INTENSITIES IN THE STARK EFFECT OF HELIUM

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ABSTRACT

Theory of the intensity of combination lines in the Stark effect.—The quantum theory of dispersion may be applied to the Stark effect in a weak field by considering the atoms exposed to external radiation of zero frequency. This makes it possible to calculate the intensity of new lines appearing in the field (combination lines) from the intensity of lines appearing in the undisturbed spectrum. The theory is applied here to the helium spectrum.

Measurements of the intensity of combination lines of He in the Stark effect.—Measurements of the intensity of the helium lines $2P-nM$, $2P-nm$, $2S-4M$, where $n=4, 5, 6, 7$; $M=P, D, F$, etc., are given. The Lo Surdo method of obtaining emission in an electric field was used, with the cathode set with its surface parallel to the slit of the spectrograph. A photographic method of measuring the intensities, using a Moll photometer to measure the photographic density was used. The intensity of the combination lines increases as the square or higher powers of the electric field, as the theory demands. The ratio of the intensity of the combination lines to the nearest lines in the spectrum emitted by the undisturbed atom is in approximate agreement with that calculated from the theory.

INTRODUCTION

ONE of the first phenomena to receive an explanation on the basis of the quantum theory of atomic structure was the Stark effect. Epstein[2] and Schwarzschild[3] showed that the observed splitting up of the Balmer lines in an electric field[4] received a simple explanation on the basis of the Bohr theory of the atom. Later Kramers[5] was able to estimate the intensities of these lines from the correspondence principle. According to Stark's[6] experiments the effect of an electric field on the spectrum of other elements is essentially different from its effect on the hydrogen spectrum. The lines of these spectra show, in general, no splitting up proportional to the first power of the electric field, but the spectra show new lines the frequencies of which are sums or differences of the frequencies of lines in the original spectrum. Bohr[7] pointed out that these facts received a simple explanation on the quantum theory,

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[1] International Fellow of Barnard College, Columbia University.

[2] Epstein, Ann. d. Physik **50**, 189 (1916).

[3] Schwarzschild, Ber. Akad. Wiss., Berlin, 548 (1916).

[4] Stark, Ber. Akad. Wiss., Berlin **47**, 932 (1913).

[5] Kramers, Det Kgl. Danske Videnskabernes Selskabs Skrifter **8**, III, 278 (1919).

[6] Stark, Monograph "Elektrische Spektralanalyse chemischer Atomen," Leipzig, 1914.

[7] Bohr, Det Kgl. Danske Videnskabernes Selskabs Skrifter **8**, IV, 1.

and that the intensity of the new lines should be, in the first approximation, proportional to the square of the field intensity. Pauli[8] has shown that for weak fields the appearance and intensity of these lines may be treated by the quantum theory of dispersion proposed by Kramers.[9] He discussed the observations of Hansen, Takamine and Werner[10] on the spectrum of mercury in an electric field on the basis of this theory and showed that they were in substantial agreement with it.

The Stark effect in helium has been subjected to careful examination by various investigators.[11] Recently Takamine and Werner[12] have given the results of measurements of the intensities of combination lines in the helium spectrum made in such a way as to obtain a direct check on the theory. Their measurements were carried out by photographic methods developed in the laboratory of the University of Utrecht. The research reported here is a continuation of their work. It is planned to give, in this and succeeding articles, measurements of the intensities of all helium lines which are of interest to the theory.

Theory of the Intensities of Combination Lines

The application of the theory of dispersion to the Stark effect allows the calculation of the intensities of combination lines from the amplitudes appearing in the quantum theory description of the undisturbed atom, or conversely the calculation of these amplitudes from measurements of the intensity of combination lines.

We shall describe the spectrum with three quantum numbers, n, k, j. The lines appearing in the absence of a field are described by $n' - n''$ arbitrary, $k' - k'' = \pm 1$, $j' - j'' = 0$ or ± 1, where n' is the value of n in the initial and n'' the value of n in the final state and k', j' are similarly defined. This is to be expected from the correspondence principle if we represent the motion in the atom by a precessing Kepler ellipse and carry this description of the motion directly into the quantum mechanics. When the atom is exposed to an external field of force the correspondence principle predicts that new harmonic components will appear in the motion because of the disturbance caused by the field. These components correspond to transitions for which the rules of selection for k and

[8] Pauli, Det Kgl. Danske Videnskabernes Selskabs Medd. **7**, 3 (1925).

[9] See especially Kramers and Heisenberg, Zeits. f. Physik **31**, 681 (1925). Cf. also Born, Heisenberg and Jordan, Ibid. **35**, 567 (1926) where the dispersion formula is derived from the quantum mechanics.

[10] Takamine and Werner, Det Kgl. Danske Vidensk. Selsk. Medd. **5**, 3 (1923).

[11] Stark, reference 5. Koch, Ann. d. Physik **48**, 98 (1919); Tschulanowsky, Zeits. f. Physik **16**, 300 (1923); Foster, Phys. Rev. **23**, 667 (1924).

[12] Takamine and Werner, Naturwiss. **14**, 348 (1925).

j do not hold. The intensity of lines whose frequencies are equal to the sum or difference of the frequencies of two lines in the original spectrum i.e., those for which *k* changes by 0 or 2, increases as the square of the field. The intensity of lines corresponding to a change of *k* of more than two increases as some higher power of the field. A quantitative determination of the probabilities of transition may be obtained by comparison with the quantum theory of dispersion, since, according to the quantum mechanics, the action of an oscillating field on an atom must approach the action of a constant field as the frequency of the oscillating field approaches zero.

