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THE DISAPPEARANCE OF THE UNMODIFIED LINE IN THE COMPTON EFFECT

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Abstract

Ratio of modified to total scattering coefficient of x-rays for carbon.—Measurements on the scattering by carbon of x-rays of wave-lengths 0.41 and 0.47A are presented in the form of curves showing the variation of the ratio of the modified to total scattering as a function of the angle of scattering. Although the curves show that the unmodified scattering does not disappear at any angle up to 120°, there are distinct decreases in slope at the angles at which Jauncey's theory of scattering by K electrons requires the disappearance of the unmodified ray. This is regarded as demonstrating the applicability of the theory to scattering by K electrons. On the other hand an extension of the theory to scattering by L_I electrons leads to results not in accord with experimental facts.

1. Experimental Results

Jauncey and De Foe[1] and De Foe[2] have described a balance method for measuring the ratio of the modified scattering coefficient per unit solid angle in a direction ϕ to the total (i.e., modified plus unmodified) scattering coefficient in the same direction ϕ. The writers have used this method to measure the ratio for various scattering angles for the wave-lengths 0.41A and 0.47A when scattered by carbon. In Fig. 1 are shown the experimental values of the ratio for the shorter wave-length, while in Fig. 2 the values for the longer wave-length are shown. It will be noted that the experimental points are so scattered that we have not attempted to draw a curve through the points but have drawn a shaded region in which the points occur. This scattering of the experimental points is due to the inaccuracy of the experimental method. However, we believe that the shaded regions do show certain tendencies, which we shall now discuss.

In Fig. 1 there seems to be a distinct elbow at from 80° to 85°. Further, on the large angle side of this elbow the experimental points are within experimental error of unity. If we may therefore assume that the ratio is unity on the large angle side of the elbow, this means that all of the scattering is of the modified type in this region and that the unmodified scattering has disappeared. The critical angle for this disappearance is therefore at about 80° to 85°. Jauncey's theory of the unmodified line[3,4,5] requires the disappearance of the unmodified at an angle given by

[1] Jauncey and De Foe, Phil. Mag. **1**, 711 (1926).
[2] O. K. De Foe, Phys. Rev. **27**, 675 (1926).
[3] G.E.M. Jauncey, Phys. Rev. **25**, 314 (1925).
[4] G.E.M. Jauncey, Phys. Rev. **25**, 723 (1925).
[5] G.E.M. Jauncey, Phys. Rev. **27**, 687 (1926).

$$\text{vers}\,\phi=(3+2\sqrt{2})\cdot(mc/h)\cdot(\lambda_0{}^2/\lambda_s), \tag{1}$$

where λ_0 is the wave-length of the primary x-rays and λ_s that of the K critical absorption wave-length of the scatterer. For a wave-length of 0.41A scattered by carbon ($\lambda_s=47.0$A) this critical angle should be 82°. This agrees very well with the position of the elbow in Fig. 1.

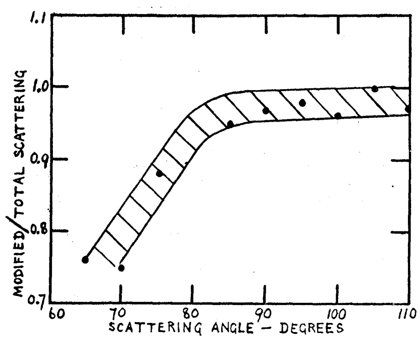

FIG. 1. Scattering by carbon—0.41A.

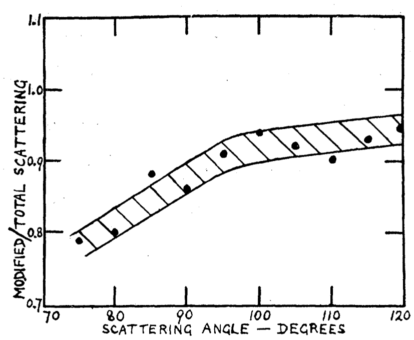

FIG. 2. Scattering by carbon—0.47A.

