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DIRECT ABSOLUTE MEASUREMENT OF ACOUSTIC IMPEDANCE

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Abstract

The uniqueness of this method of measuring acoustic impedance (ratio of pressure to volume-current, lies in the fact that it involves the measurement of no other mechanical impedance, such as that of a telephone diaphragm. The computations involve values of the acoustic frequency and of the density of and sound velocity in the medium (air in this instance), and these are assumed to be known with sufficient accuracy. The only observations actually made are the readings of a centimeter scale and of relative resistances, and the disappearance of audible intensity. All measurements can thus be referred to units of mass, length and time. The method is based upon the knowledge of the effect of acoustic impedance upon the incident wave when inserted as a branch of an acoustic conduit.

Theory.—With the impedance as a branch the values of its two components can be ascertained by $Z_1=(\rho a/2S)\cdot A/(A^2+B^2)$ and $Z_2=(\rho a/2S)\cdot B/(A^2+B^2)$ wherein ρ and a are the density of and velocity of sound in the medium, S, is the area of the conduit and A and B are known functions of the incident and transmitted pressure amplitudes and of the difference of phase between them.

Method.—A common source actuated two telephone receivers, one serving as a source of sound and the other as a comparison instrument. The apparatus was so constructed that the transmitted wave was matched in intensity by one from the comparison receiver. This was done by reducing the effect of the addition of the two waves at the ears to zero. The adjustment enabled the phase relation to be measured at the same time.

Application.—Illustrations are given of the accuracy obtainable by measuring Z_1 and Z_2 for an orifice, a Helmholtz resonator and an infinite tube. Satisfactory comparisons are made with the theoretical values. The components of a conical horn are also given. The method can be used for any sort of acoustical impedance in a fluid medium and is simple, direct and accurate.

MODERN developments emphasize the importance of acoustic measurements. That of acoustic impedance is fundamental. It is herein defined as the ratio of pressure to volume-current both expressed as complex quantities, and is in fact a "point" impedance.[1] The published work on the measurement of acoustic impedance is not extensive. Kennelly and Kurokawa[2] have employed a method wherein determinations of the mechanical impedance of a telephone receiver are necessary.

[1] The definition of acoustic impedance herein adopted is to be preferred for it is convenient in acoustic theory and it is closely analogous to the accepted electrical treatment.

[2] Kennelly, J. of Franklin Inst. **200**, 467 (1925); Kennelly and Kurokawa, Proc. Am. Acad. **56**, 1 (1901); Kurokawa, Denkigakukai, 427, 1 (1924), (Translated in June 1925 issue of Q.S.T.).

The word "direct" in the title signifies that the method herein described is strictly acoustic and does not require the measurement of any other mechanical impedance. The word "absolute" indicates that all measurements can be referred to units of mass, length and time, for only the values of density of the medium, velocity of sound and of certain lengths are involved.

Theory

A previous article[3] gives the theory of the transmission in an acoustic conduit with a branch line. The ratio of the transmitted to incident flow of energy in the conduit is independent of the nature of the branch and is given by:

$$|P'/P|^2=[(Z_1^2+Z_1\rho a/2S+Z_2^2)+(\rho a Z_2/2S)^2][(Z_1+\rho a/2S)^2+Z_2^2]^{-2} \quad (1)$$

wherein P' and P are complex pressures at the junction of the branch, the former with the branch present and the latter with it absent, Z, the impedance of the branch line, is Z_1+iZ_2, ρ and a are respectively the density and velocity in the medium and S is the area of the conduit. If measurement of the transmission, that is, of the value of (1), be made with more than one value of S, since Z_1, Z_2, and the other quantities remain constant, then it is possible to obtain values of Z_1 and Z_2.

