ACOUSTICS OF THE PIANOFORTE

By R. N. Ghosh

Abstract

The amplitudes cf fundamental and overtones of a steel string struck by a hammer as a function of time of contact. Vibration curves of a steel string struck by a tape-check system hammer taken from a piano were photographed and analyzed. The duration of contact between hammer and string was also measured photographically. The time of contact was varied by varying the striking position on the string and by varying the mass of the hammer. It was found that the amplitudes of the fundamental, the cctave and the third harmonic were greatest when the time of contact was half the free period of vibration of the string.

The factors which determine the *choice of striking point* are discussed.

THE author has shown in a series of papers[1] that the duration of contact of the hammer with the string during impact depends upon the elasticity of the hammer, its mass and the striking distance. In this paper a short account is given of the experimental work on the amplitudes of partials for (1) the same hammer at different striking distances, and (2) for the same point by different hammers. The results show that the amplitude of the fundamental is a maximum when $T/\phi=2$ or 1, where T is the free period of vibration of the string and ϕ is the duration of contact.

Experimental Methods

A steel string was fixed over two bridges in a sonometer kept vertical by suitable supports and the string was struck by a tape-check system hammer and key which were taken out of a piano for the purpose. The velocity of impact was kept constant by a mechanical device consisting of an eccentric wheel fixed just over the key. When the wheel was rotated it pressed the key once in a revolution. It was so arranged that a heavy load (1 kg) fell through a constant distance and gave only one revolution to the wheel. The point struck was photographed by a falling plate. To obtain the duration of contact in addition to the vibration curves the initial stages were simultaneously photographed with the trace of a vibrating tuning fork. In these experiments one precaution should be taken about the arrangement of the hammer system. The distance

[1] R. N. Ghosh, Phil. Mag. **47,** 1125 (1900); Prcc. Ind. Asscc., Calcutta, **9, III, 194** (1925). See also reference 3.

of the hammer from the string in the rest position should be such that when the former is slowly pressed it just comes near the string but does not press it. This is the arrangement in the piano.

The vibration curves were magnified fifty times and then a trace was taken. They were then analyzed by the schedule method given by

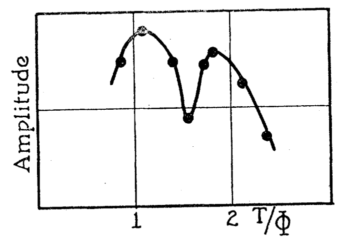

Fig. 1. Length of the string $l=100$ cm, freq. $=84$, $\rho=.096$ gr/cm, velocity of impact $=2\times10^4$ cm/sec.

Professor H. O. Taylor.[2] A large number of curves was obtained and analyzed. Below is given a graphical representation of a few typical sets, obtained with a single hammer. The time of contact (ϕ) was varied by varying the striking point.

George[3] has plotted the amplitude of the fundamental with respect to the striking distances α, but no definite results could be obtained.

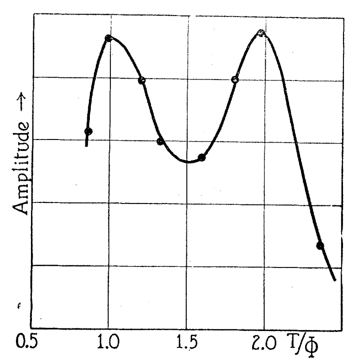

Fig. 2. $l=100$, freq. $=75$, $\rho=.0961$/cm, $\alpha=18$ cm, constant velocity $=2\times10^4$ cm/sec.

Curve (1) shows the relation between the amplitude of the fundamental and T/ϕ. In the figure the amplitude is shown qualitatively as a function of the striking distance. A large number of observations were made and graphs drawn, but in each case the curve as related to T/ϕ was the

[2] H. O. Taylor, Phys. Rev. **6**, 303 (1915).
[3] George, Phil. Mag. **50**, 491 (1925).

same. It will be observed that the amplitude reaches a maximum when $T/\phi = 2$ or 1. It has been found that the amplitude of the octave and the third harmonic also follow the same rule. But the amplitudes of the octave and the third harmonic are greater at $T/\phi = 2$ than for $T/\phi = 1$.

If the amplitude is a function of the ratio T/ϕ, then it is to be expected that a variation of the ratio keeping the striking distance the same by using different hammers should show the same result as shown in curve (1).

Fig. 2 shows the relation between the amplitude of the fundamental (upper curve), and T/ϕ for a fixed striking distance. In all these cases T/ϕ was experimentally obtained. The second curve shows the same relation for the octave.

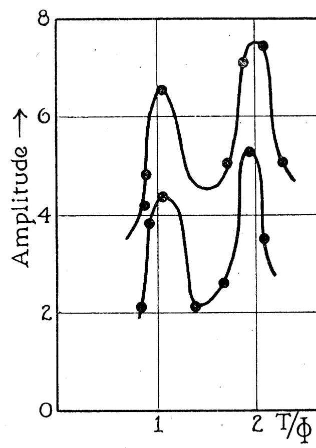

Fig. 3. $l = 100$, $\rho = .0971$/cm, $F = 88$, $\alpha = 20$ cm, constant velocity $= 2 \times 10^4$ cm/sec.

Fig. 3 shows the same relations, with a striking distance of 20 cms. From these experiments we therefore conclude that the amplitudes of the fundamental, the octave and the third harmonic are at their maxima when $T/\phi = 2$. They are large again at $T/\phi = 1$.