To carry through the application of the dispersion theory it is convenient to consider a non-degenerate system. We may remove the degeneracy with respect to orientation in space by imagining the atom exposed to a very weak magnetic field, of axis parallel to the electric field we are going to apply. The magnetic field is to be so weak that it does not change the motion in the atom nor affect the rules of selection. For a complete description of the motion we must now introduce a magnetic quantum number *m*, which describes the orientation with respect to the field and which may change by 0 or ± 1. In transitions involving no change of *m* the emitted radiation is polarised parallel to the field. The radiation emitted in transitions in which *m* changes by ± 1 is circularly polarised in a plane perpendicular to the field. The formula giving the polarisation and intensity of combination lines whose frequency is the sum or difference of two lines in the original spectrum is

$$A(p,r)\, e^{2\pi i \nu (p,r) t} = \frac{1}{2h} \Sigma_q \left[\frac{A(p,q)(EA(q,r))}{\nu(q,r)} \right. $$

$$\left. - \frac{A(q,r)(EA(q,p))}{\nu(p,q)} \right] e^{2\pi i [\nu(q,r)+\nu(p,q)] t} \qquad (1)$$

where $A(p,\ r)$ is a vector, in general complex, which represents the so-called characteristic amplitude of the transition state *p* to state *r*. It is so defined that the real part of the expression on the left side of the equation is the electric moment we must ascribe to the atom to account for the radiation of frequency $\nu(p,\ r)$ emitted. *h* is Planck's constant, the amplitudes $A(p,\ q)\ A(q,\ r)$ are defined similarly to $A(p,\ r)$ but refer to the amplitude of the corresponding transitions in the undisturbed atom. *E* is the field strength, considered as a vector quantity. The summation is to be extended over all states *r* for which the transitions, *p*, *q* and *q*, *r* are permitted by our rules of selection. The formula follows directly from the general formulas of the quantum theory of

dispersion if the frequency of the external radiation is set equal to zero.[13] The energy emitted in the form of radiation of the frequency $v(p, q)$ per atom per second is given by the formula

$$I = \frac{\lfloor 2\pi v(p,q) \rfloor^4}{c^3} A(p,q)A(q,p) \tag{2}$$

$A(q,p) = \bar{A}(p, q)$ where $\bar{A}$ represents the vector conjugate to A. Because of the appearance of a frequency v in the denominator of both terms of formula (1) certain terms in the infinite sum represented by the general formula will be of a greater order of magnitude than all of the others and need alone be considered.

In considering the new lines whose intensities vary as powers of E higher than the square we shall confine ourselves to those lines whose frequencies are combinations of three lines in the spectrum emitted in the absence of the field. In this case the polarisation and amplitude of the line are given by

$$A(p,s)e^{2\pi i v(p,s)} = \frac{1}{4h^2}\sum_r \sum_q \left[\frac{A(p,q)((EA(q,r))((EA(r,t))}{v(q,t)v(r,t)} \right.$$
$$\left. + \frac{A(q,r)(EA(r,s))(EA(p,q))}{v(p,q)v(r,s)} + \frac{A(r,s)(EA(p,q))(EA(q,r))}{v(q,r)v(q,t)} \right] \tag{3}$$
$$\cdot\, e^{2\pi i [v(p,q)+v(q,r)+v(r,s)]t}$$

This formula is obtained directly as the second approximation of the theory of perturbations of the quantum mechanics, as formula (1) may be obtained as the first approximation of this theory.[14] Because of the appearance of two frequencies in the denominator, in making the summation, only such states need be taken into consideration as give at least one term in which both v values are small.

APPLICATION OF THE THEORY

In order to illustrate the application of formula (1) let us consider the calculation of the intensity of the parallel and perpendicular components of the line $2P-4P$ in parhelium. In Fig. 1 the terms of the helium spectrum are represented by points in the usual way, the values of the terms being proportional to the distances of the points from the vertical line on the right. The transition $4P$, $2P$ is shown by a dotted line, while the pairs of transitions by combination of which the transition

[13] Kramers and Heisenberg, ref. 9, formula (32). Cf. also Pauli, ref. 8, formula (5).
[14] See Born, Jordan and Heisenberg, ref. 9, p. 566.

$4P$, $2P$ can be obtained are shown by the solid lines. It is seen that the only terms to be used in formula (1) are those whose denominators are $\nu(4P, 4D)$ and $\nu(4P, 4S)$. But $\nu(4S, 4P)$ is twelve times $\nu(4P, 4D)$ and the numerator of the term of which it is the denominator is smaller than the numerator of the other term, so we may discard the term of which $\nu(4S, 4P)$ is the denominator. In order to apply formula (1) we must remember that in the magnetic field the states $2P$ and $4P$ consist each of three states $(m = -1, 0, +1)$ and the state $4D$ consists of 5 states
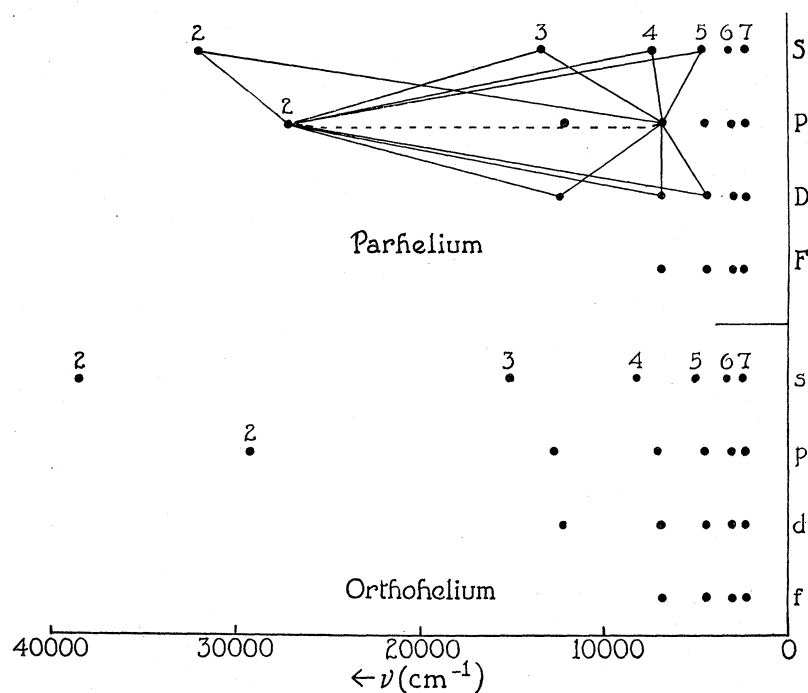