On the other hand the following leads to a more promising method. In the theory just cited[3] we have

$$P'=P-\rho a\dot{X}/2S \quad (2)$$

wherein $\dot{X}=P'/Z$, or $\dot{X}$ is the volume-current[4] in the branch at the junction. If P and P' are respectively $P_0e^{i\omega t}$ and $P'_0e^{i(\omega t-\epsilon)}$, then from (2)

$$\begin{gathered} e^{i\epsilon}P_0/P_0'-\rho a/2SZ=1 \\ (P_0/P_0')(\cos\epsilon-i\sin\epsilon)-\rho a/2S\cdot(Z_1-iZ_2)(Z_1^2+Z_2^2)^{-1}=1. \end{gathered} \quad (3)$$

From this, if $A=(P_0/P_0')\cos\epsilon-1$, and $B=-(P_0/P_0')\sin\epsilon$ readily follows that,

$$\begin{aligned} Z_1&=(\rho a/2S)[A/(A^2+B^2)] \\ Z_2&=(\rho a/2S)[B/(A^2+B^2)] \end{aligned} \quad (4)$$

If it is possible to ascertain the ratio of amplitudes P_0 and P_0' and the difference of phase, ϵ, the values of Z_1 and Z_2 readily follow from Eq. (4).

[3] Stewart, Phys. Rev. **26**, 688 (1925).

[4] Volume-current is the time rate of volume displacement.

[5] The ratio of the pressure to volume-current per unit area, sometimes called "acoustic resistance" (see Brillié, Journal Le Génie Civil, Aug. 23 and 30, and Sept. 6 (1919)) would more appropriately be called the "specific acoustic resistance."

[6] Stewart, Phys. Rev. **20**, 528 (1922).

These formulas evidently cannot produce accurate results if ϵ is small, for when it is small, usually P_0 and P_0' are nearly equal and A, being a difference of two values closely equal, will have a relatively large error. Assuming all values in c.g.s. units, the unit of Z_1 and Z_2 might well be called the "*acoustic ohm*" until a more appropriate name can be found. Z_1 is the value of the *acoustic resistance*[5] and Z_2 the value of the *acoustic reactance*. The absolute values of Z_1 and Z_2 depend upon the measurable values of ρ, a and S.

Experimental Method

The apparatus devised to obtain the experimental values of P_0/P_0' which are required for the application of Eq. (4), is a modification of the apparatus for transmission measurements previously described.[6]

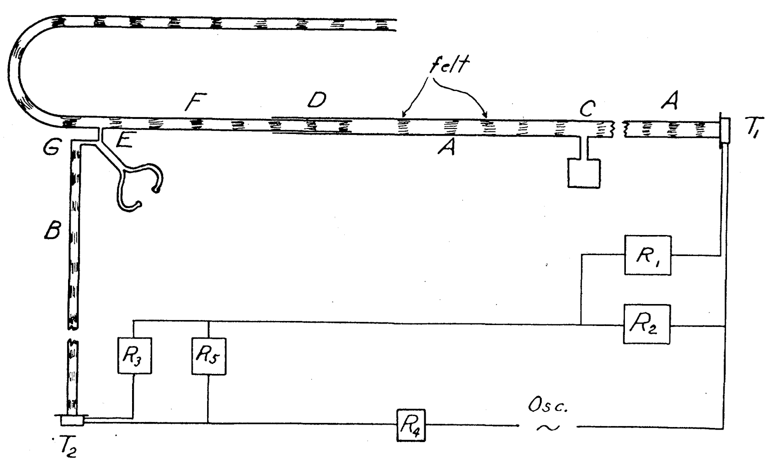

Fig. 1. The apparatus for the measurement of acoustic impedance.

The main conduit is the tube AD in Fig. 1, the source of sound a telephone receiver T_1 and the branch whose impedance is to be measured is attached at C. The tube F slides within AD. From the points E and G small branches are led to a pair of stethoscope binaurals. The tube B leads to a second telephone receiver T_2 similar to T_1. The tube B moves with the tube F as the latter telescopes into AD. The telephone receivers are connected as shown, with the filtered output of a vacuum tube oscillator used as the source of sound. The non-inductive resistances R_1 and R_3 are kept high in comparison with the impedance of the telephones. R_2 is fixed at a convenient value. The selection of dimensions of the apparatus is not important. The following measurements of length will give a sufficient description: CT_1, 274 to 84 cm; DC, 180 cm;

ED, 20 to 100 cm; GT_2, 300 cm; inside diameter of tubing, DT_1, 1.42 cm, GT_2, 1.27 cm.

