THE REFRACTION OF X-RAYS IN PRISMS OF VARIOUS MATERIALS

By C. M. Slack

Abstract

The method consisted in the use of the "double x-ray spectrometer" for measuring the extremely small changes in angle produced by refraction in prisms, which were placed between two crystals. The rocking curves from the second crystal were from 6 to 10 seconds of arc wide at half maximum and, under favorable temperature conditions, a shift of .2 seconds could be detected. Two different wave-lengths of x-rays and prisms of various materials were used. The values of $\delta \times 10^6$, where $\delta = 1 - \mu$, are as follows:

λ	silver	copper	sulfur	aluminum	carbon	celluloid	paraffin
.7078A	5.85±.3	5.95±.5	1.39±.05	1.68±.07	1.23±.15	.980±.03	.701±.03
1.537 A				8.40±.2		4.78 ±.1	3.28 ±.15

These results agree with the Lorentz dispersion formula within experimental error. In the case of certain materials, notably graphite, the curves from the second crystal were extremely broad (20 to 50 seconds), caused by internal refraction due to the granular structure. This seems of interest with respect to the possibility of comparing internal arrangements of particles in various samples, though nothing of that sort was attempted here.

Introduction

In a recent paper by Bergen Davis and C. M. Slack[1] a method was described which employed the "double x-ray spectrometer"[2] for the measurement of the refraction of x-radiation in an aluminum prism. The present paper presents a continuation of that work using a variety of substances as prisms.

Experimental Method

In all cases, except copper, silver and carbon, *double* prisms were used, in which the ray passed through the prism symmetrically, being bent the same at each surface. With copper and silver the difficulty experienced in perfecting the prism edge was overcome by backing a carefully polished face of the material against a steel plate and then milling the other face to the angle desired. The ray was then allowed to enter normally, refraction taking place on emergence. In these metals, on account of the high absorption, it was not possible to work at a distance greater than .01 cm from the edge. With carbon a right angled block was used, this being rotated to obtain the refracting angle.

[1] Bergen Davis and C. M. Slack, Phys. Rev. **27**, 18 (Jan., 1926).

[2] Bergen Davis and W. M. Stempel, Phys. Rev. **17**, 608 (May, 1921).

In measuring the refraction, the first crystal was set for the wavelength desired (MoKα or CuKα) and a direct rocking curve from the second crystal was taken. Then the prism was inserted and the rocking curve again taken. The shift in the peak measured the bending of the ray in the prism. This process was usually repeated several times at different prism positions to make sure that the prism was across the x-ray beam and to iron out slight irregularities in the prism surfaces and the larger shifts due to temperature changes.

In calculating δ from the prism angle and the measured bending a simple modification of the formula for the refractive index of a material for light was used:

$$\delta = -\theta \cot R \tag{1}$$

for single prism:

$$\delta = -\tfrac{1}{2}\theta \cot \tfrac{1}{2} R \tag{2}$$

for double prism. R is the prism angle and θ the deviation of the ray. The sign of θ is negative for bending as observed, which gives a positive value for δ in every case.

Results and Discussion

The results are shown in Table I.

TABLE I

Data on refraction of x-rays. Values of δ $(= 1-\mu)$.

Substance	prism angle	(degrees)	refraction (sec. of arc)	$\delta\times10^6$ observed	$\delta\times10^6$ Calc. by Lorentz formula complete	omitting char. freq.
			.7078A			
silver	single	63.5	2.42±0.1	5.85 ±0.3	5.80	6.33
copper	single	60.0	2.12±0.2	5.95 ±0.5	5.72	5.60
sulfur	double	168.3	5.39±0.2	1.39 ±0.05	1.37	1.37
aluminum	double	166.0	5.62±0.2	1.68 ±0.07	1.77	1.77
carbon	single	86.4	4.02±0.5	1.23 ±0.15	1.10	1.10
celluloid	double	171.0	5.12±0.1	.980±0.03	.991	.991
paraffin	double	173.0	4.71±0.15	.701±0.03	.714	.714
			$\lambda=1.537$A			
aluminum	double	116.0	5.53±0.1	8.40 ±0.2	8.46	8.44
celluloid	double	144.0	6.06±0.1	4.78 ±0.1	4.71	4.71
paraffin	double	152.0	5.40±0.15	3.28 ±0.15	3.37	3.37

