

THE RATIO OF THE MODIFIED TO THE TOTAL SCATTERING COEFFICIENT OF X-RAYS

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ABSTRACT

Use has been made of the fact that the modified portion of scattered x-rays has not only a greater wave-length but also a greater absorption coefficient than the primary x-rays in order to separate the modified and unmodified portions of a scattered x-ray beam. The method involves the change in the intensity of scattered x-rays when a given thickness of aluminum is transferred from the primary to the scattered beam. Using a balance method, measurements of the ratio of the modified to the total scattering coefficient have been made for x-rays of wave-lengths 0.27A to 0.58A scattered from carbon, aluminum and copper at angles varying from 60° to 130°. *Comparison with theory.* Jauncey's theory predicts that the ratio should become unity when $\text{vers } \phi = 242\lambda_0^2/\lambda_s$ (λ_s and λ_0 being measured in Angstroms). For carbon at 60° and 90° the experimental ratio becomes unity at wave-lengths of 0.32A and 0.46A respectively: The theory gives 0.31A and 0.44A respectively. A calculation of the modified scattering coefficient for copper at 90° for $\lambda = 0.4A$ gives $0.71s_0$ (where s_0 is the Thomson coefficient at 90°), while the theory gives $0.74s_0$. Thus as far as they go the experiments are in agreement with the theory.

1. INTRODUCTION

A. H. COMPTON¹ has shown theoretically that x-rays scattered at an angle should be changed in wave-length by an amount given by the formula

$$\lambda_\phi - \lambda_0 = h/mc \text{ vers } \phi = .024 \text{ vers } \phi \quad (1)$$

This value is derived on the assumption that the electrons which do the scattering are free and at rest. Experiments by Compton,² Ross,³ and others confirm this result but show also unmodified scattering which the theory does not predict. The existence of the unmodified line necessitates the definition of two scattering coefficients, one for the modified and the other for the unmodified portion. Previous determinations of the scattering coefficients give approximately the sum of these two coefficients. Throughout this article we shall denote by σ_1 and σ_2 the spherical linear scattering coefficient for the unmodified and modified x-rays respectively; while s_1 and s_2 will be used to denote the linear scattering coefficient per unit solid angle in the direction ϕ with respect to the forward direction of the primary rays

¹ A. H. Compton, Phys. Rev. **21**, 483 (1923).

² A. H. Compton, Phys. Rev. **22**, 408 (1923).

³ P. A. Ross, Proc. Nat. Acad. Sci. **9**, 246 (1923); **10**, 304 (1924).

for the unmodified and modified x-rays respectively. If the primary x-rays are unpolarized the quantities σ_1 and s_1 are connected by the relation

$$\sigma_1 = \int_0^\pi 2\pi s_1 \sin \phi \, d\phi \quad (2)$$

while a similar relation exists between σ_2 and s_2 .

Jauncey^{4,5} has proposed a theory which explains not only the presence of the unmodified line but also the width of the modified line. This theory takes into account both the motion and the binding energy of a scattering electron in its orbit and it gives a formula for the fraction of the total number of electrons which are in orbital positions such as to scatter modified x-rays in a direction ϕ . If we denote this fraction by $p(\phi)$ and use a subscript to denote the orbit which is occupied by the electrons considered, (for example $p_K(\phi)$ denotes the fraction of K electrons which are in a position to scatter modified x-rays in the direction ϕ then $p_K(\phi)$ is approximately given by

$$p_K(\phi) = \frac{\alpha_0 \text{vers } \phi + 2\sqrt{2\alpha_s} \sin \frac{1}{2}\phi - \lambda_0/\lambda_s}{4\sqrt{2\alpha_s} \sin \frac{1}{2}\phi} \quad (3)$$

where $\alpha_s = h/mc\lambda_s$, $\alpha_0 = h/mc\lambda_0$, λ_s denotes the K critical absorption wave-length of the scattering substance, λ_0 the wave-length of the incident radiation and ϕ the direction of scattering. The values of $p_L(\phi)$, $p_M(\phi)$, etc., are obtained by means of similar formulas, when the orbits are circular and by more complicated formulas in the case of elliptic orbits.^{5,7} $p(\phi)$ is now obtained as a weighted mean of the values for the various orbits. Jauncey and DeFoe⁶ have shown that the theoretical value of s_2 is given by

$$s_2 = \frac{Kp(\phi) \{1 + \cos^2 \phi + 2\alpha_0(1 + \alpha_0) \text{vers } \phi\}}{2(1 + \alpha_0 \text{vers } \phi)^5} \quad (4)$$

where K is given by

$$K = (NZ\rho/W)(e^4/m^2c^4). \quad (5)$$

In Eq. (5) N denotes Avogadro's number, Z the number of electrons per molecule, ρ the density, W the molecular weight of the scattering substance, e the electronic charge, m the mass of an electron, and c

⁴ G. E. M. Jauncey, Phys. Rev. **25**, 314 (1925).