Fig. 1. The terms of the helium spectrum with the transitions giving $2P-4P$.

$(m = -2, -1, 0, +1, +2)$. The only q, r transitions which contribute to the sum on the right side of formula (1) are those for which m remains unchanged, since for these transitions A and E are parallel, while for transitions in which m changes by one unit A and E are perpendicular, and their scalar product, appearing in the formula, is zero. Writing the m values as subscripts to the terms we obtain

$$A(4P_{m'}, 2P_{m''}) = \frac{E}{2h} \frac{A(4D_{m'}, 4P_{m'})A(4D_{m'}, 2P_{m''})}{\nu(4D, 4P)} \tag{4}$$

The ratios of the intensities of the transitions corresponding to different m' and m'' values for a given line are known from the formula of Kronig, Goudsmit and Honl.[15] Since it may be assumed that there are the same number of atoms in each $4P_m$ state and in each $4D_m$ state, we also know the ratio of the amplitudes to each other, and may express them all in
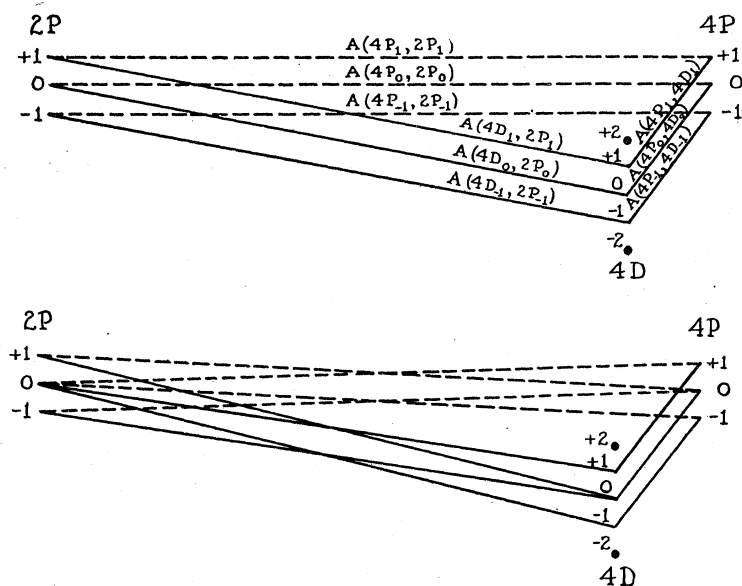

Fig. 2. Transitions used in calculating the intensity of the line $2P-4P$.

terms of $A(4D_0, 2P_0)$ and $A(4D_0, 4P_0)$. If we introduce the assumption that the number of atoms in a $4P_m$ state and a $4D_m$ state are the same, it is seen that the ratio $I^\pi(4P, 2P)/I^\pi(4D, 2P)$[16] which has been the subject of the measurements depends only on E, $\nu(4D, 4P)$ and $A(4D_0, 4P_0)$. Since we know the field this latter amplitude can be calculated from the intensity measurements and compared with the theoretical value. The numerical calculation follows.

$$A(nP\pm1,4D\pm1)^2 = \tfrac{3}{4}A(nP_0,4D_0)^2 \tag{5}$$

$$A(nP_0,4D\pm1)^2 = \tfrac{3}{8}A(nP_0,4D_0)^2 \tag{6}$$

$$A(nP\pm1,4D_0)^2 = \tfrac{1}{8}A(nP_0,4D_0)^2 \tag{7}$$

$$I^\pi(4P,2P) = K\nu(4P,2P)^4 \frac{E^2}{4h^2} \frac{A(4P_0,4D_0)^2 A(4D_0,2P_0)^2}{\nu(4P,4D)^2}[2(\tfrac{3}{4})^2+1] \tag{8}$$

[15] See, for instance, Kronig, Zeits. f. Physik **31**, 885 (1925).

[16] I^π represents the intensity of the parallel component of a line and I^σ of the perpendicular component.

$$I^\pi(4D,2P) = K\nu(4D,2P)^4 A(4P_0,2P_0)(2 \cdot \tfrac{3}{4}+1) = I^\sigma(4D,2P) \qquad (9)$$

where K is a constant containing the proportionality factor of formula (2) and the number of atoms in the initial states ($4P_m$ and $4D_m$). Since $\nu(4P, 2P) = \nu(4D, 2P)$ to a good approximation

$$\frac{I^\pi(4P,2P)}{I^\pi(4D,2P)} = \frac{E^2}{4h^2} \frac{A(4P_0,4D_0)^2}{\nu(4P,4D)^2} \frac{17}{20} \qquad (10)$$

