t_2 \cos \omega_k \Delta t \pm \sin \omega_k t_1 \sin \omega_k \Delta t$$

the integral takes the form

$$\begin{aligned} &\gamma^2 n_1^2 \cos \omega_k t_1 \int_0^T \cos \omega_k \Delta t\, \epsilon^{-\alpha \Delta t} dt' \pm \\ &\gamma^2 n_1^2 \sin^2 \omega_k t_1 \int_0^T \sin \omega_k \Delta t \epsilon^{-\alpha \Delta t} dt' \end{aligned} \tag{12}$$

plus an expression dependent on $n_1 N_0$ which will later be seen to vanish. In the two integrations of Eq. (12), we may extend the integration from $-\infty$ to $+\infty$ for any value of t_1 without perceptible error, since the integrand of t_1 soon becomes immeasurably small in both directions and since the "instants t_1" which lie near the times 0 and T play no part if T is made sufficiently great. Integrating both integrals from $-\infty$ to t_1 and setting $dt' = d(\Delta t)$, we get for the first integral

$$-\int_{-\infty}^{\Delta t=0} \cos \omega_k \Delta t\, \epsilon^{-\alpha \Delta t} d(\Delta t) = \frac{\alpha}{\alpha^2 + \omega_k^2}.$$

Carrying the integration on to $+\infty$ we double this value. The value of the sine integral is the sum of pairs of equal and opposite terms, and vanishes. Consequently the expression (12) becomes simply equal to

$$\gamma^2 n_1^2 \cos \omega_k t_1 \frac{\alpha}{\alpha^2 + \omega_k^2}.$$

If we make $t_1 = t$ and integrate over t (Eq. (8)) we obtain:

$$-\int_0^T \cos^2 \omega_k t\, n^2 \frac{\gamma^2 \alpha}{\alpha^2 + \omega_k^2} dt = \overline{n^2} \frac{\gamma^2 \alpha}{\alpha^2 + \omega_k^2} \cdot T/2 \tag{13}$$

since a mean value of each $\overline{n^2}$ for a given value of the cosine can be set equal to the general mean value, and the integral of the square of the cosine gives a value $T/2$.

Since the term in Eq. (11) which is proportional to $n_1 N_0$ contributes nothing to the integral (8) in the corresponding calculation, it follows that in the averaging corresponding to Eq. (13), the mean value of $\bar{n}$ occurs, and this by definition is zero. However, we cannot make this assertion for any *particular term* A_k^2 of the Fourier resolution any more than we could for the small-shot effect (*loc. cit.* [1], p. 559), as it might perfectly well happen that the quantities n have incidental coherence with the cosine function. It is probable, therefore, that a Fourier resolution for the individual coefficients does not lead to definite results but since we are concerned only with the mean values of A_k^2 over very many or infinitely many values of k, I regard the above calculation which gives the summary and definite result as justified. I hope that from the preceding it will be possible some time to deduce the results which follow in a much simpler manner.

Substituting (13) in the equation for A_k^2 and keeping in mind that this symbol stands for a mean value derived from many values of k, we have

$$\overline{A_k^2} = \frac{2}{T}\,\overline{n^2}\cdot\frac{\gamma^2\alpha}{\alpha^2+\omega_k^2}\ . \tag{14}$$

If we form the equation

$$A_k^2 \Delta k = A_k^2 T \Delta v\ ,$$

we obtain from (14)

$$A_k^2 \Delta k = 2\overline{n^2}\gamma^2 \frac{\alpha}{\alpha^2+\omega_k^2}$$

in which the symbol ω_k stands for a mean value of the different values of k falling within the interval Δv. Passing to the limiting case of the interval by putting $T=\infty$, and writing dv for Δv and $\omega = 2\pi v$ for ω_k, then we obtain the spectrum distribution of the flicker effect in the form

$$a^2 dv = 2\overline{n^2}\gamma^2 \frac{\alpha}{\alpha^2+\omega^2} dv\ .$$

Up to the present we have considered only the cosine function of the Fourier analysis. The sine functions naturally give the same amount so that we finally obtain the total value per unit of frequency change of the average Fourier coefficient:

$$c^2 = 4\overline{n^2}\gamma^2 \frac{\alpha}{\alpha^2+\omega^2}\ . \tag{15}$$

Discussion

Trend with frequency. In order to remain in contact with the experiments, we must first say something about the ratio of c^2 to the magnitudes observed by Johnson. Johnson referred all measurements to the formula for the small-shot effect after he had established that the constants of the resonant circuit enter into the present effect in the same manner as into that; he plotted the quantity $\overline{V^2}_{obs}/\overline{V^2}_{calc}$ which according to the theory is given by

$$\overline{V^2}_{obs}/\overline{V^2}_{calc}=(\overline{V_F^2}+\overline{V_S^2})/\overline{V_F^2}=\overline{V_F^2}/V_S^2+1 \tag{16}$$

and in which V_F stands for the contribution made by the flicker effect V_S for that of the small-shot effect.

