NOTE ON THE QUANTUM THEORY OF THE UNMODIFIED LINE IN THE COMPTON EFFECT

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Abstract

Disappearance of the unmodified line.—According to the theory developed in two previous papers (Phys. Rev. 25, 314 and 723) it is shown that the unmodified line should disappear at a scattering angle ϕ given by vers $\phi=(3+2\sqrt{2})(\lambda_0^2/\lambda_s)(mc/h)$ where λ_0 is the wave-length of the primary x-rays and λ_s the K critical absorption wave-length of the scattering substance. This formula shows that there is an unmodified line for all values of ϕ when Mo Kα rays are scattered by carbon and heavier elements, but when scattered by boron and lithium this line should disappear at 147° and 60° respectively. *Ratio of the energies in the modified and unmodified lines.*—The theory of the previous papers gives the ratio of the number of electrons in orbital positions U such as to scatter unmodified rays in a direction ϕ to the number of electrons in orbital positions M such as to scatter modified rays. A formula for the ratio of the probability of an electron scattering when in the U position to that when in the M position is given together with a simpler method for calculating the number of electrons in the M position to the number in the U and M positions for elliptic orbits.

1. Disappearance of the Unmodified Line

In two previous papers,[1,2] the writer has explained the existence of the unmodified line in the Compton effect by showing that there is a certain range of positions of the scattering electron of a given type (K, L, etc.) in its Bohr orbit from which the electron cannot be ejected from the atom by the scattering process when x-rays are scattered at an angle ϕ with the direction of the primary rays. For convenience, we shall speak of an electron which is in an orbital position such that it cannot be ejected by the scattering process as being in the U position and of an electron which is in a position such that it can be ejected as being in the M position. Jauncey's theory requires that for scattering in a direction ϕ by a given type of electrons the ratio of the number N_M of electrons in the M position to the total number (N_U+N_M) of electrons in the U and N positions is approximately given by

$$\frac{N_M}{N_U+N_M}=\frac{\alpha_0 \operatorname{vers}\phi+2\sqrt{2\alpha_s}\sin\frac{1}{2}\phi-\lambda_0/\lambda_s}{4\sqrt{2\alpha_s}\sin\frac{1}{2}\phi}\,, \tag{1}$$

[1] G. E. M. Jauncey, Phys. Rev. **25**, 314 (1925).
[2] G. E. M. Jauncey, Phys. Rev. **25**, 723 (1925).

where λ_0 is the wave-length of the primary x-rays, λ_s is the critical absorption wave-length for the particular type of electrons under consideration, $\alpha_0 = h/mc\lambda_0$ and $\alpha_s = h/mc\lambda_s$. Eq. (1) is true for those types of electrons which move in circular orbits, while a more complicated formula[2] exists for electrons which move in elliptic orbits.

The ratio $N_M/(N_U+N_M)$ is to be taken as zero when the right side of Eq. (1) is negative and as unity when the right side is greater than unity. For values of the right side between zero and unity, the ratio increases as ϕ increases and as λ_s decreases. The ratio is therefore least for the K electrons. Hence when the ratio becomes unity for the K electrons, it will be unity for all the other electrons, and we shall have no unmodified line. Therefore, equating the right side of Eq. (1) to unity and solving for ϕ, we obtain the value of ϕ for which the unmodified line in the Compton effect should disappear. This value of ϕ is given by

$$\text{vers}\,\phi = (3+2\sqrt{2})(\lambda_0^2/\lambda_s)(mc/h) \tag{2}$$

If λ_0 and λ_s are measured in angstroms, Eq. (2) may be written vers $\phi = 242(\lambda_0^2/\lambda_s)$. For the scattering of MoKα x-rays by the K electrons of carbon we have $\lambda_0 = 0.71$A and $\lambda_s = 47$A, so that the right side of Eq. (2) is greater than 2 and there is no real solution for ϕ. A similar result holds for the scattering of MoKβ x-rays by carbon. Hence there is an unmodified line for all values of ϕ when MoKα and MoKβ x-rays are scattered by carbon and elements of higher atomic number than carbon. However, for the K electrons of boron we have $\lambda_s = 66$A and there is now a real solution for ϕ. For scattering by boron the unmodified line should disappear at $\phi = 147°$, while the unmodified Kβ line should disappear at $\phi = 117°$.

Jauncey, Boyd and Nipper[3] have tested this experimentally by scattering MoK x-rays by boron at about 145° to 150°, the scattered spectrum being examined photographically by means of a Sieman-Ross x-ray spectrometer. The photograph showed the modified Kα and Kβ lines plainly but the unmodified lines were not apparent although in a comparison experiment where carbon was the scattering substance the unmodified lines were evident. As further experimental evidence Allison and Duane[4] and Woo[5] have scattered MoK x-rays from lithium at angles greater than 60° and find either very little or no evidence of the unmodified lines. For the K electrons of lithium $\lambda_s = 240$A, so that

[3] Jauncey, Boyd and Nipper, Phys. Rev. **27**, 103 (1926).
[4] Allison and Duane, Phys. Rev. **26**, 300 (1925).
[5] Y. H. Woo, Phys. Rev. **27**, 119 (1926).