Also

$$I^\sigma(4P,2P) = K\nu(4P,2P)^4 \frac{E^2}{4h^2} \frac{A(4P_0,4D_0)^2 A(4D_0,2P_0)^2}{\nu(4P,4D)^2} 2(\tfrac{3}{4} \cdot \tfrac{3}{8} + \tfrac{1}{8}) \quad (11)$$

So that

$$\frac{I^\pi(4P,2P)}{I^\sigma(4P,2P)} = 2.6 \qquad (12)$$

Similar calculations for the intensity of the line $2S-4D$, give

$$\frac{I^\pi(4D,2S)}{I^\pi(4D,2P)} = \frac{E^2}{4h^2} \frac{A(4P_0,4D_0)^2}{\nu(4P,4D)^2} \qquad (13)$$

and

$$\frac{I^\pi(4D,2S)}{I^\sigma(4D,2S)} = 1.3 \qquad (14)$$

From calculations of the intensity of the line $2P-4F$ we obtain

$$\frac{I^\pi(4F,2P)}{I^\pi(4D,2P)} = \frac{E^2}{4h^2} \frac{A(4F_0,4D_0)^2}{\nu(4D,4F)} \frac{14}{15} \qquad (15)$$

$$\frac{I^\pi(4F,2P)}{I^\sigma(4F,2P)} = 1.3 \qquad (16)$$

If we apply the second approximation formula to the intensity of $2S-4F$ we obtain

$$\frac{I^\pi(4F,2S)}{I^\pi(4P,2S)} = \frac{E^4}{16h^4} \frac{A(4P_0,4D_0)^2 A(4D_0,4F_0)^2}{\nu(4P,4D)^2 \nu(4D,4F)^2} \qquad (17)$$

To deal with the orthohelium lines we shall assume that they are unresolved triplets. In Fig. 3 the resolution of the p and d terms into their j and m values is given. The ratios of the various $A(np_{m'j'}, 4p_{m''j''})$ t, each other is found as for parhelium from the formulas of Kronig. The calculations are more complicated because of the large number of

transitions possible. Instead of going through the entire calculation for $2p-4p$ we shall give in detail only the calculation for $2p_1-4p_1$ $m'=0\to m''=0$. The transitions used in this calculation are drawn in Fig. 3. The calculations for the other transitions are made in the same way and the squares of the amplitudes summed as for parhelium. We
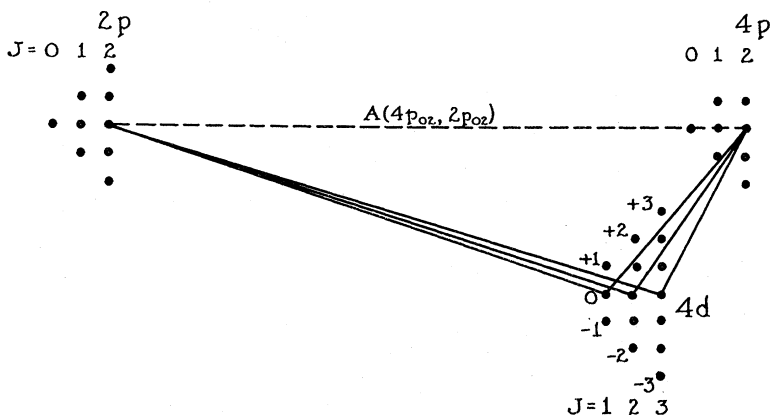

Fig. 3. Transitions used in calculating the intensity of $2p_1-4p_1$,
$m=0\to m=0$.

shall neglect the terms containing $\nu(4s, 4p)$, $\nu(5s, 4p)$ or $\nu(5d, 4p)$ as this introduces only a slightly larger error than the neglect of corresponding terms in the calculation for parhelium. If we write the j values as subscripts following the m values, we have as q states, $4d_{01}$, $4d_{02}$, $4d_{03}$, and the amplitude of the transition considered is

$$A(4p_0,2p_0)=\frac{E}{2h}\ \frac{A(4d_{01},2p_{02})A(4d_{01},4p_{01})}{\nu(4d,4p)}(1+0+54) \tag{18}$$

The sum of the squares of the amplitudes of all the $2p-4p$ transitions gives

$$\frac{I^\pi(4p,2p)}{I^\pi(4d,2p)}=\frac{E^2}{4h^2}\ \frac{A(4p_{02},4d_{01})^2}{\nu(4d,4p)^2}\ \frac{4675}{9} \tag{19}$$

For comparison with the amplitudes obtained from the intensity measurements by calculations similar to the above, calculations were made of the amplitudes of corresponding amplitudes in hydrogen by the method of Schrödinger. This author has shown[17] that the *eigenfunktionen*

[17] Schrödinger, Ann. d. Physik **79**, 734 (1926).

given in an earlier paper[18] give the Heisenberg matrices and can therefore be used to calculate amplitudes exactly, without the uncertainties of the older methods. The formulas used are: His formula for the *eigenfunktionen* in his first paper,[18] formulas (5) and (6) in the paper discussing the relation between his quantum mechanics and the Heisenberg quantum mechanics.[17] Calculations were made for all amplitudes calculated from the dispersion theory and the intensity measurements, and also for the amplitudes needed to determine which terms must be included in formula (1) to give the desired approximation. The recent work of Heisenberg[19] on the helium spectrum shows that the amplitudes calculated for hydrogen will be correct for helium to a good approximation. The amplitudes calculated in this way follow, (they are all for $m' = 0 \rightarrow m'' = 0$).