The first of the last two columns was obtained from a modification of the Lorentz dispersion formula[3] thus:

[3] R. von Nardroff, Phys. Rev. **24**, 143 (1924).

$$\delta = \frac{e^2}{2\pi m}\left[\frac{n_1}{\nu^2 - \nu_c^2} + \frac{n_2}{\nu^2}\right] \tag{3}$$

where ν_c is the critical absorption frequency for the K series.

Which for the last column was simplified to

$$\delta = \frac{e^2}{2\pi m}\,\frac{n}{\nu^2} \tag{4}$$

It will be observed that only in the cases of silver and copper did the consideration of ν_c have an appreciable effect and in these it brought the result nearer the experimental value.

The curves were extremely narrow and of almost perfect regularity. It seems of interest to note that, with the exception of a few cases mentioned below, the transmission curve had practically the same width as the direct. The peak of an individual curve could be located to within .2 seconds. A shift due to temperature caused considerable trouble, but as it affected both curves alike it could be largely eliminated by a series of "out" and "in" curves. The other errors, such as lining up of the prism in mounting, measuring of prism angle, imperfections in prisms, etc., were not so easy to determine but in no case would introduce an error greater than .2 seconds, and in most cases considerably less. The discrepancy in the case of carbon is due to curve broadening discussed below.

Fig. 1 is a direct curve which shows the accuracy with which the peak can be located under favorable conditions. It also shows to a fine degree the perfect structure of the crystals used.

Because of the relatively low absorption in celluloid and the high accuracy with which it was possible to mill the prism edge, quite large refractions could be obtained. Fig. 2 shows such a case. The prism was pushed only part way across the beam, (a) representing that which passed the edge and (b) the portion which was refracted in the prism.

Internal Refraction in Graphite

In the determination of δ for carbon a piece of brush graphite kindly supplied by Dr. Welsh of the Union Carbide and Carbon Research Laboratories was used. It was observed immediately that the curves, instead of being 6 seconds wide at the half maximum, were of the order of 30 to 50 seconds, depending on the prism position. This effect apparently came from internal refraction due to the granular structure of the material. To check this, however, the transmission through a paraffin block was compared with that through the same amount granulated and it was found that the curve width was nearly doubled for the granulated material. There was some broadening in the case of

sulfur, due to the same cause. In no case was the effect noticed where it could be attributed to the surface conditions though in every case surface irregularities were enormous as compared to the x-ray wavelength.

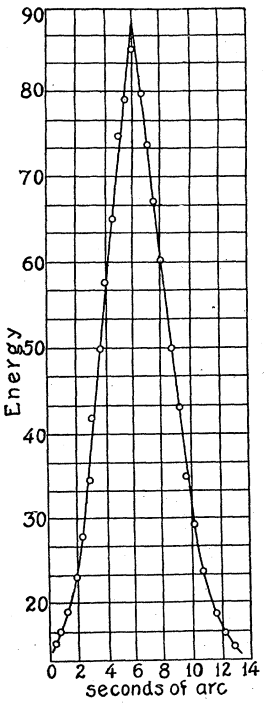


Fig. 1. Apparatus curve.

