STANDARD WAVE-LENGTHS FOR USE IN THE EXTREME ULTRA-VIOLET

By Stanley Smith and R. J. Lang

Abstract

Wave-lengths of a number of lines, chiefly of carbon, which frequently occur in the spectra of other elements have been measured in the second and third orders on plates obtained in a vacuum spectrograph with a concave grating with a radius of 6 feet ruled with 30,000 lines to the inch. The dispersion was approximatly 4.5A per mm. The carbon lines and the iron standard lines used as comparisons were obtained simultaneously by passing a condensed spark discharge between a carbon and an iron electrode. Details are given of a new method of obtaining curves to correct for (a) deviation of the spectra from normality, (b) error in the tilt of the photographic plate. These curves can be used for correcting the calculated wave-lengths obtained by interpolation between any pair of standards chosen within the range of the curves.

I. Introduction

SPECTROSCOPISTS working in the extreme ultra-violet region are well aware that at the present time the greatest obstacle to attaining a higher degree of precision in the measurement of wave-length is the lack of adequately accurate standards in this region.

The vacuum spectroscope, which has already been described in detail[1], employing a concave grating of radius 6 feet with 30,000 rulings to the inch gives a dispersion of about 4.5A per mm in the first order. This dispersion being almost four times as large as that hitherto obtained in vacuum grating work, it was thought that the instrument might well be used to measure as accurately as possible a number of well known spectral lines which may then be regarded as standards for future work in the extreme ultra-violet. The lines selected are chiefly those of carbon, since of all the lines of foreign origin found in spectra obtained from condensed sparks in vacuo, those due to carbon are by far the most frequently occurring and occur with the greatest intensity. The carbon spectrum was obtained *simultaneously* with an iron spectrum by using an electrode of each element in the spark gap. By this arrangement it was hoped to eliminate the possibility of a relative displacement between the lines of carbon and of iron as might easily occur from various causes if the spectra of the two elements were obtained by two separate exposures. Even

[1] Lang and Smith, J.O.S.A. **12**, 523 (1926).

in the case of a single exposure it has sometimes been found that the spectral lines appear doubled although it was definitely known that the focusing was extremely good. The reason for this effect is to be found in a change in the relative positions of the spark gap, slit, and grating caused by the slight adjustments to the electrodes which must necessarily be made from time to time during the exposure. It will be readily understood that the possibility of such a displacement occurring when a complete change of electrodes is made is by no means negligible.

II. Theory of Method of Measurement

In the case of a concave grating operated in a vacuum the mechanical difficulties do not permit the use of a Rowland mounting. The obvious

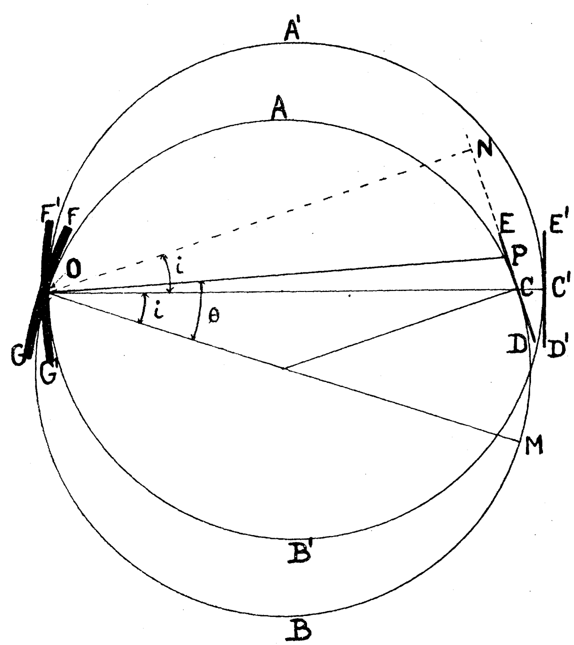

Fig. 1. Showing relative positions of grating *FG* and photographic plate *ED*.

