

THE ORIGIN OF K-RADIATION FROM THE TARGET  
OF AN X-RAY TUBE

BY MARK BALDERSTON

## ABSTRACT

The fluorescent energy transformation coefficient, the ratio of the energy of secondary K-radiation emitted by an elementary volume of the radiator to the energy absorbed by it from the exciting x-rays, is observed for Fe, Ni, Cu, Zn, Mo, and Ag. The ratio of quanta emitted as K-radiation to quanta absorbed seems to be a constant for a given metal; Fe—33 percent; Ni—39 percent; Cu—43.5 percent; Zn—50 percent; Mo—83 percent; Ag—75 percent. Making use of the experimentally determined values of the transformation coefficient the *fluorescent K-characteristic radiation* from the target of an x-ray tube is calculated for Cu and Ag, and compared with observed values of the total K-radiation. An estimate is also made from theoretical considerations of the relative amounts of direct impact K-characteristic radiation to be expected from an x-ray tube target. The calculations indicate that for silver and probably for other elements of high atomic number practically all of the K-characteristic radiation from an x-ray tube is fluorescent in origin while for copper and probably other elements of low atomic number a considerable part of the K-radiation is due to direct impact of the cathode electrons.

FOLLOWING the discovery by Kaye<sup>1</sup> that characteristic x-radiation was emitted by the target of an x-ray tube Barkla<sup>2</sup> explained the phenomenon as secondary radiation resulting from the absorption, in the target itself, of the general radiation. Beatty<sup>3</sup> examined the subject theoretically, not however publishing his method of analysis, and announced that not over ten per cent of the observed characteristic radiation could be so accounted for. An experimental investigation conducted with as much care as was possible with the facilities then available (1912) convinced him that practically all of the K-radiation from an x-ray tube target was the result of direct impact of the cathode electrons without the intermediate formation and absorption of general x-radiation.

With the x-radiation spectrometer came new evidence for Barkla's theory in the form of a sharp discontinuity in the general radiation spectrum at what was soon recognized as the K-absorption limit. Apparently the wave-lengths below the K-limit were strongly absorbed

<sup>1</sup> G. W. C. Kaye, Phil. Trans. A209, 123–151 (1909).

<sup>2</sup> C. G. Barkla and C. A. Sadler, Phil. Mag. 17, 739–760 (1909).

<sup>3</sup> R. T. Beatty, Proc. Roy. Soc. 87, 511–518 (1912).

in the target and it was but natural to assume that the K-radiation was an indirect effect resulting from this absorbed energy. Under the influence of Beatty's work, however, this evidence was largely discounted and explained as due to absorption by small bumps on the face of the target. It seems to have been generally assumed that if the target were absolutely smooth there would be no such discontinuity; that the characteristic radiation would appear as a new effect superposed on the general radiation. Beatty's<sup>3</sup> investigation of the total radiation from a copper target added weight to these conclusions; for as the velocity of the impinging electrons was increased the intensity of the resulting x-rays increased being nearly proportional to the fourth power of the velocity until the K-radiation appeared when the rate of increase became more rapid as though a new effect were superposed on the old. A similar piece of work by Brainin<sup>4</sup> confirmed Beatty's result for copper and secured similar results for cobalt, palladium and platinum. Molybdenum, however, gave a perfectly straight line showing no break at the K-absorption limit while silver showed a well defined decrease in efficiency at about the voltage corresponding to the K-absorption limit.