Now in the following section we shall see that in sufficiently selective circuits such as were used in the principal experiments of Johnson, the following equality holds:

$$\overline{V_F^2}/\overline{V_S^2}=\overline{c_F^2}/c_S^2$$

in which c_F and c_S stand for the amplitudes of the natural frequencies of the resonant circuit in the two cases. On account of (16) we may write

$$c_F^2/c_S^2=\overline{V^2}_{obs}/\overline{V^2}_{calc}-1 \tag{16a}$$

and since we know the quantity $c_S^2=4ei_0$, the curves obtained by Johnson give the absolute value of c_F almost directly. Dropping the subscript F, we have

$$c^2=4ei_0\left(\overline{V^2}_{obs}/\overline{V^2}_{calc}-1\right) \tag{17}$$

in which the second term on the right hand side may be omitted when $\overline{V^2}_{obs}$ is large compared to $\overline{V^2}_{calc}$.

Let us now turn back to the theoretical statements about c^2 and discuss the trend with frequency to be expected according to the theory. In Eq. (15), $\overline{n^2}$ is independent of the frequency and so also are γ and α. We find thus that, in contrast to the small-shot effect in which the corresponding expression c^2 had the value $4ei_0$, *the spectrum of the flicker effect depends on the frequency*; obviously, so long as the frequency ω is large as compared to the previously defined quantity $\alpha=1/\tau'$, the quantity c^2 is proportional to $1/\omega^2$. If, however, the frequency is equal to or less than $1/\tau'$ the trend with frequency is no longer of this type; and near zero frequency the expression becomes independent of frequency:

$$c^2_{\omega=0}=4\overline{n^2}\gamma/\alpha$$

Consequently, we have

$$\frac{c^2}{c^2_{\omega=0}}=\frac{1}{1+\omega^2/\alpha^2} \tag{18}$$

and we see directly that it is possible to determine the quantity α by measurements of c^2 at two or more values of frequency, for instance by a measurement near zero frequency and at a frequency $\omega > \alpha$. The relation is

$$\alpha^2 = \omega^2 \frac{c^2}{(c^2_{\omega=0} - c^2)}$$

The relative trend with frequency as given by (16) is shown in Figs. 6a and 6b, once in linear and once in double logarithmic coordinates.

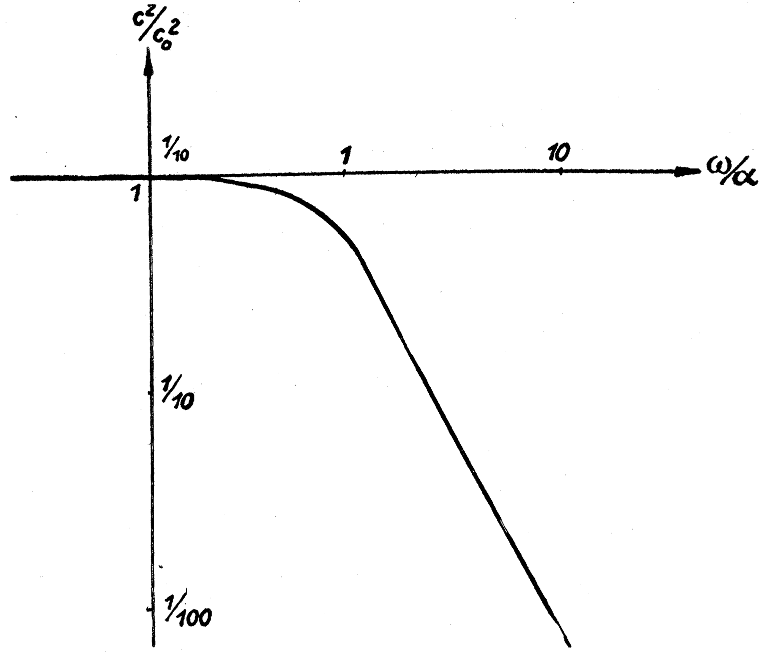

Fig. 6a.