Observations are taken as follows: with the branch at C removed and the hole closed, the position of the tube F and the value of R_5 are adjusted until no sound is heard in the stethoscope, this condition indicating that the amplitudes from E and G are equal and opposite in phase. Then the branch is attached at C and the process repeated. The ratio of the pressure amplitudes in F in the two conditions is given by the ratio of the currents in T_2 and hence by the ratio of the two values of the non-inductive shunt, R_5. This will also be the ratio of the two pressure amplitudes at C or P_0/P_0'. From the distance between the two settings of F and the wave-length, there can be computed the difference in phase of the two waves, or the phase difference of the incident and transmitted waves at C. If d is the distance between the first position and the second, measured positively toward T_1, ϵ will be given by $2\pi d/\lambda$, wherein λ is the wave-length.

With this apparatus it is possible to make very exact settings of F and R_5 over the middle range of frequencies, since, if the sound is not too greatly attenuated by the branch, one can set R_5 to 0.1 of 1 percent, and obtain ϵ to a fraction of a degree. More difficulty is experienced with the very high and low frequencies, since the sound becomes inaudible over a small range. This can be remedied by using a more powerful source of sound.

The accuracy of the results obtained was highly satisfactory, but success of the method is dependent upon the precautions observed. The theory assumes that there is only one wave in the main conduit, traveling toward the left. This is a condition which was approximated in practice by using a liberal amount of damping, in the form of tufts of hair felt, in the tubes. This damping should be sufficient to absorb all but a negligible portion of the wave reflected toward T_1 by the branch at C, so that neither the vibration of the source nor the wave from it will be affected. Also there must be sufficient damping in the tube F to the left of E to prevent the wave reflected from the distant end of the tube reaching the point E again in any appreciable intensity. The first of these conditions was not approached as closely as possible since the introduction of a large amount of damping reduced the intensity of the main wave to an undesirable extent. However, measurements were taken a number of times using different lengths of tubing between C and T_1. The wave reflected from the branch and again from T_1 combined in different phases with the main wave and thus a smooth curve drawn among the plotted values of S and transmission are probably fairly accurate. A more powerful source of sound would reduce this precaution, since it would allow the use of more damping.

A source of difficulty encountered in the measurement of values of ϵ of the order of $1°$, was in the changes of wave-length in the damped tubes by small fluctuations in temperature. This was overcome by the use of asbestos steampipe covering for the tubes, and by allowing as little time as possible to elapse between the two comparison readings.

Another source of difficulty was found in the small variations of the frequency of the oscillator, which was largely eliminated by making the length of the tube B approximately equal to the distance from E to T_1, and by allowing but a short time to elapse between the two readings.

An anticipated error which did not, however, appear appreciable, was the change of the phase relation of the sound emitted by the two telephones, as the resistance R_5 was altered. The two receivers were matched and kindly lent by the Bell Telephone Laboratories, Inc.

From the foregoing it is to be noticed that the electrical comparison involves merely relative values of resistances. The actual measurements are of a length, an area and the density of and velocity of sound in the medium. For all practical purposes the last two values may be regarded as known. The frequency is assumed to be given by the calibration of the oscillator.

Illustrations of Measurement

The method is applicable to any kind of branch whatsoever but in order that its use may be clearly understood, several illustrations will be cited where theoretical computations are possible. These cases may be otherwise instructive.