Meanwhile D. L. Webster and A. E. Hennings<sup>5</sup> have shown that for a molybdenum target, the focal spot of which had been fused so that there could not possibly be inequalities great enough to explain any discontinuity in the general radiation curve, nevertheless a well defined discontinuity appeared which they explained as wholly due to K-absorption in the target and from the amount of the discontinuity calculated the mean depth of production of the x-rays at the K-absorption limit. Their result is well in accord with the data on penetration of electrons found by Terrill.<sup>6</sup>

These results make it seem worth while to redetermine on the basis of present knowledge the amount of fluorescent or indirect K-radiation to be expected from the target of an x-ray tube. Our knowledge of absorption coefficients is adequate. The results of Terrill's work on electron penetration in metals gives at least a rough indication of the mean depth of production of general x-radiation of a given wave-length. Webster and Hennings' work mentioned above indicates its reliability. There remains to find what we have termed the "fluorescent transformation coefficient,"  $\phi$ , for a given range of wave-length of incident x-rays; i. e., the ratio of the energy of fluorescent

<sup>4</sup> C. I. Brainin, *Phys. Rev.* **10**, 461 (1917).

<sup>5</sup> D. L. Webster and A. E. Hennings, *Phys. Rev.* **21**, 301-311 (1923).

<sup>6</sup> H. M. Terrill, *Phys. Rev.* **22**, 101-108 (1923).

K-radiation from a small element of the target to the total energy absorbed by the element during the same time. The determination of this coefficient for a number of metals ranging from iron (26) to silver (47) and for a number of wave-lengths from  $.314\text{\AA}$  to  $.9\text{\AA}$  was the object of the investigation of which this paper is a report.

#### APPARATUS AND EXPERIMENTAL METHOD

The apparatus is an x-ray spectrometer with calcite crystal *C* (Fig. 1) and a 15 cm ionization chamber *A*, containing air saturated

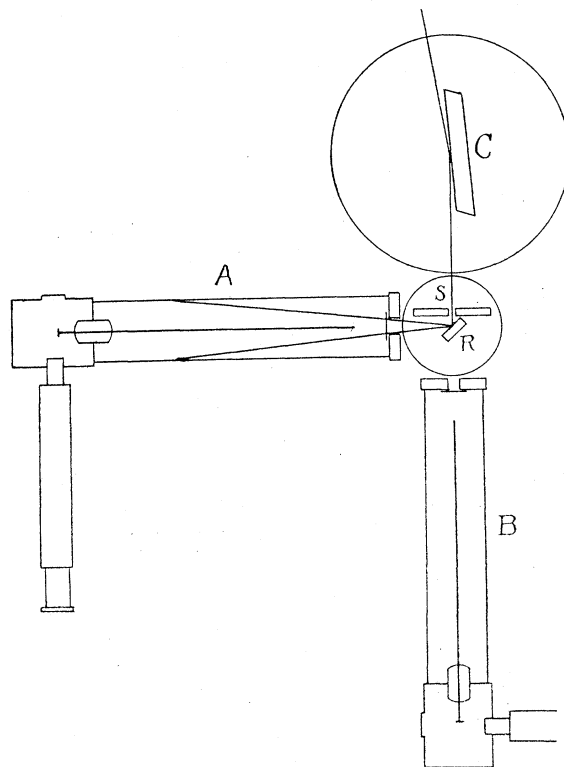


Fig. 1. Apparatus. *C* represents the calcite crystal, *A* and *B* the two positions of the movable ionization chamber, *R* the radiator, *S* the slit.

with ethyl bromide vapor. The chamber is mounted so that it may be revolved about a vertical axis 2 cm in front of the aluminum window making it possible to use the same chamber for measuring the secondary radiation from the specimen *R* and when turned through an angle of  $90^\circ$  to position *B*, the exciting radiation (*R* being then removed). The face of the radiator *R* makes an angle of  $45^\circ$  with the incident

beam so that the path of the incident x-rays in the radiator is equal to the mean path of the resulting characteristic radiation which reaches the ionization chamber. (On account of the very large solid angle subtended by the window this condition is only approximately satisfied.) The ionization current is measured by the rate of collapse of a small gold leaf electroscope mounted on the end of the chamber. The insulated electrode and the gold leaf are charged to a potential of about 60 volts. The sensitivity of the electroscope is about five divisions per volt and as the capacity is small the sensitivity per unit charge is high. The source of x-rays is a water-cooled tungsten-target tube of the Coolidge type. This tube was kindly constructed for the Laboratories by the Research Department of the Westinghouse Lamp Company. The source of power was a 500 cycle voltage rectified by two kenotrons and smoothed out by a large condenser.

