NATURAL IONIZATION IN SPHERICAL CONTAINERS; THEORETICAL CONSIDERATIONS

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ABSTRACT

Theory of the variations with pressure and size of container of the natural ionization in gases in spherical containers.—The variations with pressure and size of container of the natural ionization in gases in spherical containers are explained in terms of simple radiations. These radiations differ chiefly with regard to their origin and range. Radiations with equal propagation in all directions, uniform ionization along a path, and with ranges either greater or less than the diameter of the sphere, are considered with origin either throughout the volume or at the inner surface of the container. Radiations are also considered with origin at the inner surface of the sphere, direction normal to the surface, and absorption exponential with the distance traversed. The observed pressure variation of the natural ionization in air is given accurately by the formulas derived, upon assuming the ionization to be due entirely to the secondary and tertiary radiations excited in the gas and its container by a primary penetrating radiation. The volume variation is given qualitatively but not quantitatively in this instance, and the constants are interpreted. The pressure variation in the case of carbon dioxide is also closely approximated.

INTRODUCTION

IN a recent experimental investigation,[1] it was found by the writer that the natural ionization in gases in closed spherical vessels increased with the pressure to 70 or 75 atmospheres, but that the pressure rate of increase became very small at high pressures. With certain gases, pressure-ionization curves became practically straight lines at high pressures, while with others the curvature persisted throughout. Measurements made with spheres of different sizes showed that at pressures above a few atmospheres the ionization did not vary unidirectionally with the size of the container, but passed through a minimum.

In view of the facts mentioned above, it was thought worth while to determine as a function of the size of the container and the pressure of the gas, the ionization which would be produced in a gas in a spherical container by a few very simple radiations.

RAYS WITH ORIGIN THROUGHOUT VOLUME

Rays of Range $C \geqq 2a$. Entirely for facility of mathematical derivation, we shall separately consider radiations of certain restricted ranges.

[1] J. W. Broxon, Phys. Rev. **27**, 542 (1926).

Therefore, we shall first consider the ionization due to rays of very long range which may be supposed to travel in straight lines until they reach the walls of the container, whatever be the pressure, i.e., rays of range $\geqq 2a$, where a is the radius of the spherical ionization chamber. We shall assume that the ionization per cm of path of these rays is constant (for a fixed pressure), and that the rays are emitted uniformly in all directions from their sources. If we imagine such rays to originate in a uniform manner throughout the volume of the container, then if $4\pi\gamma_c$ is the number of rays emitted per sec. from unit volume, $\gamma_c d\tau$ is the number of rays emitted per sec. in unit solid angle from the volume element, $d\tau$.

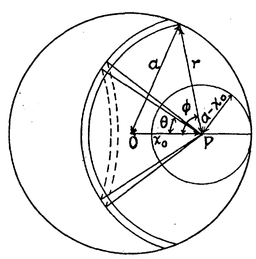

Fig. 1.

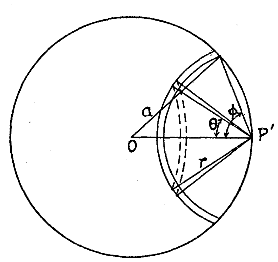

Fig. 2.

If C_c be the number of ions produced per cm of path by one "ray," $\gamma_c C_c d\tau d\Omega ds$ represents the number of ions produced per sec. along the element, ds, of their path by the rays emitted within the solid angle, $d\Omega$, from the volume element, $d\tau$.

Let us now proceed to determine N_c', the total number of ions produced per sec. within the spherical chamber by the rays emitted from a particular volume element, $d\tau$, situated at P (Fig. 1), distant x_o from O, the center of the chamber.

The number of ions produced per sec. inside a sphere of radius $(a-x_o)$ with center at P, is seen to be

$$N_{c1}' = 4\pi\gamma_c C_c d\tau(a-x_0).$$

To find N_{c2}', the number of ions produced per sec. outside this sphere but within the chamber, let us note that the number produced per sec. in the region between the two spherical surfaces with centers at P and of radii r and $(r+dr)$, cut off by all radii of the latter which make angles θ or $(\theta+d\theta)$ with OP, is

$$dN_{c2}' = \gamma_c C_c d\tau d\Omega ds = \gamma_c C_c d\tau \frac{2\pi r^2 \sin\theta d\theta}{r^2} dr.$$