advantages of working with normal spectra are therefore lost. The mechanical arrangement which naturally suggests itself on account of the facility of adjustment, is one in which the slit source is coincident with the center of the photographic plate and the plate (assumed to be short) is a tangent to the focal circle, the point of contact being the center of the plate. In Fig. 1 $F'G'$ represents the grating, C' the center of curvature, and $OA'B'$ the focal circle when the angle of incidence of the light on the grating is zero. $E'C'D'$, tangent to this circle at C' represents the photographic plate whose center C' is coincident with the slit. In order to throw another region of the spectrum on the plate the grating is rotated about O through an angle i into the position FG the focal circle moving to OAB. It is now clear that on moving the center of the grating along OC' through a distance $R(1-\cos i)$, R being the radius of curvature of the grating, the center of the plate together with the slit itself will be brought on to the focal circle at C. On rotating the plate about C through the angle i the plate will touch the focal circle and the spectrum will again be in focus. The tilting of the grating and its translation along OC' are readily effected from the outside of the evacuated spectrograph as previously described.[1] The photographic plate is given a corresponding angle of tilt by interposing a brass wedge, whose angle has the given value i, between the plate and the plate holder without moving the center of the plate relative to the grating. To simplify the mechanical construction of the spectrograph the slit S actually lies about 4 cms distant from the plane of the focal circle, and C, the center of the plate, is in the corresponding position on the other side of the plane, i. e. OS is equal to OC and the plane SOC is perpendicular to the plane of the focal circle. It is assumed that although such an arrangement will alter slightly the scale of the spectrum it will not appreciably affect the relative positions of the lines in the spectrum.

The spectrum will be practically normal only for the case in which the angle of incidence is zero. For all other angles of incidence calibration curves will have to be obtained in order to correct the estimated wave-lengths for the deviation of the spectrum from normality. As flat plates were used, it was necessary to limit their lengths if uniformly good focusing over their entire lengths was desired. Consequently it was decided to photograph the spectrum from 300A to 4300A in nine sections of about 600A each, with 600, 1000, 1400, 1800, 2200, 2600, 3100, 3600 and 4000A respectively at the center of the plate. The illustrate the method employed for constructing the calibration curves for each of these regions let us select the 700-1300A region as an example.

Let P, Fig. 1, be the position of a spectral line λ whose angle of diffraction is θ, the normal to the grating being OM and N the foot of the perpendicular drawn from O on to DE produced. It is clear that

$$NP = ON\tan(2i-\theta) = R\cos^2 i\tan(2i-\theta).$$

From the relation $\sin i + \sin\theta = N\lambda$ for the first order spectrum where N is the number of lines per centimeter of the grating the value of θ corresponding to a given λ can be found and therefore the corresponding value of NP can be calculated. This was done for every 50A

TABLE I

Sample set of data illustrating how corrections for lack of normality of spectra are made.

λ	y (cms)	$n\Delta+\delta y$ (cms)	$\delta\lambda$ in A
700	0	0	0
750	1.08571	$\Delta+.00314$	$-.145$
800	2.17046	$2\Delta+.00532$	$-.246$
850	3.25437	$3\Delta+.00665$	$-.307$
900	4.33756	$4\Delta+.00727$	$-.336$
950	5.42015	$5\Delta+.00729$	$-.336$
1000	6.50227	$6\Delta+.00684$	$-.316$
1050	7.58398	$7\Delta+.00598$	$-.276$
1100	8.66538	$8\Delta+.00481$	$-.222$
1150	9.74672	$9\Delta+.00357$	$-.165$
1200	10.82799	$10\Delta+.00227$	$-.105$
1250	11.90933	$11\Delta+.00104$	$-.048$
1300	12.99086	$12\Delta+0$	0

from 700A to 1300A. The distances, y, of the lines having these values of λ from the position of 700A were then computed and are tabulated in the second column of Table I. The average dispersion between