Line:	$2P-4D$	$2P-4S$	$2P-5D$	$2P-5S$	$4P-4D$	$4D-4F$
Amplitude:	$1.8a_0$	$0.40a_0$	$1.0a_0$	$0.29a_0$	$21a_0$	$16a_0$

Line:	$4S-4P$	$5S-4P$	$5D-4P$	$5P-5D$	$5S-5P$
Amplitude:	$27a_0$	$9.1a_0$	$17a_0$	$35a_0$	$42a_0$

where a_0 is the amplitude of the smallest orbit in hydrogen, 0.532×10^{-8} cm.

In order to compare the amplitudes in parhelium with those in orthohelium and the latter with the calculated amplitudes it is convenient to introduce a "total" amplitude defined by the relation

$$I(p,q) = \frac{[2\pi\nu(p,q)]^4}{3c^3} a^2(p,q) 3N(p) \tag{20}$$

where $N(p)$ is the total number of atoms in the state p. From any amplitude for specific values of m' and m'' this total amplitude is easily obtained by the Kronig rules.

EXPERIMENTAL METHOD

The method of Lo Surdo, in which the electric field of the cathode dark space produces the Stark effect was used. The tube was set with the surface of the cathode parallel to the slit of the spectrograph, so that the light falling on the slit was all from a portion of the tube in which the field intensity was approximately the same. A cathode of molybdenum, one-half centimeter in diameter, was used. This was enclosed in two quartz tubes fitting one over the other, the inner of which ended

[18] Schrödinger, Ann. d. Physik **79**, 361 (1926).
[19] Heisenberg, Zeits. f. Physik **38**, 421; **39**, 499 (1926).

at the surface of the cathode, while the outer tube extended about two centimeters further. A small slit was cut in the outer tube close to the surface of the cathode where the glow is brightest and the image of this slit focussed on the slit of the spectrograph. A double image prism was used to separate the parallel components of the lines from the perpendicular. Two direct current generators, each capable of supplying 4,000 volts, were used in series. The voltage supplied could be varied by varying the current through the magnets of the generator and was very constant. The helium used was purified by passing over charcoal at liquid air temperature, and a charcoal tube immersed in liquid air was always in the system when the discharge was passing. No impurity lines except the strongest lines of mercury, which appeared with low intensity, the carbon line at $\lambda = 4267$ and the Balmer lines showed on the plates. It was not found necessary to have the system of large volume when a charcoal tube immersed in liquid air was included in it. The tube usually ran so smoothly that there was no swinging of the needle of the voltmeter connected across it. A current of from 10 to 6 milliamperes with a voltage of from three to eight thousand volts was used, with exposures of from one to six hours. A plane grating spectrograph made in this laboratory and having a dispersion in the first order of 17 angstroms per millimeter was used. The blackening of the plate was measured in a Moll microphotometer of magnification about 100. The method of obtaining the intensities of the lines from the blackening of the plate was the following: The spectrum of a Phillips Argenta lamp was photographed with a constant current flowing through the lamp. A large number of exposures were taken, each two minutes long, with varying slit width. The blackening of the plate was then measured at a constant wave-length and varying slit width and the photographic density plotted against the logarithm of the slit width. The curve obtained was a straight line for all except very low blackenings. The lower, curved portion of the curve was plotted from a large number of points and appeared to give as accurate results as the straight line portion. Such curves were made for a number of wave-lengths over the range of wave-lengths used in this investigation. All curves were taken from more than one plate and points from different plates fell on the same curve with the same accuracy as did points from the same plate. Blackening curves of wave-lengths between four and five thousand were of the same form except for small differences in the curved portions but for wave-lengths shorter than four thousand the slope of the curve dropped off rapidly with decreasing wave-length. To determine the intensity of a line in the helium spectrum the blackening of the plate

was measured in the photometer and the slit width corresponding to this blackening was read off the curve taken for a wave-length near that of the line. (It is assumed that the intensity of the light falling on the plate is for a continuous spectrum proportional to the width of the slit of the spectrograph.) It is obvious that this method of comparing intensities does not allow the comparison of intensities of lines whose wave-lengths differ appreciably from each other, unless the distribution of energy with wave-length is known for the lamp used, which was not the case in this work. It is hoped later to determine the sensitivity of the plate to different wave-lengths so that lines of different wave-lengths may be compared. Ilford Special Rapid Chromatic plates were used. They were developed in one part rodinol to twenty parts water at 18°C for four minutes. Any desired slope may be obtained for the blackening curve by varying the time of development.

Where it was desired to compare the intensity of lines of very different intensities an exposure long enough to bring out the weak line with measureable intensity gave the strong line too greatly overexposed to measure its blackening. For this reason exposures of one-tenth and one-one-hundredth the time of exposure of the strong exposure were made. The blackening of as many lines as possible which could be measured on two exposures gave a value of the ratio of the intensities of the two exposures and from this ratio the intensity of the strong line on the strong exposure was calculated. Where the intensity of parallel and perpendicular lines is compared, the intensities of the exposures were compared by assuming that such lines as $2p-ms$, $2p-4d$, etc. were unpolarised. As these lines all gave the same value for the ratio of the intensity of the parallel to the intensity of the perpendicular exposure the assumption was justified by the results.

We cannot *a priori* expect the blackening curves to be of the same form for exposures of different lengths so that the procedure used here is open to obvious theoretical objections. The results show, however, that no large error was introduced by using blackening curves taken with two minute exposures to determine the intensity of lines in exposures of times up to six hours. Where a number of lines were measured in a two minute and in a twenty-five minute exposure and the ratio of the energy which had fallen on the plate during the two minute exposure to that which had fallen on the plate in the half hour exposure determined from each of these lines the values obtained from the different lines agreed fairly well with each other. The same thing was true for the twenty-five minute and four hour exposures. That is, the measured ratio of the different lines to each other was the same for a two minute,

a twenty-five minute and a four hour exposure. The largest a.d. obtained in measuring the ratio of the intensity of two exposures to each other was thirty percent and the a.d. was usually between ten and twenty percent. For the ratio of the intensity of the parallel to the intensity of the perpendicular exposures the agreement was in general better, even when values taken from exposures of different lengths but taken under the same conditions were averaged. Unless some compensation of errors takes place a systematic error in the measurements should also appear as a systematic change with field of the value of the intensity of a combination line divided by the square of the field.