The tube voltage and current were adjusted to suitable values. The crystal table was set at the proper angle to reflect the wave-length desired and, with the ionization chamber in position *B* (see Fig. 1) the proper position of the slit *S* was determined. The ionization current was then taken several times in this position giving a good mean value as a measure of the incident or exciting radiation. The chamber was then turned to position *A* at right angles to position *B*. With the tube input maintained at the same value there were then taken in turn; first, the natural leak of the electroscope and chamber; second, the ionization current with the required radiator *R*, placed in position; third, the natural leak again (the radiator *R* having been removed). This was repeated many times during a run, using different radiators. Occasionally during a run the incident radiation measurement was repeated as there was usually a slight drift in sensitivity due to change in temperature. The leak varied capriciously. Not over ten percent of it was due to stray x-rays (except at the highest voltages used) as was readily determined by shutting off the power. The amount of the leak varied a great deal with the weather but the range of variation during a given run was no greater in summer than in late winter. It was found necessary to keep the electroscope system charged throughout a run. If it were allowed to become completely discharged, for even a few seconds, it would give abnormally high values for a considerable period, the effect disappearing logarithmically with the time. Extreme precautions were taken to return the ionization chamber to exactly the original position after a reading of the incident radiation.

The ratio of the ionization current due to the secondary radiation (corrected for leak) to the ionization current due to the incident radiation is given in Table I.

TABLE I  
*Ratio of the ionization currents due to secondary and incident radiations.*

Incident Wave-length	Ag	Mo	Value of $c'/c$ for			
			Zn	Cu	Ni	Fe
.306Å	.0093	.0058	.....	.00015	.....	.....
.359	.00826	.00653	.00065	.00052	.....	.....
.466	.00736	.00637	.00115	.00109	.00067	.00057
.571	.....	.0065	.00127	.00119	.00096	.00063
.678	.....	.....	.00195	.00171	.00138	.00094
.782	.....	.....	.00233	.00204	.00158	.00125
.887	.....	.....	.00297	.00256	.00230	.00161

#### ENERGY TRANSFORMATION COEFFICIENT, $\phi$

In order to compare the values of the transformation coefficients of molybdenum and silver with those of the lighter elements, the radiations from which do not excite the K-radiation in bromine or the ethyl bromide, it is necessary to reduce the ratio of ionization currents given in Table I to the ratio which would have been obtained if the radiation could have been completely absorbed in air. This was done by making use of Barkla's<sup>7</sup> data on relative ionization in air and ethyl bromide. On account of the difference in path in the ionization chamber of different pencils of the divergent beam of K-radiation it was necessary to integrate over the solid angle  $\omega$  to get the correction factor  $A'$  for the K-radiation. The energy transformation coefficient  $\phi$ , the fraction of energy absorbed by a small elementary volume of the radiator which is re-emitted as K-radiation is given by the following expression

$$\phi = \frac{4\pi}{\omega} \left[ M \int_0^{l \sec \pi/4} \mu e^{-\mu x} e^{-\mu_K a x} dx \right]^{-1} A' c' / A c.$$

The integral accounts for absorption in the radiator of the exciting radiation before transformation and for absorption of the resulting K-radiation before emergence from the radiator.  $\omega$  is the solid angle subtended at the radiator by the window of the ionization chamber.  $\mu$  is the coefficient of absorption of the radiator for the exciting radiation.  $\mu_K$  is the coefficient of absorption of the radiator for its own K-radiation.  $A'c'$  is the total energy per second of K-radiation entering the ionization chamber, expressed in terms of the total ionization it can produce in air.  $Ac$  is the energy per second of the exciting radia-

<sup>7</sup> C. G. Barkla, *Phil. Mag.* **20**, 372 (1910).

tion falling on the radiator, expressed in the same terms.  $M$  is a correction factor for absorption of the K-radiation in air and in the aluminum window of the ionization chamber.  $t$  is the thickness of the radiator.  $x$  is the path in the radiator of the exciting radiation before transformation.  $ax$  is the path in the radiator of the resulting K-radiation before emergence.