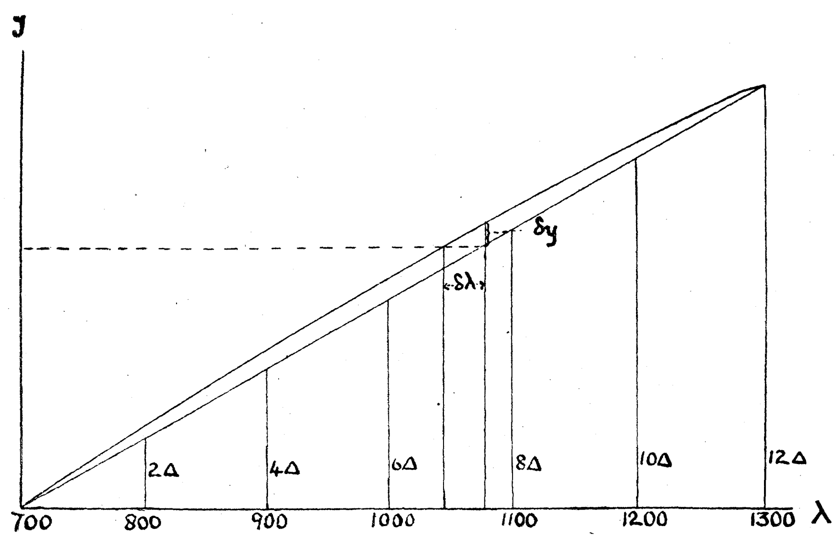

Fig. 2. Illustrating derivation of dispersion correction curve.

700A and 1300A is 1.08257 cms per 50A. Denote this quantity by Δ. Taking 700A and 1300A as standards and assuming that the spectrum is normal between these limits the values of y for 750A, 800A, 850A, etc. would be Δ, 2Δ, 3Δ etc. But as is seen in the third column of the table the actual y's are larger than these values. It is clear from Fig. 2 that the values of λ estimated on the assumption that the spectrum is normal will be too large. The necessary correction $\delta\lambda$ for each λ calculated in the above table can easily be found by dividing δy by the slope of the dispersion curve. The values of $\delta\lambda$ thus obtained are recorded in the fourth column of the table. The calibration curves $(\delta\lambda, \lambda)$ for six settings of the grating are plotted in Fig. 3. The numbers 1, 2, 3, 4, 5 and 6 refer to cases in which the wave-lengths at the center of the plate are 600, 1000, 1400, 1800, 2200 and

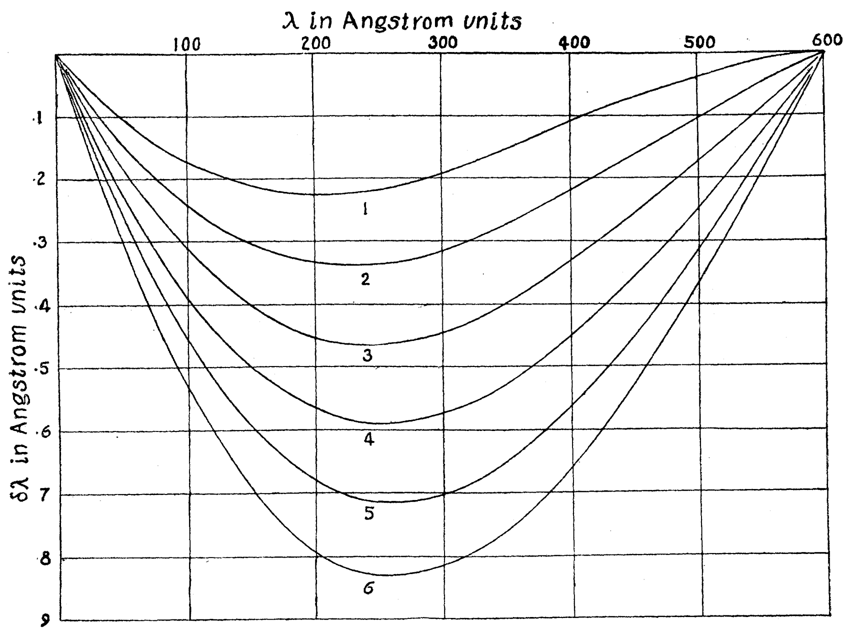

Fig. 3. Showing correction curves for abnormality of spectrum.