RESULTS

Measurements are given of the intensities of the groups of lines $2P-nM$ and $2p-nm$ for $n=4, 5, 6, 7$, $M=P, D, F, G$, etc. In addition the intensity of the combination lines $2S-4D$ and $2S-4F$ and the line $2S-4P$ have been measured. Table I gives the ratios of the intensities of the combination lines of parhelium to the nearest original line for all

TABLE I

The intensities of the combination lines of parhelium and the amplitudes calculable from them

Field $\frac{Kv}{cm}$	$\frac{I^\pi(4D,2S)}{I^\pi(4P,2S)}$	$\frac{I^\pi(4D,2S)}{I^\pi(4D,2P)} \div E^2 \times 10^{10}$ $\frac{cm^2}{v^2}$	$\frac{I^\pi(4F,2S)}{I^\pi(4P,2S)}$	$\frac{I^\pi(4F,2S)}{I^\pi(4F,2P)} \div E^4 \times 10^{19}$ $\frac{cm^4}{v^2}$	$\frac{I^\pi(4P,2P)}{I^\pi(4D,2P)}$	$\frac{I^\pi(4P,2P)}{I^\pi(4D,2P)} \div E^2 \times 10^{10}$ $\frac{cm^4}{v^2}$	$\frac{I^\pi(4F,2P)}{I^\pi(4D,2P)}$	$\frac{I^\pi(4F,2P)}{I^\pi(4D,2P)} \div E^2 \times 10^{10}$ $\frac{cm^2}{v^2}$	$\frac{I^\pi(5P,2P)}{I^\pi(5D,2P)}$	$\frac{I^\pi(5P,2P)}{I^\pi(5D,2P)} \div E^2 \times 10^{10}$ $\frac{cm^2}{v^2}$
13.2					.025*	1.4				
14.2	.026	1.3	.010	2.6	.031	1.6	.30	15	.16	8.0
16.2	.028	(1.1)	.021	3.1	.032	1.2	.36	14	.34	13.
18.6	.035	(1.0)	.33	2.8	.053	1.5	.55	16	.44	13.
19.8	.036	(.92)	.33	(2.1)	.053†	(1.3)	.54†	(14)	.49†	13.
24.3	.049	(.89)	.46	(1.6)						
Average				2.8		1.4		15		12

Amplitude (cm)	$a(4P,4D)$ 1.2×10^{-7}	$A(4P,4D)A(4D,4F)$ 8.0×10^{-15}	$a(4P,4D)$ 1.3×10^{-7}	$a(4D,4F)$ 5.1×10^{-8}	$a(5P,5D)$ 2.1×10^{-7}
Calc. Amplitudes	1.1×10^{-7}	4.9×10^{-15}	1.1×10^{-7}	4.6×10^{-8}	1.7×10^{-7}

*From the measurements of Takamine and Werner.
†At this and higher fields the D-line is probably double.
Values in parentheses were not used in calculating the amplitudes.

the combination lines measured which may be considered to be emitted in a weak field. Table II gives the same information for orthohelium. The amplitudes calculable from the measured ratios are given in both tables. It is seen at once from the tables that the measured agree approximately with the theoretical values of the amplitudes. The intensity of the combination lines divided by the square of the field shows no systematic variation except in the case of $2S-4D$. Since the $4D$ term

TABLE II

The intensities of the combination lines of orthohelium and the amplitudes calculable from them

Field $\frac{\text{Kv}}{\text{cm}}$	$\frac{I^\pi(4p,\ 2p)}{I^\pi(4d,\ 2p)}$	$\frac{I^\pi(4p,\ 2p)}{I^\pi(4d,\ 2p)} \div E^2 + 10^{10}$ $\frac{\text{cm}^2}{\text{v}^2}$	$\frac{I^\pi(4f,\ 2p)}{I^\pi(4d,\ 2p)}$	$\frac{I^\pi(4f,\ 2p)}{I^\pi(4d\ 2p)} \div E^2 \times 10^{10}$ $\frac{\text{cm}^2}{\text{v}^2}$	$\frac{I^\pi(5p,\ 2p)}{I^\pi(5d,\ 2p)}$	$\frac{I^\pi(5p,\ 2p)}{I^\pi(5d,\ 2p)} \div E^2 \times 10^{10}$ $\frac{\text{cm}^2}{\text{v}^2}$
13.2	.0015*	.086*				
14.2	.0017	.086	.16	.80	.026	1.3
16.2	.0023	.089	.17	.66	Hg line superposed	
18.6	.0028	.080	.20	.57	.043	1.2
19.8	.0031	.079	.25	.65	.040	1.0
24.3	.0049	.089	.34	.63	.12	(2.2)
Average		$.085 \pm .004$		$.66 \pm 0.05$		$1.2 \pm .1$
Amplitude (cm) Calc.		$a(4p, 4d)1.6 \times 10^{-7}$		$a(4d, 4p)6.2 \times 10^{-8}$		$a(5p, 5d)3.1 \times 10^{-7}$
Amplitudes		1.1×10^{-7}		4.6×10^{-8}		1.7×10^{-7}

*From the measurements of Takamine and Werner.

lies very close to the $4F$ term this variation is not unexpected. The deviation of the measured ratios over the square of the field from the average values gives some idea of the experimental accuracy, at least for orthohelium where more measurements may be averaged. It is particularly satisfactory that the measurements made by Takamine and Werner agree with those given here for the $p-p$ lines. Their measurements of the $p-f$ lines are not included since they did not use sufficient dispersion to separate these lines from the $p-d$ lines.