⁵ G. E. M. Jauncey, Phys. Rev. **25**, 723 (1925).

⁶ Jauncey and De Foe, Phil. Mag. **1**, 711 (1926).

⁷ G. E. M. Jauncey, Phys. Rev. **27**, 687 (1926).

the velocity of light in vacuo. Jauncey and DeFoe⁶ have pointed out that although s_2 should be given by Eq. (4) yet it is not a necessary result of the theory that s_1 should be given by replacing the factor $p(\phi)$ in the right side of Eq. (4) by $\{1-p(\phi)\}$. This could only be done if the probability of an electron's scattering in the direction ϕ were the same irrespective of whether the electron was in a position to scatter modified or unmodified rays. If the ratio s_1/s_2 can be measured experimentally, then Jauncey⁷ has shown that the ratio of the probability P_U that an electron will scatter in a direction ϕ when in a position to scatter unmodified rays to the probability P_M that it will scatter modified rays is given by

$$P_U/P_M = (s_1/s_2) \{p(\phi)/\{1-p(\phi)\}\} \quad (6)$$

The experiments of Barkla and Ayers,⁸ Hewlett,⁹ and Jauncey and May¹⁰ show that there is scattering considerably in excess of the Thomson¹¹ value at $\phi = 30^\circ$ in the case of scattering by light elements. On Jauncey's theory however it is at 30° that the fraction $\{1-p(\phi)\}$ becomes considerable and therefore a considerable portion of the scattered rays are unmodified. Further, Barkla and Dunlop¹² find considerable excess scattering from heavy elements such as copper even when $\phi = 90^\circ$. Here again the fraction $\{1-p(\phi)\}$ becomes large when $\phi = 90^\circ$ when x-rays are scattered by copper. Again Hewlett⁹ and Jauncey and May¹⁰ find that when x-rays are scattered by light elements the scattering goes to zero at about $\phi = 5^\circ$. At this angle $p(\phi)$ vanishes. It seems then that excess scattering and also the zero scattering at small angles is a property of the unmodified scattering and not of the modified scattering. It is therefore evident that it is important to measure s_1 and s_2 experimentally, so that we may test the theoretical formula, Eq. (4), and also so that we may obtain s_1 as an empirical function of ϕ . As a preliminary attack on this problem the experimental values of $s_2/(s_1+s_2)$ have been determined for different scattering substances, different wave-lengths and for different angles of scattering.

2. THEORY OF THE EXPERIMENTAL METHOD

The experimental value of the scattering coefficient σ_{exp} determined by previous observers has always been approximately the sum of the

⁸ Barkla and Ayres, *Phil. Mag.* **21**, 275 (1912).

⁹ C. W. Hewlett, *Phys. Rev.* **20**, 688 (1922).

¹⁰ Jauncey and May, *Phys. Rev.* **23**, 128 (1924).

¹¹ J. J. Thomson, *Conduction of Electricity through Gases* 2nd Ed., p. 325.

¹² Barkla and Dunlop, *Phil. Mag.* **31**, 222 (1916).

two scattering coefficients σ_1 and σ_2 . In the determination of σ_{exp} there is a slight error introduced by assuming the same absorption coefficient for the scattered as for the primary radiation, whereas in fact there is a slight increase in the absorption coefficient as shown experimentally by Sadler and Mesham.¹³ This error is small but always tends to make σ_{exp} less than the true value. The softening is of course due to the change in wave-length of the modified portion of the scattered x-rays. This suggests that, since we know the change of wave-length on scattering and therefore also the change in absorption coefficient of the modified x-rays on scattering, we can separate s_1 and s_2 at a given value of ϕ by transferring a certain thickness of absorbing material from a position P in the primary beam to a position Q in the scattered beam and observing either the intensity of the scattered x-rays if the intensity of the primary x-rays is kept constant or the change in the intensity of the primary rays necessary to keep the intensity of the scattered x-rays constant. In the present experiment the second alternative was used. Jauncey and DeFoe^{6,14} have given formulas for determining the ratio $s_2/(s_1+s_2)$ for two cases. First, for the case in which the scattering block is placed in a position such that a normal to its face makes an angle $\theta = \phi/2$ with the forward direction of the primary beam, ϕ being the scattering angle, we have

$$s_2/(s_1+s_2) = \frac{y-1}{(y-1) + [(1-e^{-g})/ge^{-g}]e^{-(kd_1+b)}(1-ye^{-kd})} . \quad (7)$$