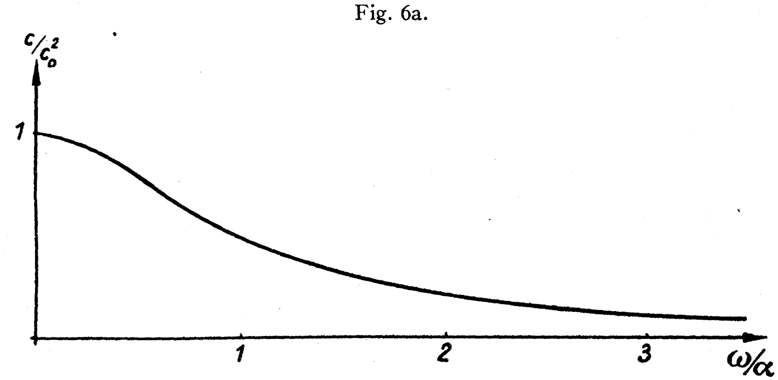

Fig. 6b.

Let us compare this theoretical curve 6b with the observations by Johnson, plotted on the same scale. In Fig. 3 one observes only an *increase* of the quantity $\overline{V^2}_{obs}/\overline{V^2}_{calc}$ at low frequencies which seems to be even more rapid the lower the frequency. The more extensive measurements shown in Fig. 4 made upon oxide coated filaments, give indications that we have both a trend roughly proportional to $1/\omega^2$ and at very low frequencies also a bending similar to that in Fig. 6b. Since in the experiments i_0 was kept constant and the second term on the right hand side of Eq. (17) is negligible almost throughout, the curves for $\overline{V^2}_{obs}/\overline{V^2}_{calc}$ plotted logarithmically as functions of frequency have the same shape as the curve for $c^2/c^2_{\omega=0}$ in Fig. 6. I have tried to draw this curve into Fig. 4 as closely as possible to the points. The agreement is clearly not very good and yet it is good enough to encourage more exact measurements as a test. If one should take the continuous curve as a sufficiently exact plot of the observations, the frequency $\omega=\alpha$ would lie somewhere near the place indicated by the arrow in Fig. 4, at the frequency of about 160. We should then infer that $\alpha=2\pi\cdot 160=1000$ and that the average time of sojourn of the foreign atoms in the surface of the oxide coated filament used by Johnson is about $\tau=.001$ sec. We do not need to emphasize that this instance is given only as an example of the kind of conclusions to be drawn without claiming the reality of the processes discussed before repetition and extensions of the measurements have been carried out.

The dependence on current. Since it has been assumed that the various foreign atoms in the surface are independent of one another, we have, according to the well-known law of fluctuations

$$\overline{n^2}=N_0\ . \tag{19}$$

For γ we substitute the value Fi_0 from Eq. (1). According to Eq. (15) we then obtain

$$c^2=4N_0F^2\frac{\alpha}{\alpha^2+\omega^2}i_0{}^2\ . \tag{20}$$

Because of the relation between c^2 and V^2 of Eq. (16) (omitting the -1), this relation gives the quadric law of variation with current displayed by the flicker effect and recorded graphically in Fig. 2. It is being assumed that N_0, F and α may be regarded as sufficiently independent of temperature. Although i_0 depends on it, the variation of i_0 with temperature is so rapid that it would be permissible to regard these quantities as sufficiently independent of temperature even though they varied as the first or second power of T or its reciprocal. We may regard this assumption as permissible for the time interval represented by $1/\alpha$ and, according to a theory yet to be given, for the factor F.