The ratio  $c'/c$  is given in Table I. The value of  $\omega$  is  $.030 \pm .004$ . This is probably by far the least accurate factor entering into the determination of  $\phi$ . The values of  $\mu$  are from data of Richtmyer<sup>8</sup> and Hewlett.<sup>9</sup> Those of  $\mu_K$  are from tables compiled by Ledoux-Lebard and Dauvillier.<sup>10</sup>

The amount of scattered radiation from the radiator entering the ionization chamber in position  $A$  is negligible. This is because of the fact that the coefficient of scattering is least at this angle ( $90^\circ$ )<sup>11</sup> and that the shortness of the ionization chamber makes it relatively much more sensitive to the K-radiation than to the scattered radiations at the short wave-lengths. (It is only at short wave-lengths that the scattered radiation becomes important.)

The experimental values of  $\phi$  are given in Table II below.

TABLE II

Values of  $\phi$ , the ratio between the energy re-emitted as K-radiation by an elementary portion of the radiator to the energy absorbed by that portion from the incident or exciting x-radiation; and the values of the quantum transformation coefficient,  $u$ .

Wave-length of incident x-radiation	Ag	Mo	Values of $\phi$ for		Ni	Fe
			Zn	Cu <sup>12</sup>		
.306Å	.398	.264	....	.021	....	....
.359	.414	.323	.077	.066	....	....
.466	.611	.488	.160	.156	.090	.086
.571	....	.632	.166	.163	.121	.089
.678	....	....	.220	.193	.142	.103
.782	....	....	.251	.218	.153	.124
.887	....	....	.311	.263	.210	.148
Values of $u$	.75	.83	.50	.435	.39	.33

#### THE QUANTUM TRANSFORMATION COEFFICIENT, $u$

If now we plot  $\phi$  versus  $\lambda$ , the wave-length of the incident x-radiation we find the relationship is apparently linear and the curves cut the

<sup>8</sup> F. K. Richtmyer, Phys. Rev. **18**, 13 (1921).

<sup>9</sup> C. W. Hewlett, Phys. Rev. **17**, 292 (1920).

<sup>10</sup> R. Ledoux-Lebard et A. Dauvillier, La Physique des Rayons X. Paris (1921).

<sup>11</sup> Barkla and Ayers, Phil. Mag. **21**, 270-278.

<sup>12</sup> At .385A G. E. M. Jauncey and O. K. DeFoe have found by a different method a single value of  $\phi$  (for Cu) = .20 Proc. Natl. Acad. Sci. **11**, 520-2 (1925).

horizontal axis at points in the neighborhood of the origin. This suggests the possibility that the ratio of the number of quanta transformed to the total number of quanta absorbed is the same for a given radiator at all frequencies and that the decrease in  $\phi$  with increase in frequency of the absorbed radiation is due to the increase, with frequency, in the energy of the absorbed quantum while the energy of the re-emitted quantum remains unchanged. The failure of the curves to pass through the origin may then be ascribed to lack of perfect proportionality between the calculated ionization and the energy of the radiation. If,