the unmodified $MoK\alpha$ and $K\beta$ lines should disappear at 60° and 53° respectively. It might be mentioned that the existence of the unmodified $MoK\alpha$ line might have been explained on the following assumptions: (1) the electrons are at rest in the atom, (2) the energy given to the electron by the scattering process is that given to a free electron, and (3) there is an unmodified line unless this energy is greater than the binding energy of the electrons. These assumptions give the condition vers $\phi=(\lambda_0^2/\lambda_s)(mc/h)$ for the disappearance of the unmodified line, which in the case of carbon should be at 75°. Experimentally, however, the unmodified $MoK\alpha$ line does not disappear at any angle when the x-rays are scattered from carbon. We are therefore left with the theory of the unmodified line as developed by Jauncey[1] as being the only available explanation.

2. Ratio of the Energies in the Modified and Unmodified Lines

Jauncey and DeFoe[6] have shown that the assumption that the chance P_U that an electron will scatter in the direction ϕ when in the U position is equal to the chance P_M that an electron will scatter when in the M position is not necessary to Jauncey's theory of the unmodified line. Experiments show that P_U is greater than P_M and we are able to calculate the ratio P_U/P_M from experimental data. Eq. (1) gives the ratio $N_M/(N_U+N_M)$ for a given type (K, L etc.) of scattering electrons so long as these electrons are moving in circular orbits. If, however, the electrons are moving in elliptic orbits the method of calculation is as follows: Referring to Fig. 1 the curves $ABCD$ and $DEFG$ are taken from Fig. 3 in Jauncey's paper.[2] The curve $ABCD$ is for circular orbits while the curve $DEFG$ is representative of the curves for elliptic orbits. If the line AD is divided at L in the ratio LD/AD such that this ratio is given by the right side of Eq. (1) and the line LM is drawn perpendicular to AD then the ratio $N_M/(N_U+N_M)$ for the particular type of electrons under consideration is given by the ratio of the area $LMFGK$ to the area $HDEFGK$. The weighted mean of the ratios $N_M/(N_U+N_M)$ for the various types of electrons is represented by $p(\phi)$. If now s_1 and s_2 represent the unmodified and modified scattering coefficients per unit solid angle in the direction ϕ, then the ratio s_2/s_1 can be found experimentally by the methods of Woo[5] and DeFoe.[7] We then have

$$P_U/P_M=(s_1/s_2)\cdot p(\phi)/\{1-p(\phi)\} \tag{3}$$

[6] Jauncey and DeFoe, Phil. Mag. **1**, 711 (1926).
[7] O. K. DeFoe, Phys. Rev. **27**, 675 (1926).

Woo in a recent paper[5] finds the ratio s_2/s_1 by obtaining the spectrum of scattered MoKα x-rays and comparing the areas under the modified and unmodified humps in his curves. Woo finds s_2/s_1 for carbon at 90° to be 1.45. However Woo compares this with a value 1.74 for the ratio of the 'intensities' calculated on theoretical grounds in a paper by the writer.[2] In this paper[2] the writer distinguishes between 'intensity' and 'energy.' In the sense in which the writer has used the words, Woo has measured the ratio of the 'energies.' The writer has used the word 'intensity' to denote the height of the ordinate of the most intense part of the modified or unmodified hump and not the area under the hump; while the word 'energy' implies the area. According to the argument of the paper[2] the ratio of the 'energies' for MoKα x-rays scattered at 90° by carbon is 10.9 and by sulfur is 4.2. These values, however, are based on the assumption that $P_U = P_M$. As s_2/s_1 is quite evidently smaller than the ratio of the 'energies,' we see that P_U is greater than P_M and from Eq. (3) the ratio P_U/P_M for MoKα x-rays scattered by carbon at 90° is $10.9/1.45 = 7.5$. Thus we see that scattering is about 7.5 times as probable when the electron is in the U position as in the M position. It should be remarked that Eq. (3) gives only the mean value of P_U/P_M. It might well be that this ratio has different values for the different orbits.

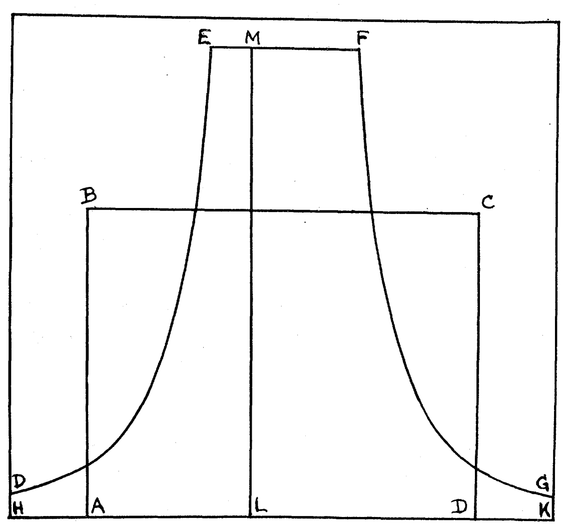

Fig. 1. Curves for obtaining the ratio $N_M/(N_U+N_M)$ for elliptic orbits.

Washington University,
St. Louis, Mo.
March 3, 1926.