It is, however, questionable whether the number of foreign atoms per unit area above designated by N_0 does not vary as rapidly with temperature as does i_0. Here we may introduce instead of N_0 the symbol $Q_0 = N_0/\tau' = N_0\alpha$, representing the number of foreign molecules arriving per second in the surface. If this quantity is determined primarily by appearance of atoms coming from the gas, its variation with temperature will be small. With certain diffusion processes from the interior of the wire, on the other hand, we may expect rapid changes with temperature for N_0 and hence variations from the i_0^2 law. So long as variation of this sort has not been observed, it seems permissible to suppose that the flicker effect is not to be attributed to processes varying for example with the temperature proportionally to $e^{-B/T}$, where B is some quantity larger than the constant b of thermionic emission. For instance, if the i_0^2 law should be found valid for tungsten, (unfortunately we have no measurements with current variation in the region of the flicker effect) it would be possible to exclude from among Johnson's suppositions, the conceptions of diffusing thorium atoms into the surface, the normal evaporation of these atoms and the reduction of thoria, since these three effects, according to Langmuir's[16] beautiful investigations, vary in the aforesaid manner, with the value of B larger than the thermionic constant.

Finally it is to be noted that upon introducing Q_0, Eq. (20) takes the form

$$c^2 = 4Q_0F^2\frac{1}{\alpha^2+\omega^2}i_0^2\ . \tag{21}$$

Accordingly, in the frequency region where $\omega >> \alpha$ and where $1/\omega^2$ gives the frequency variation, the magnitude of c^2 for any particular frequency and current is influenced only by the factor F and the rate at which new foreign particles appear in the surface. One can even conclude that this will happen when there are two or more kinds of foreign molecules; the effect will then be determined by the several values of Q, each obtained by a corresponding factor F.

Influence of the effect on a connected circuit. Substituting in Eq. (7) the expression $\overline{A_k^2}\Delta k$ by $\frac{1}{2}c^2dv$ and integrating over the complete frequency range, we obtain for the time average of the square of the voltage fluctuation in a circuit connected as in Fig. 5, the equation

$$\overline{V^2} = \int_0^\infty \tfrac{1}{2}c^2Z^2dv$$

[16] I. Langmuir, Phys. Rev. **22**, 357, 389 (1923).

in which Z signifies the absolute value of the combined impedance of the tube and circuit. Introducing expression (21) for c^2 and writing $d\omega/2\pi$ for $d\nu$ we get

$$\overline{V^2} = 1/\pi \cdot Q_0 F^2 i_0^2 \int_0^\infty \frac{Z^2 d\omega}{\alpha^2+\omega^2} . \tag{22}$$

We will consider three special cases.

1—Constant ohmic resistance outside to be designated by R_a; constant ohmic resistance in the tube to be designated by R_i, V here to be written as V_1. The impedance Z, which is the sum of R_a and R_i connected in parallel, is constant, and integrating (22) we get

$$\overline{V_1^2} = \frac{1}{2}\,\frac{Q_0}{\alpha} F^2 i_0^2 Z^2 = \frac{1}{2} N_0 F^2 i_0^2 Z^2 = \frac{1}{2} N_0 \gamma^2 Z^2 . \tag{23}$$

This result differs from that for the small-shot effect where even in the case of a pure ohmic resistance we do not get a result independent of residual natural oscillations, but on the contrary we should get infinitely great values of $\overline{V_1^2}$ when the natural frequency is infinite, no matter how great the damping.[17]

When $R_a >> R_i$

$$\overline{V_1^2} = \tfrac{1}{2} N_0 \gamma^2 R_a ; \tag{23a}$$

when $R_a = R_i$ then

$$\overline{V_1^2} = \tfrac{1}{4} N_0 \gamma^2 R_i ; \tag{23b}$$

and obviously $\overline{V_1^2} = 0$ when $R_a = 0$. Naturally, in actual measurements we must take care to separate the fluctuations of voltage from the steady voltage and to shunt out by a capacity any high frequency natural oscillation which might exceed the small-shot effect. This capacity would not influence the measurements on the flicker effect.

2—Resonant circuit with very small damping. In this case the ordinary procedure in dealing with circuits leads to an expression for the impedance:[18]

$$Z^2 = \omega_0^2 L_1^2 \frac{x^2+r^2}{(1+R/R_1-x^2)^2+r_1^2 x^2}$$

[17] Naturally the rapid decline of amplification with increasing frequency prevents anything of this sort being observed. This case might, however, be of some influence with particular experimental arrangements. One must assume that even in the small-shot effect some kind of critical time exists beyond which the effect declines as the square of the period.