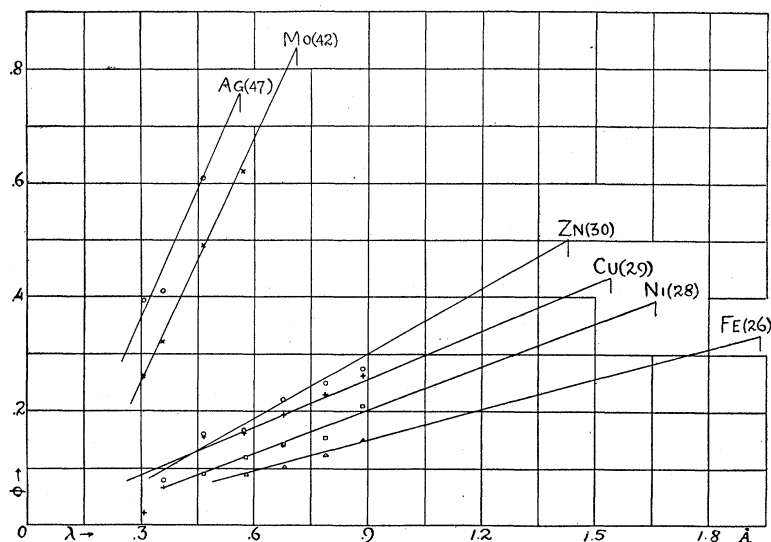


Fig. 2. Relation between  $\phi$  and the wave-length of general x-radiation absorbed by the specimen.

$$\phi = \frac{\text{Energy of K-radiation emitted}}{\text{Energy of general x-radiation absorbed}}$$

now, the line be extended to the wave-length of the K-characteristic radiation it will represent an imaginary condition in which the absorbed quantum is equal to the re-emitted quantum and in consequence the ratio of the energy absorbed to the energy emitted will be equal to the ratio of the quanta absorbed to the quanta emitted. This quantity,  $u$ , has been derived from each of the curves in Fig. 2 and the values are shown in the last column of Table II.

The values of  $u$  have been plotted against the atomic number of the element in Fig. 3. Since the K-absorption, for any given wave-length according to the formulas given by Richtmyer<sup>8</sup> for absorption coefficients above and below the K-limit, is about 87 percent of the total

absorption, we should expect 87 percent of the absorbed quanta to eject K-electrons and in consequence the same number of quanta of K-radiation to be re-emitted, i. e.,  $u$  should by this reasoning be equal to .87. The author's results, however, are in agreement with determinations of  $u$  by Barkla<sup>13</sup> and by Kossel<sup>14</sup> from data by Sadler.<sup>15</sup> These have been indicated on Fig. 3 where the author's values appear as circles, Barkla's as squares and Kossel's as crosses. Moreover, as indicated near the close of the paper in the determination of the



Fig. 3. Relation between ratio of quanta absorbed to quanta re-emitted and atomic number of element absorbing general x-radiation.

fluorescent K-radiation from the target of an x-ray tube and also of direct impact K-characteristic radiation, the use of values of  $\phi$  based on  $u = .87$  leads to an amount of fluorescent K-radiation hard to reconcile with experimental data while the use of the observed values of  $\phi$  gives very satisfactory agreement. This points to the possibility that, at least for metals of low atomic number, only one half or less of the absorbed energy goes into photo-electric ejection of K-electrons,

<sup>13</sup> C. G. Barkla, *Phil. Trans.* **A217**, 315-360 (1918).

<sup>14</sup> W. Kossel, *Zeits. f. Physik* **19**, 333-346 (1923).

<sup>15</sup> C. A. Sadler, *Phil. Mag.* **18**, 107-132 (1909).



or else that these electrons are replaced by some process which does not involve the expected K-radiation. In this connection attention is called to the recent work of Auger<sup>16</sup> and of Bothe<sup>17</sup> indicating a "special inner absorption" of K-radiation by the parent atom, an absorption much more intense than the normal absorption by the substance, the energy leaving the atom in the form of a second ejected electron.