The orifice. In the case of the orifice, the theoretical values of Z_1 and Z_2 are[7]

$$\begin{aligned} Z_1 &= \rho\omega^2/2\pi a + (2\omega\rho\mu)^{1/2}(\pi R^3)^{-1} \cdot L. \\ Z_2 &= \rho\omega/c + (2\omega\rho\mu)^{1/2}(\pi R^3)^{-1} \cdot L. \end{aligned} \tag{5}$$

Herein, the new symbols c, L and R are, respectively, the conductivity, the thickness and the radius of the orifice. μ is the coefficient of viscosity and ω is 2π times the frequency. The results of experimentation and computation with an orifice 0.2 cm in diameter and .015 cm in length are shown in Fig. 2. The experimental curves are those of the square root of transmission, $\sqrt{T}$, or P_0'/P_0, and of ϵ. The indicated points on these two curves represent actual observations. With Z_1 and Z_2 the

[7] Stewart, Phys. Rev. **27**, 487 (1926).

indicated points represent the computations by Eqs. (4) using the experimental curves of transmission and phase. The curves actually represent

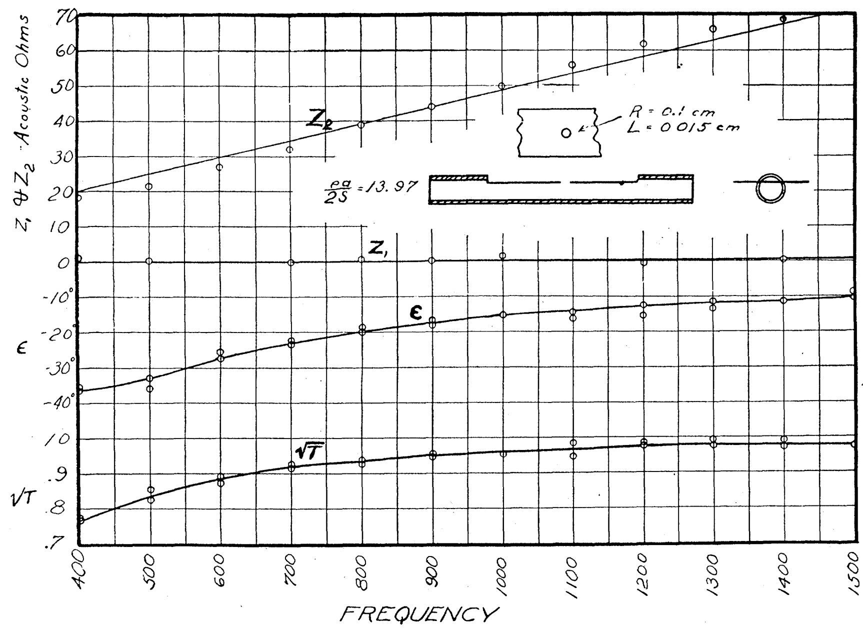

Fig. 2. Acoustic resistance and reactance of an orifice.

the values computed by the theory in Eq. (5) and are observed to be very nearly the mean of the experimental values.

The conductivity of the orifice, 0.182, is computed from the well-known expression

$$c = \pi R^2 (L + \pi R/2)^{-1}$$

The accuracy of the measurements of Z_1 and Z_2 are self-explanatory, the relative error in the former being large.

In the theoretical value of Z_1, μ is neglected because of the smallness of the term involved. The radiation, which must depend upon Z_1, is shown to be exceedingly small, in fact, clearly in evidence only at the highest frequencies. In the theoretical value of Z_2 the μ term is the cause of the variation from a strictly linear function of ω. This variation is not visible in the figure.

The Helmholtz resonator. In Fig. 3 the marked points and the full line curves have the same significance as in Fig. 2. There is close agreement between the theoretical values of Z_1 and Z_2 obtained from Eq. (6) and shown on the curves and the measured values shown by the indicated points. For $\sqrt{T}$ and ϵ the mean values of experimental data are indicated by circles, there being four series of observations with the following lengths of damped conduit between the source and the branch: 274, 285, 84 and 95 cm.