[18] In evaluating the small-shot effect Hull and Williams (l.c.) used an expression for $\overline{v_k^2}$ obtained by ignoring r^2 and R/R_1 in the above equation for Z, which in that case is quite permissible. In the expression I gave in 1922 (l.c.) I ignored the second but took account of the first of these quantities. Fry (l.c.) was the first to carry through a calculation in which account was taken of both of these quantities, using the complete expression for Z.

in which R stands for the series resistance and R_1 for the total shunted resistance of the resonant circuit, ω_0 for the natural frequency of the circuit, r for the ratio $R/\omega_0 L$, $(\omega_0^2 LC=1)$, r_1 for the expression $R/\omega_0 L + \omega_0 L/R_1$ and $x=\omega/\omega_0$. The mean square of the voltage fluctuation V across the terminals of the resonant circuit is found by Eq. (22) in the case of the flicker effect to be:

$$\overline{V^2}=\frac{1}{\pi}Q_0F^2i_0^2\omega_0L^2\int_0^\infty\frac{1}{x^2+\alpha^2/\omega_0^2}\,\frac{(x^2+r^2)dx}{(1+R/R_1-x^2)^2+r_1^2x^2}\,. \tag{24}$$

If now r_1, r^2 and R/R_1 are very small compared with unity we can set $x=1$ in the first factor of the integral and reduce the second to the expression[19]

$$\int_0^\infty\frac{x^2}{(1-x^2)^2+r_1^2x^2}=\pi/2r_1\ .$$

If for the special case we write $\overline{V_2^2}$ instead of $\overline{V^2}$ we get

$$\overline{V_2^2}=\tfrac{1}{2}Q_0F^2I_0^2\frac{\omega_0L^2}{r_1}\,\frac{1}{1+(\alpha/\omega_0)^2}$$

which by (21) becomes

$$\overline{V_2^2}=\frac{c^2}{8}\,\frac{\omega^3{}_0\,L^2}{r_1} \tag{25}$$

in which c stands for the specific Fourier amplitude at the natural frequency ω_0 of the resonant circuit. Naturally this expression is the same as that arrived at for the small-shot effect, the amplitude c being in that case the same for all frequencies. Subject to the assumptions, therefore, we may write $\overline{V_F^2}/\overline{V_S^2}=C_F^2/C_S^2$.

Combining Eqs. (21), (23) and (25) we get an expression involving α of the form

$$\alpha+\frac{\omega_0^2}{\alpha}=\overline{V_1^2}/\overline{V_2^2}\,\frac{\omega^3{}_0\,L^2}{\gamma_1}\,\frac{1}{Z^2}\,. \tag{26}$$

The quantities c and ω_0 refer to the natural frequency of the circuit for Case 2, and Z is to be found from the circuit constants for Case 1. By means of this equation we are enabled to determine α by one measurement with pure resistance and one measurement with a resonant circuit connected to the tube.

[19] W. Schottky, Ann. d. Phys. **68**, 157 (1922).

3—General form of resonant circuit. If the simplifying assumptions made in Case 2 are dropped, the integral in Eq. (24) must be calculated as it stands. We introduce the following symbols: $\alpha/\omega_0 = s$;

$x^2/(1+R/R_1) = x'^2$; $r^2/(1+R/R_1) = r'^2$; $r_1^2/(1+R/R_1) = r'^2_1$; $s^2/(1+R/R_1) = s'^2$.

Then the integral in (24) is equal to

$$1/s'^2 \cdot 1/(1+R/R_1)^{3/2} \cdot \int_0^\infty \frac{1}{x'^2/s'^2+1} \cdot \frac{r'^2+x'^2}{(1-x'^2)^2+r'^2_1 x'^2} dx'\,.$$

This integral may be reduced to the integrals S_1 and S_2 used in the simplified solution for the small-shot effect by suitable transformations; its value becomes

$$\frac{1}{(1+R/R_1)^{3/2}} \frac{\pi}{2r_1'} \frac{1+r'^2(1+r'_1/s')}{s'^2+r_1's'+1}\,.$$

We thus obtain from (24) after further transformation the value

$$\overline{V^2} = \tfrac{1}{2} Q_0 F^2 i_0^2\, \omega_0 L^2/r_1 \frac{1+r^2(1+r_1/s)}{(1+R/R_1)\,(1+R/R_1+r_1 s+s^2)}\,.$$

Calculation of the basic factor F from the image theory. Since c^2 and α can be determined from the measurements, it should be possible, if our theory is right, to make an experimental determination of $N_0 F^2 i_0^2 = N_0\gamma^2$ (in Eq. (21) or of $(N_0/\alpha) F^2 i_0^2 = Q_0 F^2 i_0^2 = Q_0 \gamma^2$. It would, however, not be possible by measurements on the flicker effect to separate the two components Q_0 and F or N_0 and F. An interesting question arises, whether one of these factors N_0 or Q_0 or F or γ, can be determined in any other way so that the other could thereupon be ascertained by measurements on the fluctuations. We will attack this problem by attempting to determine F theoretically and then, using the values of $N_0 F^2$ determined from Johnson's measurements, to determine N_0 and Q_0; whence we shall be able to make more exact deductions as to the process underlying the flicker effect.