DETERMINATION OF INDIRECT K-CHARACTERISTIC  
RADIATION FROM THE TARGET

Unnewehr<sup>18</sup> has given some data on the ratio of the intensity of K-radiation to general radiation from the target of an x-ray tube. From a copper target at 15,400 volts the ratio between the energy of the  $K\alpha$ -radiation entering the ionization chamber of a simple Bragg form x-ray spectroscopy and the energy of the general radiation entering simultaneously he finds to be 11.2. For a silver target at 36,400 volts he finds this ratio to be 2.7. We are now in a position to determine from fairly reliable data the ratio between the energy of indirect or fluorescent  $K\alpha$ -radiation entering the ionization chamber and the energy of the general radiation entering simultaneously. By comparing the two ratios we may then determine what part of the  $K\alpha$ -radiation measured by Unnewehr was fluorescent in origin, i. e., energy re-emitted by atoms of the target which have lost K-electrons by absorption of general x-radiation from other portions of the target. We will assume a general radiation curve  $I$  vs  $\lambda$  where, per unit volume of the target,  $I$  is the energy radiated having wave-length within a given range  $\lambda$  to  $\lambda + \Delta\lambda$ . We will further assume the mean depth of production of  $I$  to be given by the Whiddington law of electronic penetration—the depth at which the electron will have the same energy as the mean energy of the quantum of  $I$ . Since an error of 50 percent in the assumed depth of production will make a difference of only about 5 percent in the corresponding value determined for the fluorescent radiation this assumption of mean depth of production though not exact will certainly not cause a very large error in our results. The values of the penetration coefficients determined by Terrill<sup>6</sup> were used in the following work. It is now a simple matter to set up an expression for the energy of general radiation in a given wave-length range  $\lambda$  to  $\lambda + \Delta\lambda$  from one elementary volume,  $d\tau$ , of the target which is absorbed in another element  $d\tau$ . Multiplying this value by  $\phi$ , the fluorescent energy

<sup>16</sup> P. Auger, *Comptes Rendus* **180**, 65–68 (1925).

<sup>17</sup> Von W. Bothe, *Phys. Zeits.* **26**, 410–412 (1925).

<sup>18</sup> E. C. Unnewehr, *Phys. Rev.* **22**, 529–538 (1923).

transformation coefficient from the curve of Fig. 2 gives the resulting fluorescent K-radiation from the second element,  $d\tau'$ . A portion of the resulting radiation lies within the very small solid angle  $\delta'\omega$  necessary for reflection from the crystal—this quantity must then be corrected for absorption in reaching the surface of the target. A further correction factor  $R$  for absorption in air, reflection coefficient of the crystal, incomplete absorption in the ionization chamber, etc., is introduced. Its exact form is immaterial since identical corrections will apply to the general radiation entering the chamber with the K-radiation. It remains to integrate  $d\tau'$  over that part of the target which contributes K-radiation to the beam reflected by the crystal and  $d\tau$  over the layer of the target in which the general radiation of range  $\lambda$  to  $\lambda + \Delta\lambda$  originates. Since the  $K\alpha$ -radiation is 5/6 of the total K-radiation<sup>18</sup> we have for the energy of fluorescent  $K\alpha$ -radiation

$$E_{K\alpha} = \sum \frac{5}{6} \phi_n \frac{\delta'\omega}{4\pi} \frac{\mu_n I_n t R}{4\pi} \int_{hs} ds \int_{-\infty}^{\eta d} dx \int_0^{2\pi} d\theta \int_0^{\infty} dr \frac{r e^{-\mu_n \sqrt{r^2 + x^2}}}{r^2 + x^2} e^{-\mu_k \epsilon(\eta d - x)} \quad (2)$$

The summation is over all wave-length ranges  $\Delta\lambda$  for  $\lambda$  less than the wave-length of the K-absorption limit of the target.  $d\tau = ds dx$  and  $d\tau' = t r dr d\theta$ .  $\eta d$  is the depth of  $d\tau$  below the surface.  $\epsilon(\eta d - x)$  is the path of secondary x-rays from  $d\tau'$  to the surface.  $s$  is the slit-width of the narrower slit of the collimator.  $h$  is the height of section of the focal spot of the target. By a change of variable  $u = \mu \sqrt{r^2 + x^2}$  this reduces to

$$E_{K\alpha} = \frac{5}{6} \frac{\delta'\omega}{4\pi} \frac{t h s R}{2} \sum \phi_n \mu_n I_n \int_{-\infty}^{\eta d} -Ei(\mu x) e^{-\mu_k \epsilon(\eta d - x)} dx \quad (3)$$