Second, for the case in which the scattering block is set in such a position that the plane of its face bisects the scattering angle ϕ , we have

$$s_2/(s_1+s_2) = \frac{y-1}{(y-1) + Be^{-kd_1}(1-ye^{-kd})} . \quad (8)$$

In the above equations the symbols used are defined as follows: $y = I_2/I_1$ where I_1 and I_2 are the intensities of the primary x-rays before and after the transfer of a thickness d of absorbing material from P to Q , the values of I_1 and I_2 being such that the intensity of the scattered rays is the same after the transfer as before; d_1 is the thickness of absorbing material at Q_1 which is not transferred (e. g. the thickness of the aluminum window of the ionization chamber); $k = (\mu_2 - \mu_1)$; $b = (\mu_6 - \mu_5)R + g$; $g = (\mu_4 - \mu_3)t \sec \frac{1}{2}\phi$, where t is the thickness of the scattering block and ϕ is the angle of scattering; $B =$

¹³ Sadler and Mesham, *Phil. Mag.* **24**, 138 (1912).

¹⁴ Jauncey and DeFoe, *Proc. Nat. Acad. Sci.* **11**, 517 (1925).

$e^{-hR}/\{1+(\mu_4-\mu_3)/2\mu_3\}$; $h=(\mu_6-\mu_5)$; R is the distance from scattering block to ionization chamber window; μ_1 is the absorption coefficient of primary x-rays in absorbing material; μ_2 is the absorption coefficient of modified x-rays in absorbing material; μ_3 is the absorption coefficient of primary x-rays in scattering material; μ_4 is the absorption coefficient of modified x-rays in scattering material; μ_5 is the absorption coefficient of primary x-rays in air; and μ_6 is the absorption coefficient of modified x-rays in air.

The change in absorption coefficient $k=(\mu_2-\mu_1)$ may be determined, either by plotting the experimental value of μ for the particular absorbing substance used against λ and reading from the curve the two values of μ corresponding to λ_ϕ and λ_0 , or by making use of the relation

$$\mu/\rho = \text{const} \times \lambda^3 + \sigma/\rho \quad (9)$$

In the present experiments aluminum was used for the absorbing material which was transferred from P to Q and Hewlett¹⁵ gives the constant for aluminum in Eq. (9) as 12.4. If Eq. (9) is differentiated with respect to λ we obtain $\delta(\mu/\rho)$. Since σ/ρ varies slowly with the the wave-length we can treat it as a constant. We then have for aluminum

$$(\mu_2-\mu_1) = 100.6\lambda_0^2(\lambda_\phi-\lambda_0), \quad (10)$$

$(\lambda_\phi-\lambda_0)$ being given by Eq. (1).

3. EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The equations giving the ratio $s_2/(s_1+s_2)$ suggest the use of a balance method. The experimental arrangement is shown in Figs. 1 and 2. Two primary beams 1 and 2 which are in the same vertical plane are taken from the x-ray tube R which was placed at an angle of approximately 45° with the plane containing the two beams so as to eliminate any effect due to polarization in the primary rays. These were passed through the two slits C and D . C was about one square centimeter in area and its size was fixed. D could be varied from zero to about two square centimeters in area. The area of D was measured to .01 mm² by means of a micrometer screw. Absorbing material was placed at P_1 and P_2 . Aluminum was used for absorbing material because the fluorescent radiation of aluminum does not interfere and because aluminum can be obtained commercially in a much higher state of purity than any other suitable substance. The scattered beams 1' and 2' from AB after passing through the absorbing material at Q_1 and Q_2 entered separate ionization chambers. These ionization

¹⁵ C. W. Hewlett, Phys. Rev. 17, 284 (1921).