Therefore,

$$N_{c2}' = 2\pi\gamma_c C_c d\tau \int_{r=a-x_0}^{r=a+x_0} \int_{\theta=0}^{\theta=\phi} \sin\theta d\theta dr, \text{ where } \cos\phi = \frac{x_0^2 + r^2 - a^2}{2x_0 r}$$

$$= \pi\gamma_c C_c d\tau \left[\frac{(a^2 - x_0^2)}{x_0} \log\frac{a+x_0}{a-x_0} + 4x_0 - 2a \right]$$

$$N_c' = N_{c1}' + N_{c2}' = \pi\gamma_c C_c \left[\frac{(a^2 - x_0^2)}{x_0} \log\frac{a+x_0}{a-x_0} + 2a \right] d\tau$$

The first term in the above bracket approaches 0 as $x_o \to a$, and $2a$ as $x_o \to 0$. We are therefore justified in using the value found for N_c' in calculating N_c, the total number of ions produced per sec. by all the rays of range $\geqq 2a$ produced throughout the volume of the sphere.

We have assumed that the rays originate uniformly throughout the chamber, which is the equivalent of assuming that γ_c is the same for all volume elements. We shall also consider C_c to be the same for rays from all volume elements. Therefore, the number of ions produced within the ionization chamber by rays emitted from the region between spherical surfaces of radii r_o and $(r_o + dr_o)$ with centers at O, is given by

$$dN_c = \pi\gamma_c C_c \left[\frac{(a^2 - r_0^2)}{r_0} \log\frac{a+r_0}{a-r_0} + 2a \right] 4\pi r_0^2 dr_0 .$$

Therefore,

$$\frac{N_c}{4\pi^2\gamma_c C_c} = \int_{r_0=0}^{r_0=a} \left[(a^2 r_0 - r_0^3) \log\frac{a+r_0}{a-r_0} + 2ar_0^2 \right] dr_0 = a^4 ,$$

as is seen after proper evaluation of the first term. Hence $N_c = 4\pi^2\gamma_c C_c a^4$. The number of ions per cc per sec. produced by rays of this range originating throughout the volume of the sphere is therefore given by

$$n_c = \pi\gamma_c C_c (3a) .$$

Rays of Range B, where $a \leqq B \leqq 2a$. We shall now proceed to find expressions similar to the preceding, for ionization due to radiations resembling the ones first discussed but of definite range of a magnitude between that of the radius and the diameter of the sphere. Let us note that the longest path of the radiations from any volume element at a distance x_o from the center of the chamber is B, providing $B \leqq (a + x_o)$, or $x_o \geqq (B - a)$; whereas the longest path is $(a + x_o)$, in case $B \geqq (a + x_o)$, or $x_o \leqq (B - a)$.

The number of ions produced per sec. by rays originating in a certain volume element, $d\tau$, situated inside a spherical surface concentric with the chamber wall and of radius, $(B-a)$, is therefore

$$N_B' = 4\pi\gamma_B C_B(a-x_0)d\tau + 2\pi\gamma_B C_B d\tau \int_{r=a-x_0}^{r=a+x_0} \int_{\theta=0}^{\theta=\phi} \sin\theta d\theta dr, \text{ where } \cos\phi = \frac{x_0^2+r^2-a^2}{2x_0 r}$$

$$= \pi\gamma_B C_B d\tau \left[\frac{(a^2-x_0^2)}{x_0} \log\frac{a+x_0}{a-x_0} + 2a \right] = dN_{B1},$$

where γ_B and C_B correspond to γ_c and C_c, respectively, and where N_{B1} represents the total number of ions produced per sec. by rays originating within the sphere of radius $(B-a)$ concentric with the chamber wall. Then

$$N_{B1} = \pi\gamma_B C_B \int_{r_0=0}^{r_0=B-a} \left[\frac{(a^2-r_0^2)}{r_0} \log\frac{a+r_0}{a-r_0} + 2a \right] 4\pi r_0^2 dr_0$$

$$= 4\pi^2\gamma_B C_B \left[\frac{(a^2-r_0^2)^2}{4} \log\frac{a-r_0}{a+r_0} + \tfrac{1}{2}(a^3 r_0 + a r_0^3) \right]_{r_0=0}^{r_0=B-a} .$$