Space permits only an approximate statement of the theory. The essential points are as follows. The action of a single foreign atom on the surface is calculated by means of Langmuir's theory[16] from the doublet effect which it produces in the surface, every foreign atom in the surface being more or less strongly polarized. The determining factor is the increment in local potential created by the atom at the image threshold where the distance, h, from the surface is 10^{-5} cm.

At any point within this threshold distance (Fig. 7) the foreign atoms of a certain area act together; the foreign atoms which lie outside of a circle of area approximately equal to πh^2, directly underneath the point in question, produce only a constant contribution to the effect at the point, while the spatial and temporal irregularities in the distribution of the foreign atoms are without effect except within the area of the circle. We represent by g the electrical moment of the average double layer for the circular area σ in question (note that this is subject to fluctuation), by g_0 the average value of electrical moment for the entire surface and by y the difference between g and g_0. The effect of the foreign atoms upon the potential at the point P is made up of two parts; first the constant part contemplated in the ordinary Langmuir theory, dependent on the value g_0 for uniform distribution of atoms over the entire

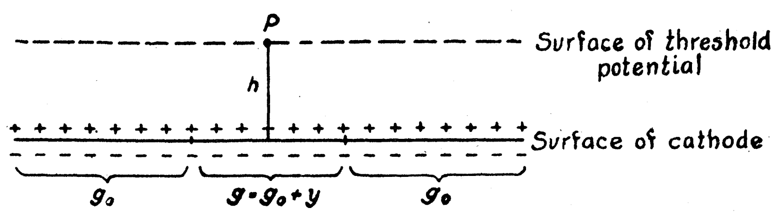

Fig. 7.

surface; and second, a contribution y to be regarded as coming from an additional uniform distribution of atoms over the circular area σ. We will now consider this second contribution. The effect of the additional layer (σy) upon the potential in P is to be calculated approximately from the potential drop which the layer y would produce if infinitely extended (approximation 1). Representing by δ the extra potential measured in electrostatic units produced at P by the layer we have $\delta = 4\pi y$.

Accordingly the effect of a *single* additional foreign atom arriving in the circular area and having the moment p, consists in producing a potential change $\theta = 4\pi p/\sigma$ in the neighborhood of P. This effect will extend nearly uniformly over a surface of the area of the aforesaid circle drawn around the foreign atom as center. A change in the potential at the potential threshold along the surface σ produces an alteration of the current σi flowing out through this surface (i = average momentary current density in the area σ) by the factor $\epsilon^{e\theta/kT}$; or when $e\theta$ is much less than kT as is always the case for one single foreign atom, a change by the ratio $1 + e\theta/kT$. The additional current is then given by

$$\gamma = \sigma i\, e\theta/kT = ie\, 4\pi p/kT\ . \tag{27}$$

If we wish to relate γ not to the local current density i, but to the average current density i_{00} determined from the measured electron current i_0 divided by the cathode area, we must know the relation between i and i_{00}; according to the preceding equation this is

$$i = i_{00}\epsilon^{e\delta/kT} = i_{00}\epsilon^{4\pi e y/kT}.$$

The larger the area σ the smaller is the deviation of the average electrical moment g from the value g_0, in other words the smaller is y. We assume (approximation 2) that the relative change of the electron current flowing out in the vicinity of P due to the spatial change in potential along the surface at the potential threshold is small compared to unity.[20] We can then set $i = i_{00}$ with sufficient approximation and obtain from (27) the equation

$$\gamma = i_{00}e4\pi/kT$$

which signifies that (subject to our assumptions) γ *becomes independent of the magnitude of the region in question*, that is, of the critical distance h and therefore *of the field strength*. Consequently our factor F, to be designated by f when referred to the current i_{00}, becomes