Choice of the Striking Point

In general the striking point in the piano varies from $l/7$ to $l/9$ in the bass section, and $l/20$ in the treble section. The hammers also vary in mass in different octaves, they are lighter in the higher octaves. From a musical point of view the choice of the striking point should be such that the first few harmonics should be present in considerable force. We have seen the first three components have maximum amplitude when $T/\phi=2$. Hence the choice of the hammer mass and the striking distance should be such as to insure this condition. Now

$$T/\phi=\frac{T}{\pi}\left(\frac{T}{M}\right)^{\frac{1}{2}}\left(\frac{1}{\alpha(1+T/\mu\alpha)}-\frac{\rho}{4M}\right)^{\frac{1}{2}} \qquad (1)^4$$

where T is the tension, M the mass of hammer, μ its elasticity. Now from Eq. (1) we see that T/ϕ mainly depends upon the product $M\alpha$ which is now fixed for the particular ratio, *viz.* $T/\phi=2$.

Now α should not be very small for then the resulting amplitude of vibration of the string will be small since it depends upon the factor $\sin(s\pi\alpha/l)$.[5] It cannot be very large for then the reflections from the farther end will produce double contact which is detrimental to the production of a good note. Hence α is fixed between two wide limits. But since the character of the musical note depends also upon the convergency of the component tones, α is fixed. The convergency factor or the music factor for $T/\phi=2$ is given below

$$F=\sin m,\ \frac{\sin 2m}{6},\ \frac{\sin 4m}{60},\ \frac{\sin 6m}{210}\cdots \qquad (2)^5$$

$$m=\pi\alpha/l.$$

Once the music factor is determined from the tone quality, α/l is fixed, and then M is calculated from Eq. (1). Thus we see that the choice of the striking point and the mass of the hammer must satisfy the following conditions

$$T/\phi=2,\quad M\alpha=\text{const},\quad F=\text{music factor} \qquad (3)$$

The striking point is usually closer to the fixed bridge. The reason for this is to minimize the free vibrations of the bridge and the board as far as possible, and to prevent the reflections from the bridge, and the slight variations[6] of pressure during contact, the effect of which is to render the tone unmusical.

The author has taken a large number of photographs of the vibration curves when the hammer, the striking point, and the velocity of impact were all kept constant, *but the nature of the impact was changed.* No change in the vibration was found. This is in accordance with the conclusions arrived at by Professor Miller that the tone quality cannot be changed by emotional touch of the key. As long as the position of the hammer is invariable, tone quality cannot be changed.

[4] R. N. Ghosh, Phys. Rev. **24**, 456 (1924).

[5] Rayleigh's Theory of Sound, Vol. 1, p. 190.

Double Contact

Fig. 4 shows that the hammer once left the string and again came in contact with it finally leaving it when the pressure became zero. When the distance of the striking point from the nearer end is small, the duration of contact is less than $T/2$, and the hammer remains in contact with the string throughout the time of contact. Under this condition the wave generated by impact does not reach the farther end before the impact is over. The reflections from the nearer end may lead to slight variations of pressure as mentioned before. It is easy to show[6] that the string attains a velocity greater than the hammer when reflection takes place from the farther end provided the duration of contact is so large.

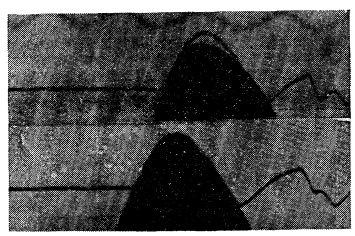

Fig. 4.

Let us take a case when the total duration of contact is approximately the same as the free period of vibration of the string, so that the hammer remains in contact when the wave is reflected from the farther end. At the point of impact the displacement of the string is given by

$$y = Ae^{-Kt/2} \sin qt \quad \pi/q = T(\text{approx}) \tag{4}$$

The equation of the wave motion towards the longer side before reflection is

$$y = Ae^{-K(ct-x)/2c} \sin q(ct-x)/c \tag{5}$$

As long as the impact lasts the velocity of the hammer is given by

$$v_h = ARe^{-Kt/2} \cos(qt+\psi) \tag{6}$$

The equation to the reflected wave is given by

$$y_r = -Ae^{-K(ct+x-2l)/2c} \sin q(ct+x-2l)/c \tag{7}$$

[6] Curves by Professors Raman and Banerji, Proc. Roy. Soc. **97**, 99 (1920).

The velocity of the hammer continues to be represented by (6), but after reflection the velocity of the string at the point of impact is $\dot{y}+\dot{y}_\gamma = v_s$ and

$$v_s = ARe^{-Kt/2}(2+KT/2)\cos qt(\text{neglecting }\psi) \tag{8}$$

From (6) and (8) we find that the velocity of the point on the string is greater than that of the hammer and therefore leaves it. When the point returns after approximately a time $T/2$ the velocity

$$v_s = -ARe^{-K(t_1+T/2)}(2+KT/2)\sin \pi t_1/T \tag{9}$$

$$t_1 = t - T/2$$

The velocity of the hammer at the same instant is

$$v_h = -ARe^{-K(t_1+T/2)}\sin \pi t_1/\mathrm{T} \tag{10}$$

Thus we see from (9) and (10) that the point moves faster than the hammer, and therefore overtakes it.

In a subsequent paper the author proposes to discuss the nature of vibration of the piano string, and the sound board as coupled together.

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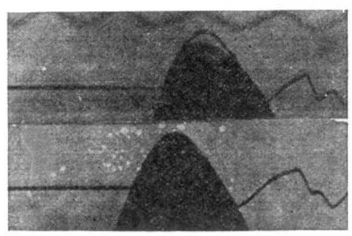

Fig. 4.