The number of ions produced per sec. by rays originating in a volume element situated without the designated spherical surface of radius $(B-a)$ but within the chamber, is given by

$$N_{B2}' = 4\pi\gamma_B C_B(a-x_0)d\tau + 2\pi\gamma_B C_B d\tau \int_{r=a-x_0}^{r=B} \int_{\theta=0}^{\theta=\phi} \sin\theta d\theta dr$$

$$= \pi\gamma_B C_B d\tau \left[\frac{(a^2-x_0^2)}{x_0} \log\frac{B}{a-x_0} + \frac{a^2-B^2}{2x_0} - 3x_0/2 + 2B + a \right] = dN_{B2},$$

where N_{B2} represents the total number of ions per sec. due to rays originating in the entire region within the chamber and outside the surface of radius $(B-a)$. Therefore,

$$N_{B2} = \int_{r_0=B-a}^{r_0=a} dN_{B2} = 4\pi^2\gamma_B C_B \left[\left(B^3 a - B^2 a^2 - \frac{1}{4} B^4 \right) \log(2a-B) + \left(\frac{1}{4} B^4 + B^2 a^2 - B^3 a \right) \log B + a^4 - \frac{2}{3} B a^3 + B^2 a^2 - \frac{1}{2} B^3 a + \frac{B^4}{48} \right].$$

The total numbei of ions per sec. produced by rays of range B originating throughout the volume of the chamber in the manner described, is

$$N_B = N_{B1} + N_{B2} = \pi^2\gamma_B C_B\left[\frac{16}{3}Ba^3 - 2B^2a^2 + \frac{1}{12}B^4\right].$$

Therefore, the number of ions per cc per sec. due to these rays is

$$n_B = \pi\gamma_B C_B\left[4B - \frac{3}{2}B^2a^{-1} + \frac{1}{16}B^4a^{-3}\right]$$

Rays of Range A, where $0 \leqq A \leqq a$. To find expressions similar to the preceding, for ionization due to similar rays of range not greater than the radius of the chamber, let us note that all such rays which originate within a sphere concentric with the container and of radius $(a-A)$, will complete their range. Hence the number of ions produced per sec. by rays of range A from a volume element within this region is $N_{A1}' = 4\pi\gamma_A C_A A d\tau$, and the total number of ions produced per sec. by rays originating inside this sphere is $N_{A1} = 4\pi\gamma_A C_A A \cdot \frac{4}{3}\pi(a-A)^3$, where γ_A and C_A correspond to previous similar symbols.

Let us next determine the number of ions produced per sec. by rays originating in the region outside the designated sphere of radius $(a-A)$. The number of ions produced per sec. by rays originating in a volume element, $d\tau$, in this region at a distance, x_o, from O, where $(a-A) < x_o \leqq a$, is

$$N_{A2}' = 4\pi\gamma_A C_A(a-x_0)d\tau + 2\pi\gamma_A C_A d\tau\int_{r=a-x_0}^{r=A}\int_{\theta=0}^{\theta=\phi}\sin\theta d\theta dr, \text{ where } \cos\phi = \frac{x_0^2+r^2-a^2}{2x_0 r}$$

$$= \pi\gamma_A C_A d\tau\left[\frac{(a^2-x_0^2)}{x_0}\log\frac{A}{a-x_0} + \frac{a^2-A^2}{2x_0} - 3x_0/2 + 2A + a\right] = dN_{A2},$$

where N_{A2} represents the number of ions per sec. due to all rays of Range A originating outside the sphere of radius, $(a-A)$. Therefore,

$$N_{A2} = \pi\gamma_A C_A\int_{r_0=a-A}^{r_0=a}\left[\frac{(r_0\ -a^2)}{r_0}\log\frac{a-r_0}{A} + \frac{a^2-A^2}{2r_0} - 3r_0/2 + 2A + a\right]4\pi r_0^2 dr_0$$

$$= 4\pi^2\gamma_A C_A\left[7A^2a^2/2 - 4A^3a + \frac{65}{48}A^4\right]$$

The number of ions per sec. due to all such rays originating throughout the spherical container is therefore

$$N_A = N_{A1} + N_{A2} = \pi^2\gamma_A C_A[16Aa^3/3 - 2A^2a^2 + A^4/12]$$

and the number of ions per cc per sec. due to these rays is

$$n_A = \pi\gamma_A C_A[4A - 3A^2a^{-1}/2 + A^4a^{-3}/16]$$

Rays with Origin at Surface of Container

Rays of Range $C \geqq 2a$. Let us next consider the effect due to rays of the same range and ionizing ability as those first considered, but which are supposed to originate uniformly over the interior surface of the chamber and to be emitted radially and uniformly in all directions from their sources. If $4\pi\gamma_c'$ represent the total number of rays of this range (effective and otherwise) which originate from unit area of the surface, and C_c', the number of ions produced per cm of path by one of these rays, then the number of ions produced per sec. along the element, ds, of their path by the rays emitted within the solid angle, $d\Omega$, from the surface element, dS, is $\gamma_c' C_c' dS d\Omega ds$, providing $d\Omega$ and ds be within the chamber.