The amplitudes of $4D-4P$, $4F-4D$ etc. calculated from these measurements show a systematic deviation from those calculated for hydrogen. More weight is to be attached to the values for $2D-4D$, $2p-4p$ and $2p-5p$ than to the others since the condition for a weak field is better fulfilled for these lines. It is seen that the amplitudes calculated from the intensities of these lines are consistently high. The agreement among the values themselves for parhelium is better, the amplitude $a(4P,\ 4D)$

calculated from the intensity of $2S-4D$ agreeing within fifteen percent with the amplitude $a(4P, 4D)$ calculated from the intensity of $2P-4D$ and the product $a(4P, 4D)\ a(4D, 4F)$ calculated from $2S-4F$ agreeing fairly well with the product calculated from the intensities of $2P-4P$ and $2P-4F$. It will at first be thought that the agreement could be improved by taking into account more terms in the formulas used in

TABLE III

The ratios of the intensities of the parallel components of the combination lines to perpendicular components

Field $\dfrac{Kv}{cm}$	$\dfrac{I^\pi(4P, 2P)}{I^\sigma(4P, 2P)}$	$\dfrac{I^\pi(5P, 2P)}{I^\sigma(5P, 2P)}$	$\dfrac{I^\pi(6P, 2P)}{I^\sigma(6P, 2P)}$	$\dfrac{I^\pi(7P, 2P)}{I^\sigma(7P, 2P)}$	$\dfrac{I^\pi(4D, 2S)}{I^\pi(4D, 2S)}$	$\dfrac{I^\pi(4F, 2P)}{I^\sigma(4F, 2P)}$
			Parhelium			
14.2	2.9	2.6	2.5	2.4	1.3	1.1
16.2	2.5	1.8			1.1	1.4
18.6	2.5	2.6			1.4	1.2
19.8	2.6				1.6	(.95)
24.3	(3.3)				(2.1)	
Av.	2.6	2.3	2.5	2.4	1.3	1.2
Calc.	2.6	2.6	2.6	2.6	1.3	1.3

Field $\dfrac{Kv}{cm}$	$\dfrac{I^\pi(4p, 2p)}{I^\sigma(4p, 2p)}$	$\dfrac{I^\pi(5p, 2p)}{I^\sigma(5p, 2p)}$	$\dfrac{I^\pi(6p, 2p)}{I\delta(6p, 2p)}$	$\dfrac{I^\pi(7p, 2p)}{I^\sigma(7p, 2p)}$	$\dfrac{I^\pi(4f, 2p)}{I^\sigma(4f, 2p)}$
			Orthohelium		
14.2	2.0	2.0	2.0	2.5	1.5
16.2			2.4		1.2
18.6	1.7	1.9	2.2	2.6	1.6
19.8	1.6	1.7	2.6	1.5	1.5
24.3	1.5	1.7	1.6	1.8	
Av.	1.7	1.8	2.2	2.1	1.4

calculating the amplitudes, since the sum of a large number of small terms may become appreciable compared with one large term. Additional terms, however, will not all be of the same sign and will tend to cancel out. It is more likely that the disagreement is due almost entirely to experimental error, introduced from the source discussed in the last section or in other ways. It is unlikely that the experimental error is larger than forty percent, but it may be as large as this, at least where lines of very different intensities are compared. The comparison of lines of nearly the same intensity and of the same wave-length is considerably more accurate. (Compare, for instance, the agreement shown by the values given in Table III with the agreement shown by the values in the first two tables.) It is possible that the large disagreement shown for $a(4p-4d)$ is connected with the fact that the frequencies of the lines

$2p-4p$ and $2p-4d$ differ by more than do the frequencies of any other pair of lines of which the intensities have been compared.

In Table III the ratios of the intensity of the parallel components of the combination lines to the perpendicular components of the lines are given, together with the calculated value of this ratio. The agreement between measured and calculated values is better than is to be expected, even lines which must be considered to be emitted in a very high field such as $2p-2p$ giving approximately the calculated ratio. The ratios for the triplet system do not seem to approach those for the
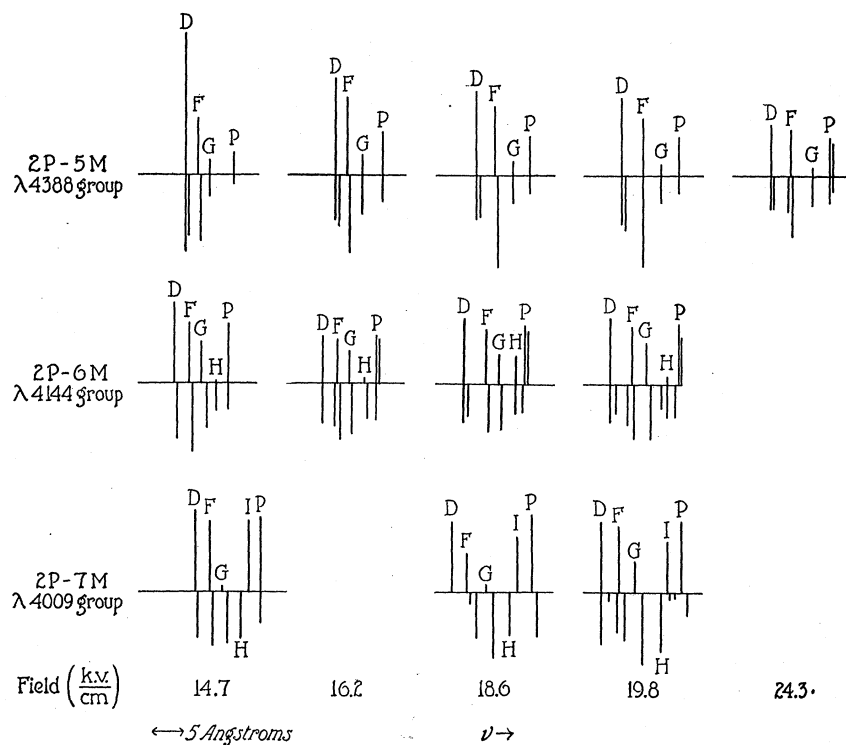