The value of ϵ is large enough to make accurate measurements possible. Z_1 is very small, and Z_2 and ϵ are practically zero at resonance. The reasons can be seen from the following equations[7]:

$$Z_1 = (2\omega\rho\mu)^{1/2}(\pi R^3)^{-1} \cdot L$$
$$Z = \rho\omega/c - \rho a^2/V\omega + (2\omega\rho\mu)^{1/2}(\pi R^3)^{-1} \cdot L. \tag{6}$$

If the μ term were zero, Z_1 and Z_2 and ϵ would all vanish at a frequency $f = \frac{a}{2\pi} \cdot (c/V)^{\frac{1}{2}}$. Z_1 is not a linear function of ω but approximately so over the range of frequencies shown. The value of c in this experiment is selected as the one which makes theory and experiment agree in regard

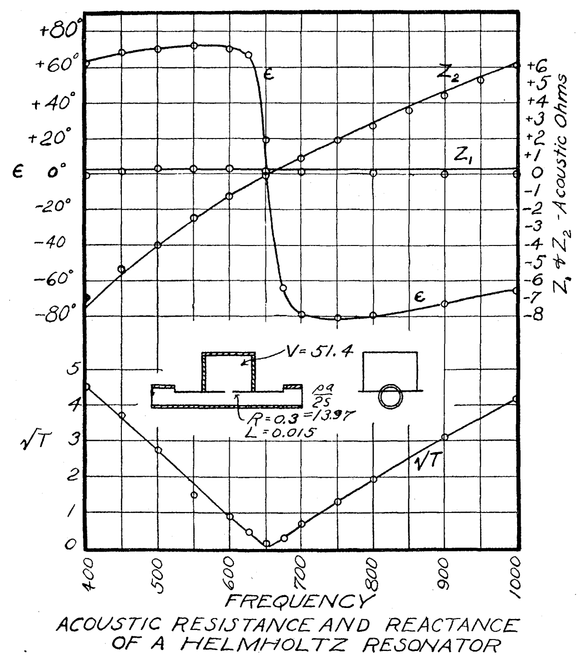

Fig. 3. Acoustic resistance and reactance of a Helmholtz resonator.

to the frequency of minimum transmission. This arbitrary selection is made necessary by the fact that, with an orifice having a diameter relatively so large in comparison with that of the conduit, the condition of location in an "infinite plane" is violated. The value computed from the usual expression for c is 0.581, whereas the selected value is 0.741.

The mean power input into the resonator at any frequency can be computed from the curves as will be shown in later illustrations. In the case of the resonator this input is very small.

The agreement between theory and experiment is remarkable. The observations of ϵ are in themselves interesting.

An infinite tube. In such a tube the ratio between the pressure and the volume current is real and is $\rho a/S$, where S is the area of the tube. If an infinite tube be attached as a branch, then there will be an inertance at the opening of $\rho\omega/c$. Therefore the values of impedances are:

$$\begin{aligned} Z_1 &= \rho a/S, \\ Z_2 &= \rho\omega/c \end{aligned} \tag{7}$$

Experimentally this case is difficult so far as accuracy is concerned, for ϵ is very small. Fig. 4 gives the results expressed as in Fig. 2. There is close agreement between the theoretical values of Z_1 and Z_2 obtained from Eq. (7) and shown in the curves and the measured values shown by

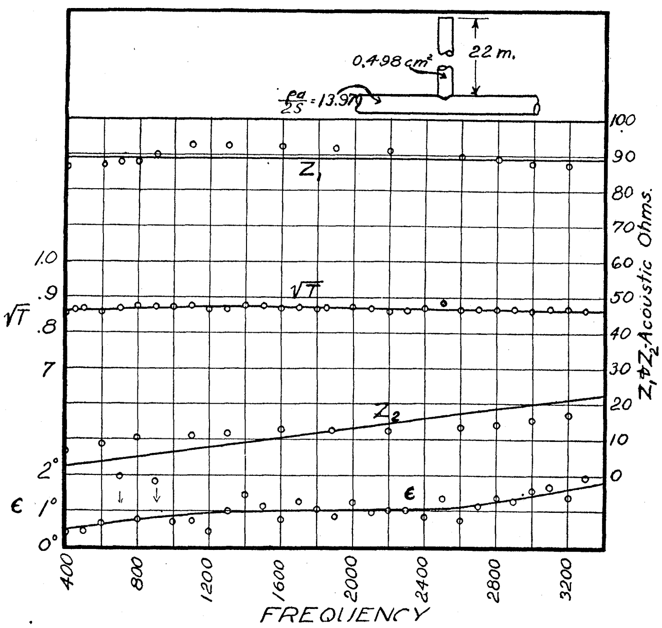

Fig. 4. Acoustic resistance and reactance of an infinite tube.