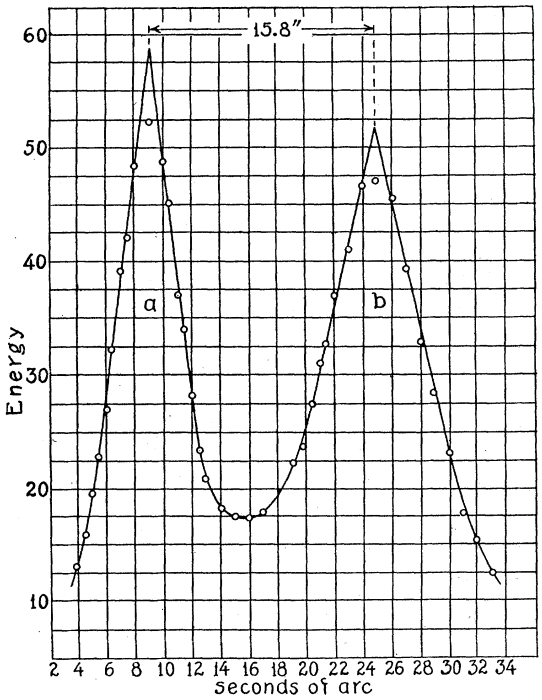


Fig. 2. Refraction in celluloid.

An investigation was made of the transmission through graphite strips of different thickness and the results obtained are given in Table II.

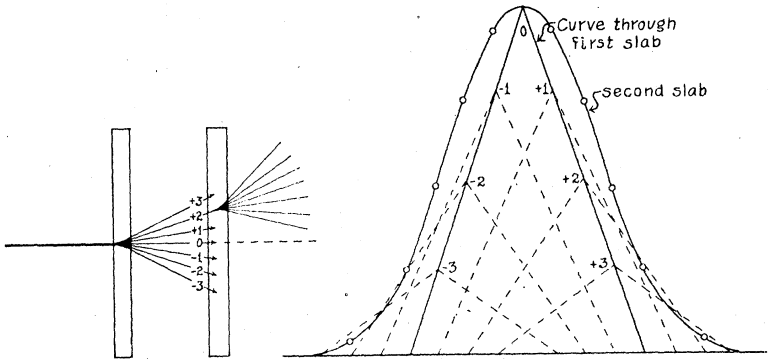
TABLE II

Data on the broadening of refraction curves in graphite.

Thickness of graphite	Curve width at half max.	Broadening	
	MoKα		
13.8 mm	58 sec.	52 sec.	±5
8.4	46	40	±5
4.3	30	24	±3
2.2	19	13	±3
1.2	15	9	±2
.62	10	4	±2
0.	6	0	...
	CuKα		
1.2	58	47	±5
.62	45	34	±4
0.	10.8	0	...

This variation of the broadening with thickness may be explained as follows. Consider the original beam as plane parallel and that in traversing a slab of known thickness it is divided into a large number of smaller elements (see Fig. 3) angularly dispersed in a manner which can be determined from a corresponding rocking curve. On traversing a second slab of equal thickness each of these small elements will be angularly dispersed in a manner exactly as was the original beam through the first slab. The total effect is then to be obtained by summation over all the elements.

The numerical value of the intensity at any angle is obtained experimentally as the height of that particular ordinate on the rocking curve. Thus to obtain the broadening graphically, curves were drawn, having the same distribution as the original, from a number of points



Figs. 3 and 4. Diagrams to illustrate how the internal refraction of x-rays in graphite broadens the rocking curves.

on the original. Then the ordinates of the elementary curves were added and the scale reduced for comparison. (See Fig. 4). This gave a curve showing distribution of intensity after traversing a second unit slab. This process was repeated for each thickness. In practice some eight points were taken on each curve and the check with the experimental results was within the error of observation. To explain quantitatively the original broadening would demand a knowledge of the shape, size and distribution of the carbon particles, which is not available.

In conclusion I wish to express my appreciation to the members of the Physics Department for their cooperation in this work, and especially to Professor Davis for the suggestion of the problem and his constant aid in overcoming difficulties encountered.

PHOENIX PHYSICAL LABORATORIES,
COLUMBIA UNIVERSITY,
January 7, 1926.