$$f = \sigma e\theta/kT = 4\pi pe/kT \cdot \tag{28}$$

The quantity p may be determined from known data with sufficient accuracy in such cases as those Langmuir's theory covers. Let N_1 be the number of foreign atoms per unit area when the surface is filled with foreign atoms;[21] let $\Delta\varphi$ represent the change of potential at the threshold produced by this number of atoms. Any smaller number $n = \theta_1 N_1$ of foreign atoms will produce a smaller change of potential $\theta_1\Delta\varphi$ in the same area. Considering once more an area σ containing σN atoms we find that when an additional foreign atom arrives at this surface the change in θ, equals $1/\sigma N_1$, independent of the value of n; the corresponding change in potential at P is given by

$$\theta = \Delta\varphi/\sigma N_1 \ . \tag{29}$$

Since $\theta = 4\pi p/\sigma$,
we have

$$4\pi p = \Delta\varphi/N_1 \ ,$$

[20] This assumption is the more justified the larger the area σ and the distance h, in other words the smaller the applied field strength. The investigation of the fluctuation effects, both secular and spatial, at extremely high field strength requires particular consideration for which the foregoing serves as a preparation.

[21] In the case of thorium atoms of tungsten, Langmuir assumes this number equal to about one half the number of tungsten atoms in the surface, $N_1 = 7.6 \times 10^{14}$.

and equation (28) becomes

$$f = e\Delta\varphi/N_1kT\ . \tag{30}$$

The quantity $e\Delta\varphi/k$ is equal to the difference Δb between the exponential constants of the clean and the completely covered metal. This quantity is positive or negative according as the foreign atoms favor or hinder the outflow of electrons (examples are thorium and oxygen atoms on tungsten). It may be calculated from the saturation current I_0 of the clean metal and I_1 of the covered metal by means of the equation

$$\Delta b = e\Delta\varphi/k = T(\log I_1 - \log I_0)\ .$$

Its order of magnitude is $\pm 10{,}000$ to $20{,}000$. We have thus obtained a value for

$$f = \Delta b/N_1T\ ^{22} \tag{31}$$

which is of the order of magnitude

$$\frac{15000}{7.6\times 10^{14}}\,\frac{1}{T}$$

or roughly $\pm 2\times 10^{-14}$. For γ, with current densities of about 0.1 ampere per square centimeter, we obtain a magnitude of about 2×10^{-15} amp. or 10^4 electrons per second.

Computation from the flicker effect of the number of foreign atoms in the surface. The following computation is meant to serve only as an example and to test the theory so far as order of magnitude goes. According to (17) and (20) we have

$$4N_0F^2\frac{\alpha}{\alpha^2+\omega^2}i_0^2 = 4ei_0\left(\overline{V^2}_{obs-}\overline{V^2}_{calc} - 1\right).$$

We introduce the following symbols for the above quantities when referred to unit area: $fi_{00} = Fi_0 = \gamma$; $i_{00} = i_0/S$ (S is the total surface); $N_{00} = N_0/S$ (N_0 is the number of foreign atoms per unit area). We get

$$N_{00}f^2i_{00}\cdot\frac{\alpha}{\alpha^2+\omega^2} = e(\overline{V^2}_{obs}/\overline{V^2}_{calc} - 1)\ .^{23}$$

[22] We observe that the same expression for f would be obtained if the effect of each individual foreign atom were calculated as if its electrical moment were distributed uniformly over the surface.

[23] In this equation let me point out incidentally that both sides in a certain sense signify the "elementary electrical quantum of the flicker effect." In case the second term (-1) can be neglected these expressions are identical with that designated by Johnson as ϵ'.

It follows that

$$N_{00}f^2=\frac{\alpha^2+\omega^2}{\alpha}\cdot\frac{e}{i_{00}}(\overline{V^2}_{obs}/\overline{V^2}_{calc})-1). \tag{32}$$

Let us calculate the right-hand member of this equation for *oxide coated filament* using the point on the curve in Fig. 4 indicated by the arrow, at which point we assume $\alpha=\omega=1000$. For this point the ratio between $\overline{V^2}_{obs}$ and $\overline{V^2}_{calc}$ is about 100, so that the second term in the parentheses (-1) may be neglected. The current i_{00} may be estimated at about 0.3 amp/cm² while the electronic charge, e, is 1.56×10^{-19} coulomb. We have, therefore,

$$N_{00}f^2=2000\times1/0.3\times1.56\times10^{-19}\times100=10^{-13}.$$

Assuming $f\doteq2.10^{-14}$ we obtain $N_{00}=10^{-13}\times10^{28}/4=2.5\times10^{14}$. Since as we have seen N_1 is of the order of magnitude 7.5×10^{14} when the surface is completely covered, it follows that,

$$\theta_1=N_{00}/N_1=1/3\ ,$$

that is to say, the flicker effect of oxide coated filaments is due to foreign atoms the number of which is of the same order of magnitude as the number of atoms of the underlying metal in the surface.