Tables for the  $Ei$  function are given by Glaisher.<sup>19</sup> We wish to compare  $E_{K\alpha}$  with the energy of the general radiation entering the ionization chamber at the same time. If we have so chosen  $\Delta\lambda$  that it is equal to the complete range of wave-lengths entering the ionization chamber when it is set on the peak of the  $K\alpha$  line we may designate the value of  $I$  for this band  $I_G$ . The energy from an element  $d\tau$  will then be  $I_G d\tau$ . The energy reaching the ionization chamber is given by

$$E_G = \int R I_G \frac{\delta'\omega}{4\pi} \frac{\delta\theta}{\Delta\theta} e^{-\mu' \epsilon \eta d'} d\tau \quad (4)$$

where  $\delta\theta$  is the angle subtended at  $d\tau$  by the slits of the collimator,  $\Delta\theta$  is the maximum value of  $\delta\theta$ ,  $\epsilon \eta d'$  is the mean path of x-rays before emergence from the target, and  $\delta\theta$  and  $\Delta\theta$  may be expressed in terms

<sup>19</sup> J. W. L. Glaisher, Phil. Trans. 109, 367-387 (1870).

of the dimensions of the collimator and its distance from the target. The ratio  $E_{K\alpha}/E_G$  has been calculated for copper and silver from the above expressions.

Assuming similar ideal general radiation curves for silver at  $V=36,400$  volts and copper at  $V=15,400$  volts in each case with the "cut off" point as prescribed by the condition  $h\nu_{max} = Ve$  and the peak at the position  $\nu = \nu_{max}/1.5$  we have the following for the two ratios

$$\frac{E_{K\alpha}}{E_G} = \frac{\text{Expected indirect K-radiation}}{\text{General radiation entering ionization chamber with it}}$$

For copper  $E_{K\alpha}/E_G = 5.03/.54 = 9.3$ . Unnewehr finds this ratio 11.2 so that we have accounted for about 83 percent of the  $K\alpha$ -radiation. For silver  $E_{K\alpha}/E_G = 12.6/4.8 = 2.62$ . Unnewehr finds this ratio to be 2.7 so that in the case of silver we have accounted by this method for 97 percent of the  $K\alpha$ -radiation.

The discrepancy between the results for the two metals suggests that direct impact characteristic radiation may play a more important role in the case of copper than it does in the case of silver.

#### ESTIMATE OF DIRECT IMPACT K-CHARACTERISTIC RADIATION FROM THE TARGET

A relative measure of the direct impact K-radiation is given by the probability that an electron of given initial velocity will approach close enough to a K-electron, before its energy has been reduced below the K-quantum, to lift it entirely out of the atom. This is the condition for direct K-characteristic radiation. To satisfy this condition the path of the electron must lie within a distance  $p$  of the K-electron where  $p$  is given by the expression<sup>20</sup>

$$h\nu_k = \frac{E}{1 + p^2 E^2 / e^4} \quad (5)$$

where  $h\nu_k$  is the energy necessary to lift out the K-electron,  $E$  is the energy of the impacting electron, and  $e$  is the electronic charge.

<sup>20</sup> It may be shown that if a body of mass  $m$  and charge  $e$  moving with velocity  $v$  approaches another body of mass  $m$  and charge  $e$  at rest that the second body will acquire a velocity  $u$  given by

$$u = v / \sqrt{1 + (p^2/e^4)(\frac{1}{2}mv^2)} \quad (6)$$

where  $p$  is the perpendicular distance from the line of flight of the first particle to the second. This is derived by the mechanics of two bodies acting by the inverse law of force. Squaring and multiplying each member by  $\frac{1}{2}m$  and setting  $\frac{1}{2}mv^2 = E$  and letting the energy acquired by the second body be  $\frac{1}{2}mu^2 = h\nu_k$  we have Eq. (5) above. The author is indebted to Prof. Bergen Davis for Eq. (6).