chambers were connected by a glass tube and methyl iodide introduced by means of evaporation from a reservoir. This kept the concentration of methyl iodide vapor the same in the two chambers. The outer electrodes of the two ionization chambers were respectively connected to the opposite poles of a battery giving 180 volts, the middle point of the battery being grounded. The inner electrodes of the chambers were connected to each other and to a Compton electrometer. If the intensities of the scattered rays entering the two chambers is the same the electrometer shows no deflection. In these experiments the same thickness of aluminum was first placed at P_1 and P_2 also the thickness at Q_1 was the same as at Q_2 . A balance was then obtained by varying the slit C by means of the micrometer screw and the reading of the

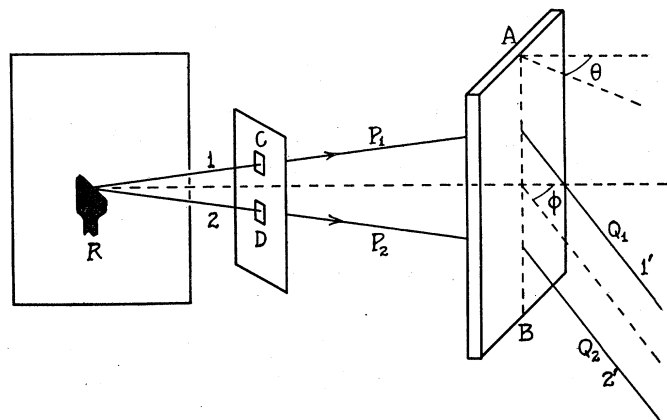


Fig. 1. Diagram of apparatus showing scattering block in the position of Method I, Eq. (7).

micrometer recorded. Second, a certain thickness of aluminum d was then transferred from P_1 to Q_1 . In order to restore the balance it was then necessary to open the slit C by a definite amount. After the balance had been restored, the micrometer was again read. These two readings were taken as proportional to the intensities I_1 and I_2 in Eqs. (7) and (8). It remained necessary to find the wave-length of the x-rays used. This was done by opening the slit C to double its value when the amount of aluminum at P_1 and P_2 was equal, (the aluminum which was transferred from P_1 to Q_1 was replaced prior to this determination) then sufficient additional aluminum was placed at P_1 to restore a balance. From this added thickness, the average wave-length of the primary beam was calculated.

The electrometer used in determining the balance had a sensitivity of about 3000 to 5000 scale divisions per volt. All wires leading

to the electrometer were insulated by means of quartz and were placed inside grounded conductors. In this way it was possible to avoid insulation leaks and electrostatic fluctuations. A further precaution was taken to avoid any steady change in conditions which might occur during a set of observations by taking readings in groups of five. For example, let I_1 denote the micrometer reading when the aluminum was at P_1 and I_2 the reading at Q_1 , then consider a group of five readings, say three of I_1 and two of I_2 , in which case the first, third and fifth were of I_1 and the second and fourth of I_2 or vice versa. The average values of I_1 and I_2 were then determined and a value for $y = I_2/I_1$ calculated. Three such groups of readings were made for each angle in the case of any given wave-length and substance. The values of y so obtained were then averaged and the ratio calculated from this

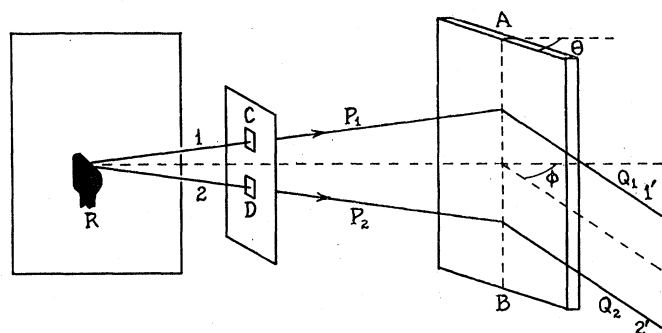


Fig. 2. Diagram of apparatus showing scattering block in the position of Method II, Eq. (8).

average value. Even with these precautions the error in some cases is probably as much as ten percent. The carbon block used was .5 cm thick and was placed in the position of Method I, Eq. (7), Fig. 1. For aluminum the position was the same but the thickness was reduced to .166 cm. In the case of copper the block was placed in the position of Method II, Eq. (8), Fig. 2, and was thick enough to absorb all the x-rays. The distance R of the block from the ionization chamber window was always 13 cm.