The total number of ions produced per sec. within the spherical chamber by the rays emitted from a particular element, dS, situated at P' (Fig. 2), is

$$N_c'' = 2\pi\gamma_c' C_c' dS \int_{r=0}^{r=2a} \int_{\theta=0}^{\theta=\phi} \sin\theta d\theta dr, \text{ where } \cos\phi = r/2a$$

$$= 2\pi\gamma_c' C_c' a dS.$$

The total number of ions produced within the chamber in one second by rays of this type emitted from the entire surface is therefore $N_c''' = 8\pi^2\gamma_c' C_c' a^3$, and the number of ions per cc per sec. produced by these rays is $n_c' = 6\pi\gamma_c' C_c'$.

Rays of Range B, *where* $a \leqq B \leqq 2a$. Let γ_B' correspond to γ_c', and C_B' to C_c', and as in the case of the long range radiations, let us proceed to find a value for n_B', corresponding to n_c'. The number of ions produced per sec. by rays from an element of area, dS, is given by

$$N_B'' = 2\pi\gamma_B' C_B' dS \int_{r=0}^{r=B} \int_{\theta=0}^{\theta=\phi} \sin\theta d\theta dr, \text{ where } \cos\phi = r/2a$$

$$= 2\pi\gamma_B' C_B' (B - B^2/4a) dS.$$

The number of ions produced per sec. by the radiation from the entire surface is therefore $N_B''' = 8\pi^2\gamma_B' C_B' (Ba^2 - B^2 a/4)$ and the number of ions per cc per sec. due to these rays is

$$n_B' = 6\pi\gamma_B' C_B' (Ba^{-1} - \tfrac{1}{4}B^2 a^{-2}).$$

Rays of Range A, *where* $0 \leqq A \leqq a$. Exactly as in the case of rays of range B, the number of ions per cc per sec. due to rays of range A, originating at the inner surface of the container is found to be $n_A' = 6\pi\gamma_A' C_A' (Aa^{-1} - \tfrac{1}{4}A^2 a^{-2})$.

It will be noticed that the expressions found for "n" and "n'" exactly correspond in the two cases of ionization due to radiations of definite range less than the diameter of the ionization chamber. The only reason for considering the two sets of ranges separately was to facilitate the choosing of the proper definite limits of integration. We may therefore sum up these two cases by noticing that if $0 \leqq D \leqq 2a$, the number of ions per cc per sec. produced by radiations of range D originating uniformly throughout the volume, proceeding uniformly in all directions from the volume elements in which they originate, and ionizing uniformly along their paths, is given by

$$n_D = \pi\gamma_D C_D(4D - 3D^2a^{-1}/2 + D^4a^{-3}/16).$$

The number of ions per cc per sec. produced by such rays originating uniformly at the inner surface of the container is

$$n_D' = 6\pi\gamma_D' C_D' (Da^{-1} - D^2a^{-2}/4).$$

Rays of Exponentially Decreasing Ionizing Ability

Proceeding as does W. Wilson,[2] we may derive an expression for the ionization which would result from a radiation given off normally and uniformly from the interior surface of the container and being absorbed according to an exponential law with the distance traversed.

If dI' represents the number of ions per sec. produced in a layer of thickness dx at a normal distance x, by the rays originating on an element of area, dS, of the inner surface of the sphere, the total number of ions produced per sec. within the spherical chamber by the radiation from this element is given by

$$I' = \int dI' = \int_{x=0}^{x=2a} I_0 e^{-\lambda' x} dx dS = \frac{I_0}{\lambda'}(1 - e^{-2a\lambda'})dS.$$

Then the total ions per sec. due to the rays from the entire surface is

$$N_F = \frac{4\pi I_0}{\lambda'} a^2(1 - e^{-2a\lambda'})$$

and the number of ions per cc per sec.,

$$n_F = 3\frac{I_0}{\lambda'} a^{-1}(1 - e^{-2a\lambda'}).$$

Assuming $I_0 \propto \lambda' \propto P$, we may write

$$n_F = Ba^{-1}(1 - e^{-2\lambda a P}),$$

[2] W. Wilson, Phil. Mag. **6**, 216 (1909).

where B is a constant, P the pressure (in atmospheres) and λ the coefficient of absorption at atmospheric pressure.