In Fig. 2, the elbow is not nearly so distinct as in Fig. 1 and furthermore, the experimental values of the ratio appear to be less than unity by an amount greater than the experimental error. We have repeated the readings between 100° and 120° several times and we have not been able to obtain a value of unity. We are therefore forced to the conclusion that the unmodified scattering is still present at $\phi=120°$, although according to Eq. (1) the unmodified scattering should disappear at $\phi=98°$. Upon examining Fig. 1 again, it is seen that although on the large angle side of the elbow the value of the ratio is within experimental error of unity, yet there are no values greater than unity. Hence even in Fig. 1 it seems that the ratio is on the average less than unity on the large angle side of the elbow and that therefore unmodified scattering does not entirely disappear at $\phi=82°$ as required by the theory. There is some evidence of the existence of an elbow in Fig. 2, which comes at about $\phi=100°$. This agrees with the theoretical value of 98°.

In confirmation of our results which indicate that unmodified scattering does not disappear at the angle given by Eq. (1), may be mentioned the recent results reported by Woo.[6] In Woo's experiments the $K\alpha$ x-rays of silver ($\lambda_0=0.5604$A) were scattered by beryllium, boron and carbon at angles greater than the respective critical angles given by Eq. (1). Woo finds by his method, which differs from ours, that, although the un-

[6] Y. H. Woo, Phys. Rev. **28**, 426A (1926).

modified line is weak compared with the modified line, yet the unmodified line is unmistakably present. De Foe[2] in a recent paper finds that the ratio of the modified to the total scattering becomes unity at angles agreeing with the theoretical value. De Foe, however, obtained this result by extrapolation from ratios which were less than unity. If the writers had only obtained points between 65° and 80° in Fig. 1, then extrapolation to unity would have given the critical angle as about 82° in agreement with theory. Hence De Foe's results are not at variance with the results of either Woo or the present writers.

2. Theoretical Discussion

The reasons that our experimental values of the ratio of the modified to the total scattering are less than unity at angles greater than the theoretical critical angle may be two-fold. First, in our experimental method we do not use monochromatic x-rays but rather a band of wave-lengths of a certain mean wave-length determined by half-value absorption in aluminum. The beam of x-rays, however, is filtered through 0.33 cm of aluminum and this makes it fairly homogeneous. This lack of homogeneity does not apply in Woo's experiments and hence we must abandon any explanation along these lines. Second, the theoretical formula may not be sufficiently exact and this possibility we shall proceed to discuss.

Eq. (1) is obtained on Jauncey's theory when the scattering of x-rays by the K electrons of the scattering substance is considered. The formulas for the disappearance of the unmodified line when x-rays are scattered by the L, M, etc., electrons which move in circular orbits are similar to that in Eq. (1) excepting that λ_s is now the respective L, M, etc., critical absorption wave-length. However, since the angles at which the unmodified scattering disappears for the L, M, etc., electrons which move in circular orbits are smaller than for the K electrons, the theory gives Eq. (1) for the critical angle when the electrons in all circular orbits are considered. Jauncey has extended the theory to elliptic orbits.[4] However, this extended theory rests upon the hypothesis that the L_I electrons (for instance) can be treated as moving in an inverse square central field of force. The speed of the L_I electrons at perihelion is then greater than that of the K electrons. If the major-axes of the L_I orbits are directed at random in space, the velocities at perihelion are also directed at random. Let A be the azimuthal angle of the electron in its orbit relative to the line joining the center of the force field to the perihelion of the orbit. At any instant of time there is a number δN of the L_I electrons of the scattering substance which are between the azimuthal angles $A \pm \delta A/2$.