It should be mentioned that in the case of carbon it was necessary for the aluminum window of the ionization chamber to have a thickness of .045 cm in order to remove the characteristic rays of iron from the scattered beam. These characteristic rays were due to impurities in the carbon block. Calculations showed that a small percentage of iron as an impurity would entirely vitiate the results unless the window used was sufficiently thick to absorb all the characteristic radiation

of iron. The thickness used was known to be sufficient since it was the same as that used for copper, and previous tests had shown that this thickness was sufficient to reduce the characteristic radiation of copper to a negligible amount.

4. RESULTS AND DISCUSSION

The experimental results for carbon, aluminum and copper are shown in Tables I, II, and III.

TABLE I
Carbon

ϕ	λ	y	d	d_1	$\frac{s_2}{s_1+s_2}$	$p(\phi)$	P_U/P_M
60	.345	1.047	.332	.045	.965	.977	1.1
60	.385	1.052	.322	.045	.857	.958	3.8
60	.435	1.052	.332	.045	.676	.920	5.5
60	.58	1.022	.083	.045	.623	.877	4.3
90	.325	1.087	.332	.045	.99	1.00	...
90	.475	1.192	.332	.045	.975	.980	1.3
90	.53	1.198	.332	.045	.82	.96	5.2
110	.31	1.109	.332	.045	1.00	1.00	...
110	.41	1.209	.332	.045	1.00	1.00	...
110	.45	1.222	.322	.045	.965	1.00	...
130	.33	1.157	.332	.045	1.00	1.00	...
130	.355	1.173	.322	.045	.975	1.00	...
130	.42	1.290	.332	.045	1.00	1.00	...

In these tables ϕ is the angle of scattering, λ the wave-length in angstroms, y the ratio I_2/I_1 , d the thickness of the aluminum transferred. It is to be noted that the ratio $s_2/(s_1+s_2)$ is always less than

TABLE II
Aluminum

ϕ	λ	y	d	d_1	$\frac{s_2}{s_1+s_2}$	$p(\phi)$	P_U/P_M
90	.31	1.047	.332	.045	.67	.950	9.5
90	.36	1.065	.332	.045	.625	.939	9.0
90	.50	1.033	.083	.045	.537	.910	8.5
110	.30	1.079	.332	.045	.814	.965	6.1
110	.36	1.112	.332	.045	.783	.952	3.6
110	.50	1.051	.083	.045	.760	.920	5.0
130	.28	1.100	.332	.045	.926	.974	3.0
130	.33	1.134	.322	.045	.895	.964	3.1
130	.385	1.180	.332	.045	.865	.953	3.2

$p(\phi)$ except where each is equal to unity. This means that the probability P_U of an electron's scattering unmodified rays in a direction ϕ is always greater than the probability P_M of an electron's scattering modified rays. In the last columns of the tables are shown the ratio

of these two probabilities. There are several blanks in the last column where the value of P_U/P_M is indeterminate. It might be mentioned that Woo¹⁶ also finds $s_2/(s_1+s_2)$ to be less than $p(\phi)$ for MoK α x-rays scattered from various elements.

TABLE III
Copper

ϕ	λ	y	d	d_1	$\frac{s_2}{s_1+s_2}$	$p(\phi)$	P_U/P_M
90	.27	1.035	.332	.045	.62	.90	5.5
90	.32	1.049	.332	.045	.609	.79	5.1
90	.36	1.059	.332	.045	.58	.67	5.2
100	.27	1.050	.332	.045	.74	.923	4.2
100	.32	1.071	.332	.045	.73	.901	3.3
100	.36	1.083	.332	.045	.68	.884	3.6
100	.41	1.096	.332	.045	.63	.849	3.3
130	.27	1.093	.332	.045	.94	.948	1.2
130	.345	1.144	.332	.045	.89	.916	1.4
130	.48	1.061	.083	.045	.81	.867	1.5

Jauncey⁷ has shown that the unmodified scattering should disappear at an angle and a wave-length such that $p(\phi)$ becomes unity. This occurs at a value of ϕ given by

$$\text{vers } \phi = (3 + 2\sqrt{2}) (\lambda_0^2 / \lambda_s) (mc/h) \quad (11)$$