In the above, no account has been taken of the possibility of the emission of the radiations in various directions or of their production within the metal beneath the inner surface of the container. The method of procedure in that case, as well as a certain justification for not considering the case here, may be found in a paper by Campbell.[3]

Dependence upon Pressure

The ionization formulas derived are collected in the following table:

Radiation Number.	*Origin.*	*Range.*	*Ions per cc per sec.*
C	throughout volume	$\geqq 2a$	$n_c = \pi\gamma_c C_c(3a)$.
D	throughout volume	$\leqq 2a$	$n_D = \pi\gamma_D C_D(4D - 3D^2a^{-1}/2 + D^4a^{-3}/16)$.
C'	at surface	$\geqq 2a$	$n_c' = 6\pi\gamma_c' C_c'$.
D'	at surface	$\leqq 2a$	$n_D' = 6\pi\gamma_D' C_D'(Da^{-1} - D^2a^{-2}/4)$.
F	at surface	—	$n_F = Ba^{-1}(1 - e^{-2\lambda aP})$

The above expressions show how the ionization would depend upon the radius of the container if it were due to any of the radiations considered, and the dependence upon the pressure also is shown in the case of n_F. It is now desired to arrive at a conclusion as to how the remaining expressions may be expected to depend upon the pressure. With this in view, the following suggestions as to the possible causes of the radiations are made: (1) a very penetrating radiation from a considerable distance; (2) spontaneous ionization of the gas; (3) a radioactive substance admitted with the gas; (4) chemical action between the gas and the walls of the container; (5) chemical action between two component gases; (6) radioactive contamination of the walls of the container; (7) disintegration of the walls of the container. Other possibilities readily suggest themselves, of course, but it is desired to restrict attention to a few simple ones.

Resulting γ's would probably be proportional to P if the ionization were due to causes (1), (2) or (3), and to P^2 if due to (5). If the ionization were due to (4), the resulting γ' would probably be proportional to P, while constant γ''s would be expected to result from (1), (6) or (7).

In all cases of uniform ionization along the paths of the rays, it seems reasonable to suppose that the ions per cm of path (the C's) would be

[3] N. R. Campbell, Phil. Mag. 9, 531 (1905).

proportional to the pressure, and that the ranges would be inversely proportional to the pressure. We therefore might have any $\gamma \propto P$, or $\propto P^2$; $\gamma' \propto P$, or constant; $C \propto C' \propto P$; $D \propto 1/P$. The corresponding expressions for n would then become:

$$n_c = K_1 P^2 a \quad , \text{ or } \quad n_c = K_2 P^3 a \quad ;$$
$$n_D = K_3 P - K_4 a^{-1} + K_5 P^{-2} a^{-3} , \text{ or } \quad n_D = K_6 P^2 - K_7 a^{-1} P + K_8 P^{-1} a^{-3} ;$$
$$n_c' = K_9 P \quad , \text{ or } \quad n_c' = K_{10} P^2 \quad ;$$
$$n_D' = K_{11} a^{-1} - K_{12} P^{-1} a^{-2} \quad , \text{ or } \quad n_D' = K_{13} P a^{-1} - K_{14} a^{-2} \quad ;$$

where the K's are constants.

Primary effects due to (1), (2), (3) and (4), apart from resulting radiations, would probably result in ions per cc per sec. proportional to the pressure at constant volume, and independent of changes in volume at constant pressure. Due to a similar primary effect of (5), the ions per cc per sec. might be proportional to the square of the pressure for constant volume, and vary directly with the volume at constant pressure.

Application to Experimental Observations

Let us now attempt by means of the formulas we have derived, to approximate the variation of the natural ionization with volume of chamber and pressure of gas as already determined experimentally.[1]

As a specific example, let us consider the ionization in air in the 9-inch sphere. Suppose this ionization is due entirely to a very hard radiation, the primary effect of which is negligible in comparison to its secondary and tertiary effects, so far as its contribution to the total ionization is concerned. Let us further suppose that the electrons ejected by the penetrating radiation from the gas itself have very long ranges (C-number), and that these in turn produce tertiary rays of short range (D-number); and that those electrons ejected by the penetrating radiation at the surface of the container are of two sorts, one emitted normally and being absorbed exponentially along its path (F-number), and the other being emitted diffusely and ionizing uniformly along its path which we shall consider to be 24 ft. in air at atmospheric pressure. This last radiation would then be termed C'-number at pressures below 32 atmospheres and D'-number at pressures above 32 atmospheres.