2600A respectively. It is interesting to note that a point of inflexion occurs near the upper limit of λ for curves 1 and 2. This arises from the fact that $dy/d\lambda$ has a minimum value at these points, giving points of inflexion in the (λ, y) curves. The values of λ at which these occur can also be found theoretically, for it may be shown that the angle of diffraction θ for such points is given by the following equation

$$\tan^2\theta \mp 3\tan\theta/\tan 2i = 2$$

For the first three curves in Fig. 3 the λ's corresponding to the values of θ obtained from this equation fall within the range of wave-lengths plotted. The practical importance of this fact is that if standards are selected which are fairly symmetrically placed with regard to a point of inflexion and not too far from it, the spectrum between these standards will be practically normal.

Each curve can be used to correct for the deviation of the dispersion from that of a normal spectrum between *any* two standard lines lying within the range of the curve in the following way. A straight line is drawn joining the two points on the curve corresponding to the two standard λ's; the required corrections are then given by the portions of the ordinates intercepted between this straight line and the curve. The corrections obtained in this way have in a number of cases been compared with $\delta\lambda$ calculated directly and the agreement between the two sets of values has been found to be very close. Table II

TABLE II

Comparison of wave-length corrections obtained by direct computation and by graphical means.

λ	$-\delta\lambda$ in A (computed)	$-\delta\lambda$ in A from 3700–4300 calibration curve
3800	0	0
3825	.151	.152
3850	.278	.280
3875	.382	.384
3900	.466	.467
3925	.527	.527
3950	.568	.569
3975	.588	.589
4000	.592	.592
4025	.575	.574
4050	.540	.541
4075	.489	.488
4100	.421	.421
4125	.338	.341
4150	.240	.241
4175	.126	.127
4200	0	0

shows such a comparison. The calibration curve was computed and plotted between the limits 3700A and 4300A. The intercepts of the ordinates between this curve and a straight line joining points on the curve corresponding to 3800A and 4200A are tabulated in the third column.

The effect produced by an error in the tilt of the plate must now be considered. In general such an error would alter the scale of the spectrum and would also change the relative positions of the lines. The first effect will, of course, introduce no error into the computed values of the wave-lengths if an interpolation method between standards is employed. Correction curves for the second effect i. e. the change in the relative positions of the lines, can be obtained in the following manner. In Fig. 4, O represents the pole of the grating, AB the correct position of the plate, A and B being the positions of the two standards between which the interpolation is to be made. $A'B'$ represents the plate when the tilt exceeds the correct value, by a small angle ϕ. Let y and y' denote the distances from A and A' respectively of the same spectral line situated at P and P'. In order

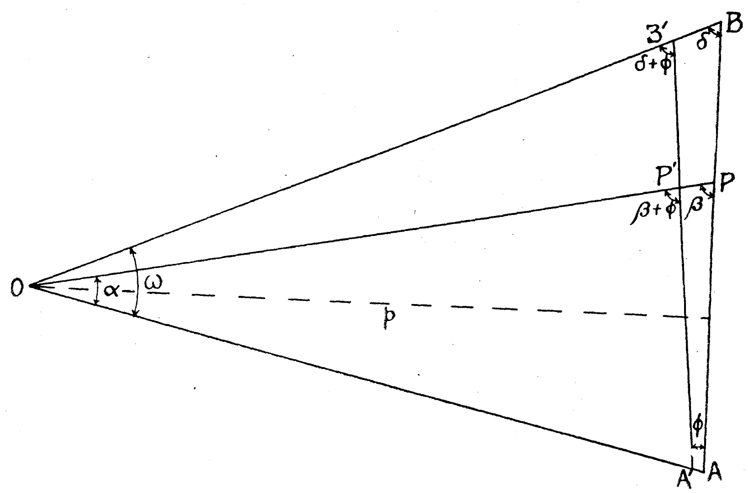

Fig. 4. Illustrating correction for error in tilt of plate.