Fig. 4. The relative intensities of parhelium lines in various fields.

singlet system as the field increases, as would be expected from analogy with the Zeeman effect. The ratios not given in the table are of lines believed to be unresolved doublets or of lines too weak to measure. All the measurements on parhelium lines of principal quantum number higher than four are affected by insufficient resolution.

In Figs. 4 and 5 the results of the measurements on $2P-nM$ and $2P-nm$ are given for $n=5$, 6 and 7. The length of the lines represents

the intensities, the unit being arbitrarily chosen so that the sum of the intensities of all lines in one group is the same for different fields and different groups of lines. It is therefore only possible to use the figures for comparison within one group. The parallel and perpendicular lines are reduced to the same units, and the parallel lines represented by lines above and the perpendicular ones by lines below the base line. The
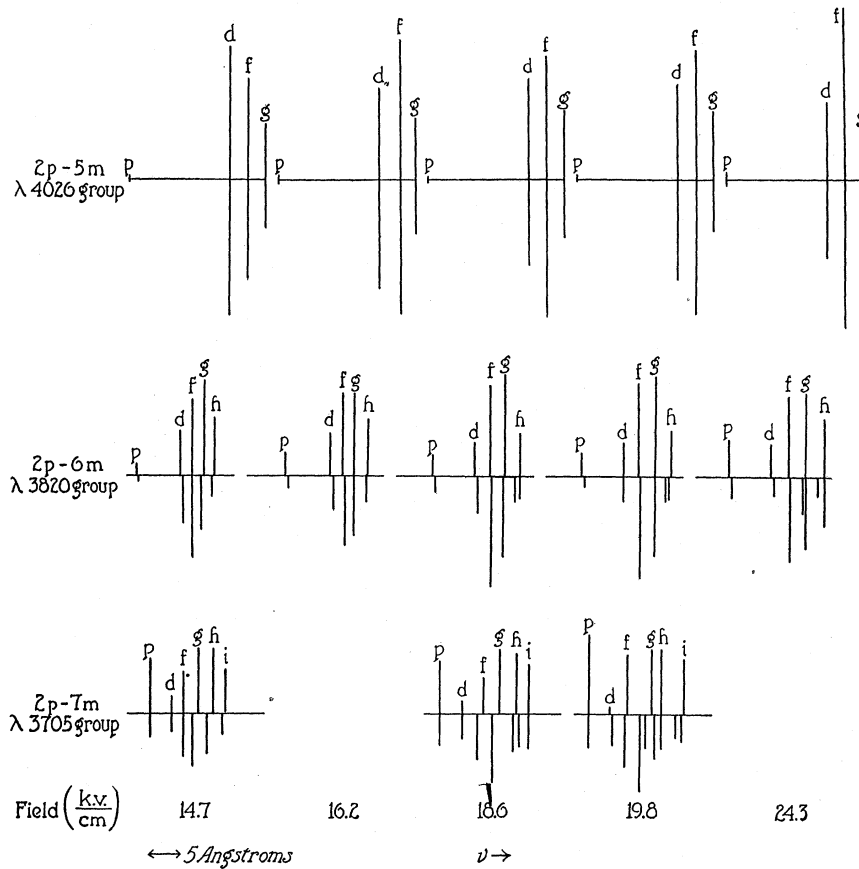

Fig. 5. The relative intensities of orthohelium lines in various fields.

groups omitted were too weak to measure. More weight is to be given Fig. 4 than Fig. 5, because of the error in the parhelium lines just referred to. When a doublet is only partially separated the measured intensity lies between the sum of the intensities of the two lines composing it and the intensity of the strongest line of the doublet. The results show an increasing resemblance between the hydrogen and the helium Stark

effect as we increase the field or go to lines of higher quantum number. It does not seem possible to find a detailed connection between the intensities in hydrogen and helium, however. In general the lines which are weak in hydrogen in strong fields are those which diminish in intensity with increasing field in helium, but the rate of decrease does not seem to be that to be expected from comparison with hydrogen. Nor do parhelium and orthohelium resemble each other closely. A striking example of this is that the d lines disappear very much more rapidly in orthohelium than in parhelium although parhelium, because of the smaller separation of its terms must be considered to be emitting in the stronger field.

The experiments reported here were carried out in the University Institute for Theoretical Physics in Copenhagen. The author wishes to express her thanks to Professor Niels Bohr for the suggestion of the problem and for his interest and advice throughout the work, to Mr. Sven Werner for placing at her disposal the results of his experience in the same work, and to Professor Kramers and Dr. Heisenberg for valuable assistance in handling the theory. To the undergraduates of Barnard College are due her best thanks for the award of the fellowship which made her stay in Copenhagen possible.