Apparently n_F and hence B must be proportional to S, the intensity of the primary penetrating radiation.

Let us suppose that the number of long-range or C-number rays produced per sec. is proportional to S, but not to the pressure, P, being independent of the latter. This might be the situation if they arose almost entirely from certain nuclei (as unremoved dust particles) distributed throughout the volume of the sphere, no new nuclei being admitted with additional gas. Thus we may assume $\gamma_c \propto S$ and $C_c \propto P$.

If the D-number rays are almost entirely of tertiary origin, we might expect their rate of production to be proportional to n_c and also to the pressure. It might conceivably be, however, although we are postulating that the C-number rays ionize uniformly along their paths, that the fraction of its ions which result in D-number rays decreases in some manner with the distance from its origin along the path of a C-number ray. The effective paths of the latter increase, in general, with the size of the sphere. We shall therefore assume that $\gamma_D \propto Pn_c/a$ and for the D-number rays, $D \propto 1/P$ and $C_D \propto P$.

Concerning the D'-number rays we shall simply assume $\gamma_D' \propto S$ and independent of P, $C_D' \propto P$, and $D \propto 1/P$.

Since the D'-number rays become C'-number rays at pressures below 32 atmospheres, we must likewise have $\gamma_c' = \gamma_D'$ and $C_c' = C_D'$.

In accordance with the foregoing hypotheses, we may now write our ionization formulas in the forms:

(1) $n_F = Ba^{-1}(1-e^{-2\lambda aP})$, (2) $n_c = G_1 aP$,

(3) $n_D = G_2P^2 - G_3a^{-1}P + G_4a^{-3}P^{-1}$, (4) $n_D' = G_5a^{-1} - G_6a^{-2}P^{-1}$,

(5) $n_c' = G_7P$,

where

(6) $G_1 = 3\pi K_2K_3S$, (7) $G_2 = 4\pi G_1K_4K_5K_6$,

(8) $G_3 = 3\pi G_1K_4K_5K_6^2/2$, (9) $G_4 = \pi G_1K_4K_5K_6^4/16$,

(10) $G_5 = 6\pi K_7K_8K_9S$, (11) $G_6 = 3\pi K_7K_8K_9^2S/2$,

(12) $G_7 = 6\pi K_7K_8S$,

where

(13) S represents intensity of the primary penetrating radiation.

(14) λ is coeff. of absorption of F-number rays in air at atm. press.

(15) $B(1-e^{-2\lambda})/S$ gives the ionization due to F-number rays per unit intensity of the primary radiation in air at atm. press. in a sphere of unit radius.

(16) $4\pi K_2$ is no. of C-number rays per unit vol. per sec. per unit S.

(17) $4\pi K_7$ is no. of C'- or D'-number rays per unit area per sec. per unit S.

(18) $4\pi K_4$ is no. of D-number rays per unit vol. per ion of the C-number at atm. press. in a sphere of unit radius.

(19) K_3 is no. of ions produced by a C-number ray per cm path at atm. press.

(20) K_5 is no. of ions produced by a D-number ray per cm path at atm. press.

(21) K_8 is no. of ions produced by a C'- or D'-number ray per cm path at atm. press.

(22) K_6 is range of D-number rays in air at atmospheric pressure.

(23) K_9 is range of D'-number rays in air at atmospheric pressure.

In Eqs. (1)–(5), let us put $\lambda=.081$; $B=34.0$; $G_1=.552-$; $G_2=.0001$; $G_3=.458$; $G_4=7.0$; $G_5=66.7$; $G_6=1057.8$; $G_7=1.066$. We have then