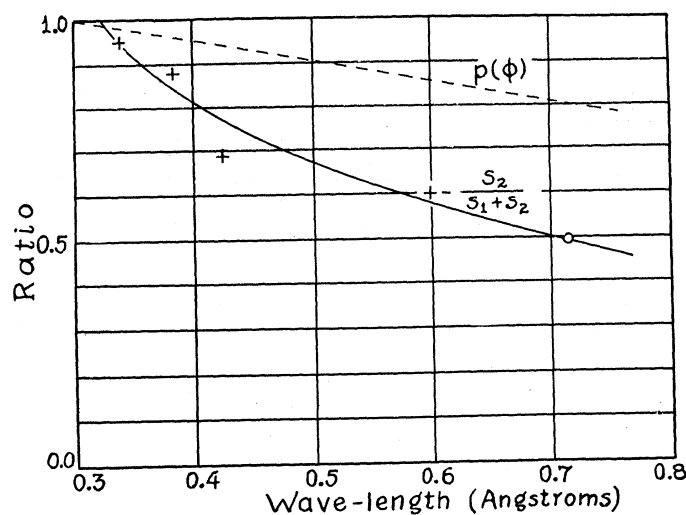


Fig. 3. Scattering from carbon at 60°.

A test of this formula has been made for scattering by carbon. The experimental values of $s_2/(s_1+s_2)$ are plotted against λ for the angles

¹⁶ Y. H. Woo, Phys. Rev. 27, 119 (1926).

60° and 90° and are shown as the full curves in Figs. 3 and 4 respectively. It is seen that $s_2/(s_1+s_2)$ becomes unity at wave-lengths of 0.32A and 0.46A for angles of 60° and 90° respectively. For carbon the K absorption limit is 47A and solving Eq. (11) at values of $\phi = 60^\circ$ and 90° we obtain $\lambda_0 = .31A$ and $.44A$ respectively. These agree very well with the experimental values. In the case of carbon at $\phi = 110^\circ$ the solution of Eq. (11) for λ_0 is 0.51A. As all the wave-lengths examined were less than 0.51A the ratio $s_2/(s_1+s_2)$ is approximately 100 percent. It will be noted that experimental values of $s_2/(s_1+s_2)$ are shown in Figs. 3, 4 and 5 for $\lambda = 0.71A$. These values are taken from results obtained by Woo¹⁶ for the scattering of MoK x-rays by

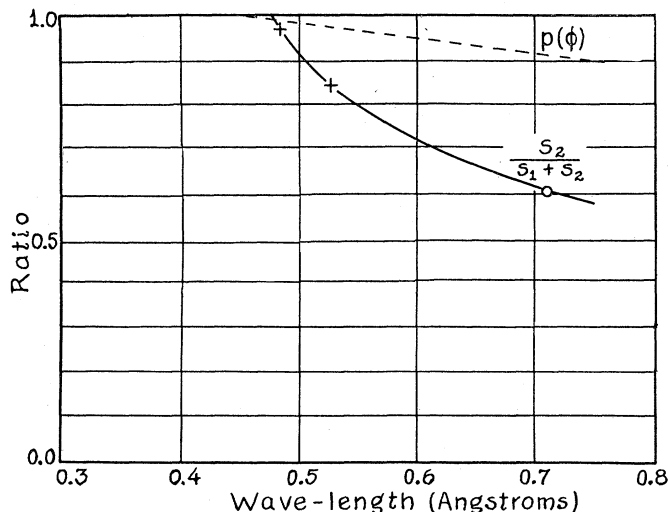


Fig. 4. Scattering from carbon at 90°.

carbon. It is seen that Woo's values fall upon the extrapolated portion of the curve as determined by the writer. The writer's values are therefore consistent with those of Woo. The broken curves in Figs. 3 and 4 are the graphs of $p(\phi)$ against λ for 60° and 90° respectively. It is seen that the curves for $s_2/(s_1+s_2)$ fall below the curves for $p(\phi)$ except where each becomes unity.

The smooth curves of Figs. 5 and 6 are the graphs of $s_2/(s_1+s_2)$ against λ for aluminum and copper at $\phi = 90^\circ$. These curves are not extrapolated to cut the 100 percent line because the amount of the extrapolation would be so large that the results would be of little value. We may however use the results for copper to test the truth of Eq. (4). The experimental value of (s_1+s_2) has been determined for copper at 90° and a wave-length of .4A. Interpolating the results

of Barkla and Dunlop¹² we find the value of (s_1+s_2) to be $1.40s_0$ where s_0 is the Thomson value at 90° . A similar determination by DeFoe and Jauncey¹⁴ gives $(s_1+s_2) = 1.24s_0$. The mean of these values