to make y and y' comparable it is, of course, necessary to put $A'B'$ equal to AB. Using the notation indicated in Fig. 4 $y/OA = \sin\alpha/\sin\beta$ and $y'/OA' = \sin\alpha/\sin(\beta+\phi)$ also $OA/AB = \sin\delta/\sin\omega$ and $OA'/A'B' = \sin(\delta+\phi)/\sin\omega$. Remembering that $A'B' = AB$ it follows that

$$y'/y = 1 + \cot\delta\tan\phi/(1+\cot\beta\tan\phi).$$

If ϕ is small this reduces to

$$y' - y = y\tan\phi(\cot\delta - \cot\beta)$$

or

$$y' - y = \delta y = y(AB-y)\tan\phi/p = \phi y(AB-y)/p$$

where p is the perpendicular distance of O from AB. If ϕ is positive, i. e. the plate is tilted more than it should be, the computed wavelengths will be too large by an amount $\delta\lambda$ where

$$\delta\lambda = \frac{\delta y}{\text{slope of dispersion curve}} = \frac{\phi y(AB-y)}{p \times \text{slope of dispersion curve}}$$

By using the values of y already computed as explained above $(\delta\lambda, \lambda)$ curves can easily be plotted for any given values of ϕ. These curves are almost parabolic. They would, of course, be truly parabolic if the spectrum were normal. As in the case of the curves in Fig. 3 they can in a precisely similar manner be used for *any* two standards provided these standards have wave-lengths intermediate between those for which the curves were originally constructed. In practice ϕ is not directly known but its value can be estimated very easily. Suppose that a correction curve for $\phi = 1°$ has already been plotted. Then using any two given standards the wave-lengths of a number of intermediate lines, accurate values of whose wave-lengths have previously been determined, are computed by using the dispersion correction curves in Fig. 3. On comparing the differences between the computed and known values of these wave-lengths with the corresponding $\delta\lambda$'s read off from the tilt correction curve, the actual value of ϕ can be estimated because the error $\delta\lambda$ for a given λ is directly proportional to ϕ. The corresponding correction for any wave-length within the region investigated can now be found from the correction curve for $\phi = 1°$.

III Experimental Procedure and Results

For a general description of the operation of the spectrograph reference should be made to the authors' previous paper already mentioned. Very great care was exercised in the focusing of the spectra on the plates. In the neighborhood of the position of best focus it was found that on moving the grating 1/64 inch towards or away from the photographic plate an appreciable difference in the definition of the images could be observed. Having once carefully focused the grating for one particular value of the angle of incidence i_1, the distance through which the grating had to be moved to bring the spectrum in focus for any other value i_2 of the angle to incidence was calculated from the expression $R(\cos i_1 - \cos i_2)$. In order to give the plate the necessary tilt, i, a brass wedge cut to the angle i as accurately as possible was interposed between the plate holder and the plate. In effecting this tilt the plate was always rotated about its center i. e. C in Fig. 1. A spectrum was then photographed and the wave-lengths of a number of standard lines were measured corrections being applied for the abnormality of the spectrum. An investigation of the residuals between the computed and known wave-lengths of these standards enabled the error in the angle of tilt to be estimated by using the tilt error curve as explained above. The angle of the wedge was accordingly altered until on an average the residuals attained the value zero. The plates were measured on a comparator reading to .001 mm. Ten settings were made on each line to be measured and on each standard line used. The plate was then reversed and the process repeated. Care was taken to keep the temperature of the plate as nearly constant as possible throughout the measurements.

The mean values of the computed wave-lengths are given in the third column of Table III and the mean deviations from the mean are tabulated in the fourth column. The number of plates on which the

TABLE III

Wave-lengths of some lines of a carbon-iron spark in the far ultra-violet.