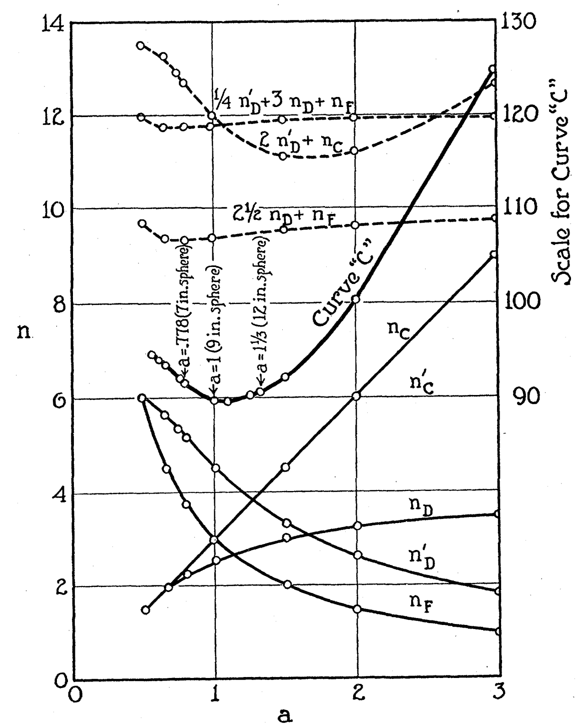

Fig. 3. In these curves (excepting curve C): $\pi\gamma_D C_D=\pi\gamma_C C_C=\pi\gamma'_D C'_D=\pi\gamma'_C C'_C=I=D$; $B=3$; $2\lambda P=10$.

as our pressure-ionization formulas (approximately):

(for $1\leqq P\leqq 32$)

$$n=n_F+n_c'+n_c+n_D=34(1-e^{-.162P})+1.16P+7P^{-1}+.0001P^2, \quad (24)$$

(for $P>32$)

$$n=n_F+n_D'+n_c+n_D=34(1-e^{-.162P})+\frac{3}{32}P$$
$$+.0001P^2+66.7-1050.8P^{-1}, \quad (25)$$

where the radius, a, has been chosen as unity. With the same values of the above λ, B, G's, we have as the relation between the ionization and the radius of the chamber at a pressure of 61.73 atmospheres, the approximate equation

$$n=n_F+n_D'+n_c+n_D=34a^{-1}(1-e^{-10a})+38.43a^{-1}+34a-17.136a^{-2}$$
$$+.38+.113a^{-3}. \tag{26}$$

In Fig. 4 (Air—9-in. S.) n as given by Eqs. (24) and (25) has been plotted as a function of P. It is seen that the observed pressure variation is very closely approximated.

In Fig. 3 (Curve C) n as given by Eq. (26) is plotted as a function of a. An inspection of this curve shows that by using the same constants (G's, etc.) as in the $n-P$ formulas, we were able to predict that the ionization at a given high pressure would pass through a minimum as the radius of the sphere was varied, and in fact this minimum is so situated that we should have less ionization in the 9-inch sphere than in either the 7-inch or the 12-inch sphere. Also, this difference is greater at higher pressures. However, this agreement is not quantitative, as the observed ionizations in the 7-inch and 12-inch spheres were much greater than our formula indicates. Thus according to Fig. 3 we should have $n=91.97$, 89.78, and 90.44 for the 7-, 9-, and 12-inch spheres respectively. The observed values, however, were approximately 100.0, 89.8 and 126.0— in the three cases, respectively. The difference here mentioned is so large that we must consider our n-a formulas quantitatively inadequate in spite of the fact that observations were taken with only three different spheres. It is possible that there were other variables such as the condition of the surfaces of the particular spheres used, which served to accentuate the variation in this instance. The matter could be determined definitely, of course, only by making observations with a considerable number of spheres of various sizes, care being taken that they should be very similar in all other respects. In this regard it is worth noting that the volume variations were much smaller when the spheres were lined with aluminum, although the ionization was still least in the 9-inch sphere.

Let us now turn to an inspection of our constants and their interpretation in terms of the fundamental phenomena. We have already assigned the value 0.081 to λ which gives the coefficient of absorption of the F-number radiation at atmospheric pressure, and the value 34.0 to B which with λ is a measure of the intensity of the F-number radiation in terms of the intensity of the penetrating primary radiation under specific conditions.

According to Eq. (6) we may choose K_2 which is a measure of the intensity of the C-number radiation, and K_3 which is a measure of the