Of these δN electrons there are δN_n which are moving in such directions as to scatter modified x-rays in the direction ϕ. Jauncey's theory[4] gives

$$\frac{\delta N_M}{\delta N}=\frac{\alpha_0 \text{vers}\phi+2f\sqrt{2\alpha_s}\sin\frac{1}{2}\phi-\lambda_0/\lambda_s}{4f\sqrt{2\alpha_s}\sin\frac{1}{2}\phi}, \quad (2)$$

$$f=(n/k)\times\sqrt{1+2\epsilon\cos A+\epsilon^2} \quad (3)$$

and

$$\epsilon^2=(n^2-k^2)/n^2 \quad (4)$$

where $\alpha_o=h/mc\lambda_o$, $\alpha_s=h/mc\lambda_s$, k is the azimuthal and n the total quantum number of the L_I orbit. Hence for the unmodified line to disappear for the scattering by L_I electrons between the azimuths $A\pm\delta A/2$ the right side of Eq. (2) must be unity. Solving we obtain

$$\text{vers}\phi=(1+2f^2+2f\sqrt{1+f^2})\cdot(mc/h)\cdot(\lambda_0^2/\lambda_s). \quad (5)$$

In order for unmodified scattering by the L_I electrons in all azimuths to disappear the value of f at perihelion ($A=0°$) must be placed in Eq. (5). Using this value of f and considering the scattering of Ag $K\alpha$ x-rays by the L_I electrons of carbon ($\lambda_s=350$A) we obtain vers $\phi=2.1$. There is thus unmodified scattering at all angles, including the angle $\phi=140°$, which was the angle used by Woo.[6] In Figs. 1 and 2 the unmodified scattering by the L_I electrons should vanish at 98° and 120° respectively. Experiment shows, however, that there is still unmodified scattering at these angles.

Let us now consider the relative intensity of this unmodified scattering when Ag $K\alpha$ x-rays are scattered by the L_I electrons of carbon at $\phi=140°$. The value of f which will make the right side of Eq. (2) equal to unity for these conditions is 3.36 and therefore the value of A from Eq. (3) is 52°. Hence all L_I electrons whose azimuths are between 0° and $\pm 52°$ may scatter unmodified x-rays if they are travelling in suitable directions. The fraction of the total number of L_I electrons which at any instant have their azimuths in this range is 0.011. However, only a small fraction of these are travelling in such directions as to scatter unmodified x-rays. For those whose azimuth is 52° the fraction is zero, while for those whose azimuth is 0° the fraction is about 0.05. If as an approximation we average these, we obtain 0.025. Hence the ratio of the number of L_I electrons which are in such azimuths and are travelling in such directions as to scatter unmodified x-rays to the total number of L_I electrons is 0.011×0.025 or .0003. Seeing that there are supposed to be two K, two L_{III} and two L_I electrons in the carbon atom and that in the case under consideration the K and L_{III} electrons do not scatter unmodified rays, the fraction of all the electrons which can scatter unmodified rays is .0001. Now the chance of an electron scattering when in the U position[5] is greater than when in the M position, so that the ratio of the energy of unmodified to that of the total scattering will be greater than the ratio of the number of electrons in the M position to the total number as has been shown by De Foe.[2] However, even allowing for this it is doubtful whether the energy in the unmodified line in Woo's experiment should be greater than one fifth of one percent of that in the modified line and it is doubtful whether such a small ratio could be discerned in Woo's experiment. Certainly a ratio of this order could not be discovered in the experiments of the writers.

It is interesting that the elbows in our Figs. 1 and 2, if there are elbows, seem to come at the angle where according to the theory unmodified scattering by the K electrons should cease. The theory, however, does not explain quantitatively the presence of unmodified scattering at angles greater than the critical angle for the K electrons. The theory of the scattering by the L_I electrons, however, is based upon the assumption that these electrons move in a coulomb field of force. These electrons at perihelion approach closer to the nucleus than the K electrons. So far the Bohr model has only been successfully applied to the hydrogen atom and to the helium ion in the quantitative explanation of series spectra. Very little is known at present of intra-atomic mechanics where the atom contains more than one electron. Then, too, there is very little known as to how the outer electrons of an atom move when the atom is part of a solid. It seems therefore that Jauncey's theory of the unmodified line should be expected to hold quantitatively for scattering by the K electrons but not for scattering by the outer electrons. A test of the theory as applied to the K electrons is to look for elbows of the type shown in Figs. 1 and 2. These elbows will probably be rounded even when homogeneous primary x-rays are scattered because the time of action between the quantum and the electron in the scattering process is not infinitesimally small.

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