Authors				Simeon	Hopfield and Leifson	Millikan and Bowen
Number of plates	Order of spectrum	λ (IA vac.)	Mean deviation	λ (IA vac.)	λ (IA vac.)	λ (IA vac.)
7	2 and 3	1335.70	.009	1335.66	1335.75	1335.72
7	2 and 3	1334.51	.007	1334.44	1334.57	1334.54
3	2	1329.60	.010	1329.60	....	1329.60
2	2	1329.12	.010	....	....	1329.14
6	2	1323.93	.015	1323.79	....	1323.96
6	2 and 3	1247.37	.006	1247.2	....	1247.43
3	2	1215.68	.010	1215.53	1215.68	....
5	3	904.47	.016	....	....	904.48
5	3	904.17	.006	....	....	904.17
5	3	903.95	.012	....	....	903.98
5	3	903.62	.010	....	....	903.63
2	2	1561.46	.000	1561.32	1561.39	1561.47
2	2	1560.71	.005	1560.67	1560.46	1560.76
2	2	1560.33	.000	1560.16	....	1560.34
2	2	1550.80	.010	1550.8	....	1550.84
2	2	1548.22	.005	1548.3	....	1548.26
5	2	1402.75	.018	....	....	....
5	2	1393.75	.016	....	....	....
5	2	1176.36	.010	....	....	1176.40
5	2	1175.98	.012	....	....	1176.05
5	2	1175.66	.016	....	....	1175.72
5	2	1175.25	.012	....	....	1175.31
5	2	1174.92	.004	....	....	1174.96
2	3	1037.03	.005	1036.84	1037.03	1037.03
2	3	1036.35	.000	1036.22	1036.33	1036.35
2	3	1010.38	.000	....	....	1010.38
2	3	1010.08	.005	1010.09	....	1010.10
2	3	977.03	.005	....	....	977.02

lines were measured is stated in the first column. The values found by Simeon[2], Hopfield and Leifson[3], and Millikan and Bowen[4] for the corresponding lines are also tabulated for comparison. Included with the carbon lines are the Si IV doublet 1402A, 1393A and the first member of the Lyman series of hydrogen 1215A. It will be observed that the table is divided into two sections. The lines in the upper part of the table were measured in the 2600A region using as comparison

[2] Simeon, Proc. Roy. Soc. **102,** 484 (1922).
[3] Hopfield and Leifson, Astrophys. J. **58,** 59 (1923).
[4] Millikan and Bowen, Phil. Trans. A **225,** 394 (1926).

lines the primary international iron arc standards λ(I. A. vac) 2740.359, 2715.223, 2563.309, 2414.044 and 2374.462. The remaining lines in the table were measured in the 3100A region. It was found that in this region primary iron standards were difficult to obtain and if they appeared on the plates at all they were extremely faint. Consequently it was necessary in some cases to use other iron lines as comparisons. The following are some of those used; λ(I. A. vac) 3228.72 3155.11, 2985.70, 2768.33 and 2344.22, the values being taken from Kayser's Handbuch der Spectroscopie Vol. 7 and reduced to λ(I. A. vac) by the tables given by Meggers and Peters.[5]

To test the accuracy of the method of measurement the same lines were measured in some cases by using different pairs of standards. The agreement between the values computed from the different sets of standards was found to be very satisfactory. The results given

TABLE IV

Comparison of wave-lengths computed using different pairs of standards.

Spectral line	using standards 2740.359 and 2374.462	using standards 2740.359 and 2563.309
Fe. Int. Std. 2715.223	2715.22	2715.22
C. second order 1335 doublet	2671.38	2671.37
	2669.02	2669.02
C. second order 1329	2659.22	2659.21
C? second order?	2658.27	2658.26
C. second order 1323	2647.87	2647.88
Fe. Int. Std. 2629.079	2629.07	2629.07

in Table IV will serve as an example. The grating used in this work was found to be so inefficient below 900A that the higher orders for lines of such wave-lengths could not be obtained on the plates. However by employing a more suitable grating it is hoped that in the near future carbon lines in the extreme ultra-violet below 900A may be measured with an accuracy which will permit of their use as standards in this region.

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UNIVERSITY OF ALBERTA,
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[5] Meggers and Peters, Bull. Bur. Stand., No. 327 (1918).