the indicated points. The "infinite tube" consisted of a tube 22 m long and 0.498 cm² in area. It seemed better experimentally to damp the vibration by the viscosity of the air, and not by the insertion of damping felt. The value of c for the opening is not the theoretical value 1.6, but the value 1.3. This experiment illustrates the difficulty in the method when the impedance is mostly resistance. The method would be improved by the use of a known additional Z_2 in parallel with the branch. This would make the values of ϵ larger and the measurements more accurate.

The conical horn. These measurements apply to the case of any trumpet, the only condition being that its exposure is just what is expected in practice. The theoretical formulas for the values of Z_1 and Z_2 in the simplest cases are found to be complicated and exceedingly tedious in computation.

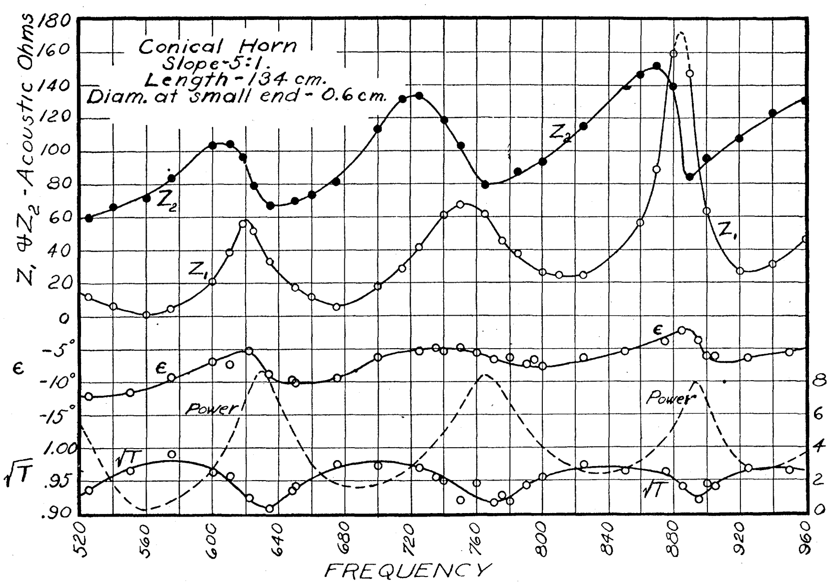

Fig. 5. Acoustic resistance and reactance of a conical horn

Fig. 5 shows the values of Z_1 and Z_2 for a conical horn as obtained from the experimental curves for ϵ and square root of transmission. The interesting technical points are: (1) Z_1 and Z_2 oscillate about ascending mean values, the former due to increased radiation and the latter due to increased inertance; (2) Z_1 has maximum and minimum values at practically those frequencies where Z_2 passes through its mean value; (3) the increase of the mean value of Z_2 may be considered as caused by the inertance at the junction end and at the exposed end, both of these increasing directly with ω but not being merely added because of the intervening complicated impedance.

It can be shown that a fair estimate of the performance of a trumpet, whether used as a receiver or as a transmitter, can be obtained from the value of Z_1. If, for example, a fixed source, or a constant value of maximum volume current, be placed at the small end, the output is proportional to Z_1. Likewise the amplification, used as a receiver, is proportional to Z_1. In a laboratory method such as here proposed, there is a limit to the size of trumpet used, but the theory of dimensions can be invoked to enable measurements to be made upon small models. In Fig. 5 the curve indicated "power" is computed from the expression $|P'|^2 Z_1/(Z_1^2+Z_2^2)$. It is the relative power output assuming the input in the incident wave in the tube is constant.

Results of tests. The above illustrations show that the method is capable of accurate and general use where the acoustic impedance is in the medium, air. It is, however, applicable to any fluid medium.

The writer is indebted to Mr. W. D. Crozier, research assistant, for the actual tests of the method described.

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