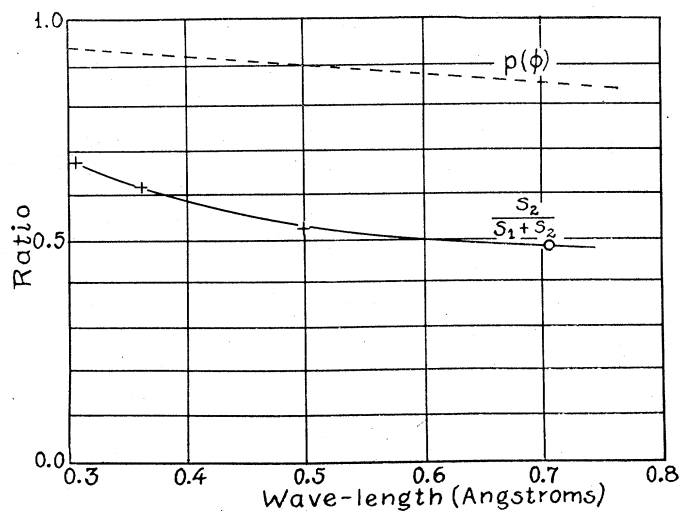


Fig. 5. Scattering from aluminum at 90° .

is $1.32s_0$. The experimental value of $s_2/(s_1+s_2)$ for $\lambda=0.4\text{A}$ at $\phi=90^\circ$ is from Fig. 6 found to be 0.54. Hence $s_2=0.54 \times 1.32s_0=0.71s_0$.

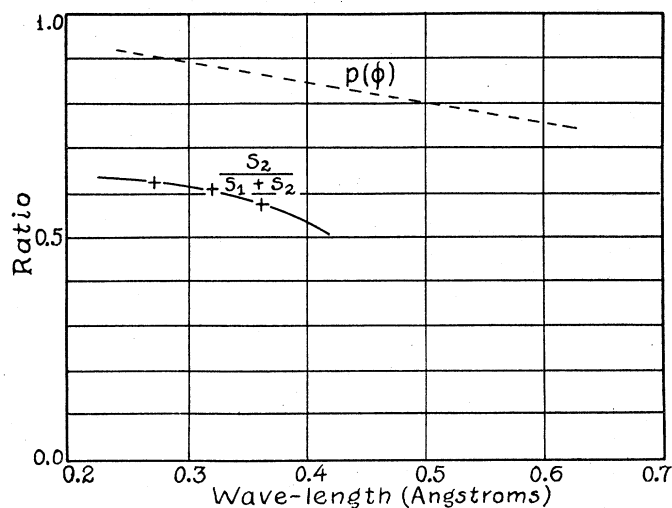


Fig. 6. Scattering from copper at 90° .

When unmodified rays are present the theoretical value of s_2 is given by Eq. (4). In the present case $p(\phi)$ has the numerical value of 0.85.

The value of the remainder of the right side of Eq. (4) is $0.87s_0$ for $\lambda = 0.4A$. Hence $s_2 = 0.74s_0$ which agrees well with the experimental value $0.71s_0$. The use of s_1 is of course the difference between $(s_1 + s_2)$ and s_2 is therefore $0.58s_0$. It is unfortunate that more values of $(s_1 + s_2)$ are not available for various scattering substances, wave-lengths and angles of scattering for then we could arrive at a relation between s_1 , λ and ϕ .

On examining the values of the ratio of P_U/P_M as given in the last columns of Tables I, II and III, we note a general tendency for the ratio for a given wave-length to become less as the angle increases. In fact in the case of copper the ratio seems to approach unity at an angle of about 130° . The results for carbon and aluminum are not quite so definite, but it must be remembered that the ratios, $s_2/(s_1 + s_2)$, as given in the sixth column are not so accurate as could be desired and it might well be that the values of P_U/P_M even for carbon and aluminum approach unity for large angles. This approach to unity implies that the unmodified scattering per electron in a direction ϕ approaches the modified scattering per electron when ϕ becomes large.

The experimental results so far as they go support Jauncey's theory of the unmodified line in that the unmodified scattering disappears for a given angle at the wave-length demanded by the theory and also in that the experimental value of the modified scattering coefficient at 90° for copper is in agreement with the theoretical value.

In conclusion, the writer wishes to express his thanks to Professor G. E. M. Jauncey who suggested this problem, for his aid and interest in the carrying out of this research.

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