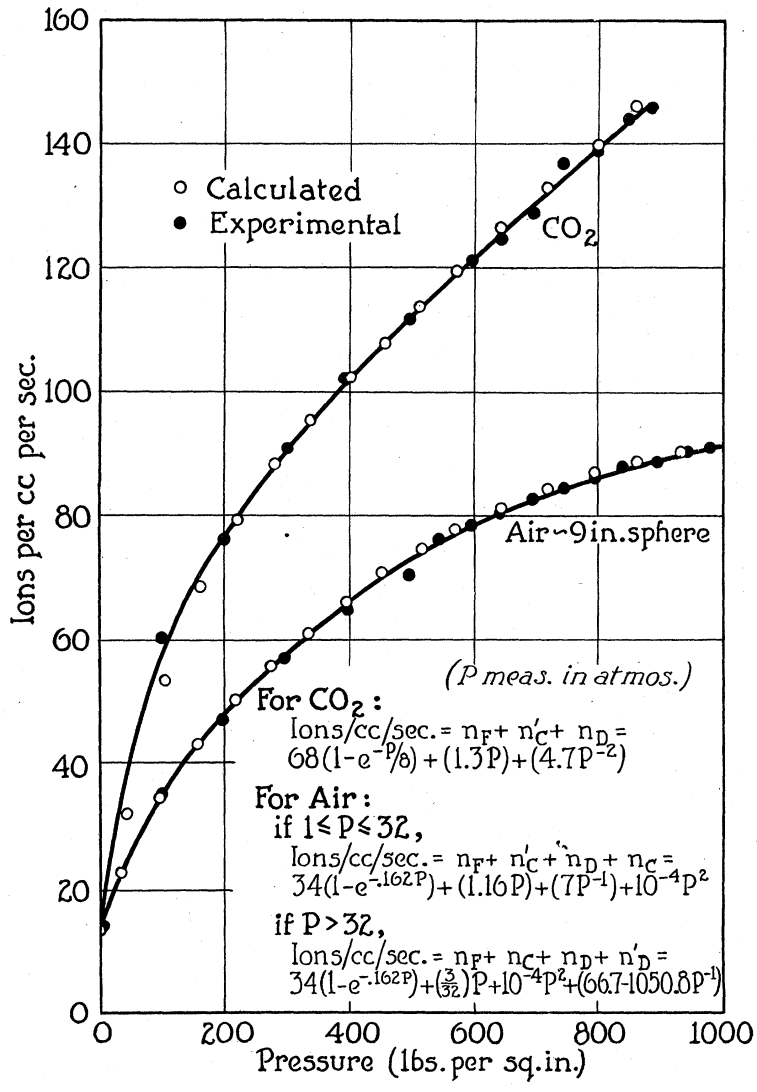

Fig. 4.

ionizing ability of the C-number radiation, in any way we like so long as we make $3\pi K_2 K_3 S = 0.552-$.

It is seen that Eqs. (10), (11) and (12) are not three equations defining three different K's, but that each pair defines K_9 if we assign values to the G's. Luckily there is no conflict with the values we have had to assign to G_5, G_6 and G_7, and we are only required to let $K_9 = 63.44-$. Then K_7 and K_8 may be chosen as we like so long as $K_7K_8S = 0.056$. It is very comforting to note that K_9 which represents the range of the D'-number rays at atmospheric pressure is very nearly 64 in terms of the radius of the sphere as unity. In accordance with our assumption, the D'-number radiation would therefore change to C'-number radiation at pressures below 32 atmospheres.

Turning to Eqs. (7), (8) and (9), we observe that we have here the same sort of relation as that existing among the group (10), (11) and (12). Here, however, we are not so fortunate. With the values we have found it advisable to assign to G_2, G_3 and G_4, there are no constant values for K_4, K_5 and K_6 which will satisfy the three equations simultaneously. We are led to conclude, therefore, that the D-number or tertiary radiations are themselves produced (and probably ionize) in a manner more complicated than we initially assumed. K_4, K_5 and K_6 may then be regarded not as constants, but as more complicated functions of the experimental conditions, which functions are so constituted that the G-functions of these may be regarded as constants. It should be noted that putting $G_2 = .0001$ merely indicates that the term in P^2 has very little significance in the $P-n$ formulas.

By way of summary, we may recall that we have derived ionization formulas by assuming the ionization to be due to a single penetrating primary radiation and its consequent effects. These formulas give the pressure variation in air quite accurately, and at the same time they give the volume variation qualitatively. With the exception of those pertaining to the tertiary radiation, all the constants occurring in these formulas have been found capable of simple interpretation in terms of characteristics of the radiations at atmospheric pressure.

Oxygen was found to behave in much the same way as air. $P-n$ curves for carbon dioxide and nitrogen were found to differ from those for oxygen and air in that they became straight lines at their high pressure ends. In Fig. 4 is included an approximation for the case of carbon dioxide. It is seen that the observed pressure variation could be approximated closely throughout by the assumption of radiations of the F-, C'-, and D-numbers. The volume variation was not investigated in this case, nor was it determined experimentally. In Fig. 3 are included several curves which show how the ionization might, in general, be expected to depend upon the size of the spherical chamber if due to the various radiations considered, or to certain combinations of these.

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