For an impacting electron of a given initial energy  $E_0$  the probability that it will lift out a K-electron is clearly the ratio of the sum of the areas of all circles of radius  $\rho$  about the K-electrons to the total area of the target face. The value of this ratio is

$$2\pi n \frac{e^2}{\beta} \left[ \frac{2V_0 - V_k}{V_k} - \log \frac{V_0^2}{V_k^2} \right] \quad (7)$$

where  $\beta$  is the Whiddington coefficient of penetration in the expression  $V_x^2 = V_0^2 - \beta x$ ,  $V_0$  is the energy in abvolts of the penetrating electron,  $V_K$  is the energy just sufficient to lift out a K-electron.  $e$  is the electronic charge, and  $n$  is the number of atoms per cc in the target.

Substituting appropriate values for the two cases investigated above, silver at  $V_0 = 36,400$  volts gives .00028 and copper at  $V_0 = 15,400$  volts gives .00108. We should expect then about four times as many quanta of direct K-radiation from copper at 15,400 volts as from silver at 36,400 volts. In terms of energy we should expect 1.4 times as much energy from the copper as from the silver. In view of this it seems obvious that the greater part of the K-radiation is not direct impact radiation as Beatty claims for how then explain that the observed ratio of  $K\alpha$ -radiation of copper and silver for the above voltages is 6 to 13 when from the calculation above it should be 1.4 to 1? On the other hand we now can explain the fact that our calculated value for the  $K\alpha$ -line of copper was too low if for metals of low atomic number the direct K-radiation forms a considerable part of the total K-radiation but for metals of high atomic number the direct radiation becomes relatively of much less importance. This may be the explanation for the failure of molybdenum and silver to show increase in radiating efficiency in the curves for total energy as found by Brainin.

Returning to Unnewehr's data and our calculated values of  $K\alpha$ -radiation we have the following:

TABLE III

Target	Cu	Ag
Tube voltage	15,400	36,400
Total $K\alpha$ -radiation (Unnewehr)	6.0	13.0
Indirect (fluorescent) $K\alpha$ -radiation	5.03	12.6
Difference = direct impact $K\alpha$ -radiation	1.0	.4

The ratio  $1.0/0.4 = 2.5$  is in good agreement with the expected value 1.4.

As suggested above we may now make use of these results to check the validity of our determination of  $\phi$ , the fluorescent energy transformation coefficient. If we use in place of  $\phi$  in the calculation of indirect K-radiation from the target a value of the transformation co-

efficient derived from the assumption that  $u = .87$  we shall have to substitute in Table III above 10.1 and 14.6 for the energy of the fluorescent  $K\alpha$ -radiation of copper and silver respectively. These values are actually larger than the total  $K\alpha$ -radiation observed by Unnewehr and copper now shows relatively more indirect radiation than silver. This leaves no room for any appreciable amount of the direct impact radiation which seems necessary to explain Brainin's results referred to in the preceding paragraph. These facts then seem to point to the reliability of the observed values of  $u$  as given in Table II.

#### CONCLUSIONS

It seems established that for any given element the ratio between the number of quanta of x-rays of any wave-length absorbed to the number of quanta of secondary K-characteristic x-rays emitted is a constant. Experimental values indicate that this quantum transformation coefficient is much less than unity and increases markedly with atomic number. This increase is further indicated by the comparison of calculated with observed values of K-radiation from the target of an x-ray tube. For copper and probably other elements of low atomic number a considerable part, but probably much less than fifty percent, of the K-radiation originates from direct impact of the cathode electrons. For silver on the other hand, and probably other elements of high atomic number, a very small part of the total K-radiation has such an origin.

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