In this calculation the factor which is most uncertain is the value for α, that deduced from the mere indication of saturation in Johnson's curve. It is therefore important to be able, without knowing the value of α to calculate the number of foreign atoms Q_{00} which strike unit area per second from measurements made in the region where $\overline{V^2}_{obs}/\overline{V^2}_{calc}$ varies approximately as $1/\omega^2$. Since $N_{00}\alpha=q_0$, it follows from (32) that

$$Q_{00}f^2=(\alpha^2+\omega^2)e/i_{00}\cdot(\overline{V^2}_{obs}/\overline{V^2}_{calc}-1)\ , \tag{32'}$$

and in the region in question α^2 is negligible in comparison with ω^2. At the frequency 1000, $(\omega=6300)$ the value of $\overline{V^2}_{obs}/\overline{V^2}_{calc}$ given by the curve in Fig. 4 is about 7. Hence $Q_{00}f^2=(6300)^2\times1/0.3\times1.56\times10^{-19}\times6=1.2\times10^{-10}$ and with the foregoing value of f we find $Q_{00}=3.5\times10^{17}$.

Suppose a gas in which Q_{00} atoms pass through unit area per unit time; the pressure of the gas would be

$$p=8.75\times10^{-21}\sqrt{T/1000}\ \sqrt{14}\,Q_{00} \tag{33}$$

in millimeters of mercury. Taking the above value of Q_{00} and assuming $T=1300$, $M=32$ (oxygen), we get the value 2×10^{-2} for the pressure. Now the pressure in ordinary tubes is 10^{-4} mm or lower. Since the quantities which enter into the calculation can scarcely be thought uncertain by as much as a factor of 100 (f is least accurately known), we may conclude from the theory as given, that the flicker effect is not due to collisions of molecules of the gas with the surface of the filament, but results from some other and much more frequently occurring process. From the number of atoms per unit area which we have computed it seems natural to imagine that the ions of both kinds which form the oxide are continually exchanging their places in the surface; but any uniform sequence is not to be thought of, and the occurrence of metal and oxygen ions in the surface are to be considered as independent events. Johnson observed that foreign gases admitted to the tube did not affect the phenomenon very much, which may be taken to support the foregoing conclusion.

In the experiment with a *tungsten filament* (Fig. 3) it is necessary to assume a value for α at least 50 times smaller than with the oxide coated filament, that is, a value less than 20. Considering how uncertain the actual value of α is, we can do no more than calculate Q_{00} approximately from Eq. (32′) neglecting α^2 in comparison to ω^2. At the frequency of 10 cycles per second ($\omega=63$) the quantity $\overline{V^2}_{obs}/\overline{V^2}_{calc}-1$ is equal to about 15. The current i_{00} is probably about 0.1 amp/cm.^{2} Using the same value of f as before, we get

$$Q_{00}=10^{28}/4\times 63^2\times 1/0.1\times 1.56\times 10^{-19}\times 15=2.10^{13},$$

which value is about 20,000 times smaller than that for the oxide coated filament. Were atoms of residual gas responsible for this, the pressure need only be 10^{-6} mm Hg. This is about the lowest limit of residual gas pressure in ordinary tubes. The impact and gradual absorption of gas atoms on the surface may therefore easily account for the effect with tungsten. A repetition with tungsten filaments of the experiments made by Johnson upon the effect of gas upon oxide coated filaments would either confirm or confute this interpretation.

Taking a value greater than 1/20 second for τ' together with the calculated value of Q_{00} we get a value greater than 10^{12} for the number of foreign atoms per unit area, or the relative covering θ_1 is more than 1/750. A more accurate value can be obtained by means of this theory only after extending the measurements in the direction of lower frevencies or using a pure resistance in place of the resonant circuit.

Physikalisches Institut der Universität Rostock, Germany.
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